Computational Model to find out Acceptance Angles with Possible Layers in Graded Index Fibre Optics

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Abstract— The fibre optics is one of the most reliable medium of communication. Optical fibre works on the simple principle of Total Internal Reflection. In this paper we had discussed a model which is capable to provide information about maximum acceptance angle (theta) & refraction angle for different layers in graded index fibre. This model is capable of finding maximum acceptance angle and is capable of find out the layers where Total Internal Refraction will occur i.e where the critical angle is achieved, when we consider the maximum acceptance angle. This model is programmed using c++ language. The program is user friendly so that any manufacturer of a graded optical fibre can easily utilized basic information about graded optical fibre.

Key words: Total Internal Reflection, maximum acceptance angle, refraction in each layer, critical angle

I. Introduction

[1]An optical fibre is a flexible, transparent fibre & has a diameter slightly thicker than that of a hair. [2]Optical fibres are used mainly to transfer data between the two ends of the fibre and find wide usage in fibre optic communication, where they permit transmission over longer distances and at higher bandwidths (data rates) than wire cables. Optical fibre works on the principal of total internal reflection. Due to this property of optical fibre, it has less Attenuation loss. Loss can be as low as 0.2 dB/km in optical fibre cables, allowing transmission over long distances without any data loss & the need for repeaters to rectify it. There are mainly two types of optical fibre,

1) step index fibre optics
2) graded index fibre optics

Graded optic fibre: -[3]In this type of optical fibre the fibre is made out of materials of varying refractive index. It is divided in to several parts, but the inner most part is called the core and the outer most part is called the cladding. The refractive index of different layers of material is in such a way that it decreases form core to cladding i.e core has the higher refractive index and cladding has the least refractive index.

The signal which enters in to the graded optic fibre follows a curve path which resembles to the path followed by the sine function because it can be said that the signal after entering the cable becomes the sine function.

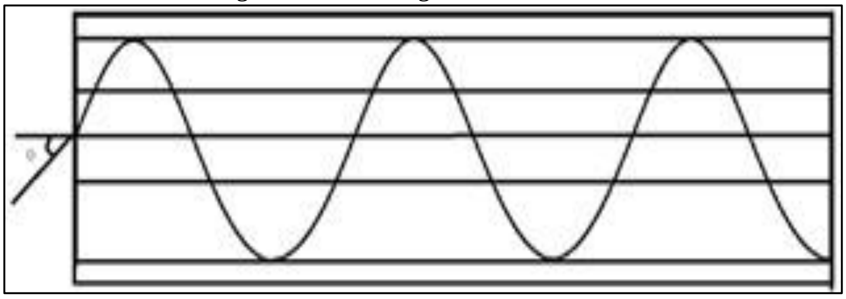

Fig. 1: Path followed by the ray in graded optic fibre

The following model is used to find the maximum acceptance angle for the signal to enter and gives the angle it makes in each layer before it undergoes Total Internal Reflection.

II. Construction of Model

Now we know that [4] optical fibre works on the principle of total internal refraction, and also uses Snell's law.

[5] Snell's law states that the ratio of the sine's of the angles of incidence and refraction is equivalent to the ratio of phase velocities in the two media, or equivalent to the reciprocal of the ratio of the indices of refraction:

$$\frac{sin(i)}{sin(r)} = \frac{v1}{v2} = \frac{\lambda 1}{\lambda 2} = \frac{n2}{n1}$$

Now,

Consider a graded index optic fibre made up of material containing decreasing refractive index from core to cladding.

Let n_0=refractive index of air ≈1

Let refractive indexes of the materials be $n_1,n_2,n_3,n_4,n_5,n_6,n_7,\ldots\ldots\ldots,n_{z-1},n_z$;

Where n=refractive index of the material
n_1=refractive index of core
n_z=refractive index of cladding
n_{z-1}=refractive index of the material where critical angle is achieved (total internal refraction is achieved)
Let the angle of refraction in each medium be $r_1,r_2,r_3,r_4,r_5,r_{6,7},\ldots\ldots\ldots,r_{z-1},r_z$ respectively.
Let ic be the be the critical angle for which total internal refraction occur

When the ray of light enters the first layer (i.e core) making an angle with the normal of the surface.
Let the angle made by the ray of light to the normal of the surface be '⊖'
Let the angle of refraction when the ray of light enters in first layer be be 'r_1'
Let ⊖ be the maximum acceptance angle
Therefore, by Snell's law

$$\frac{sin(\Theta)}{sin(r1)} = \frac{n1}{n0} \quad \ldots\ldots\ldots\ldots(1)$$

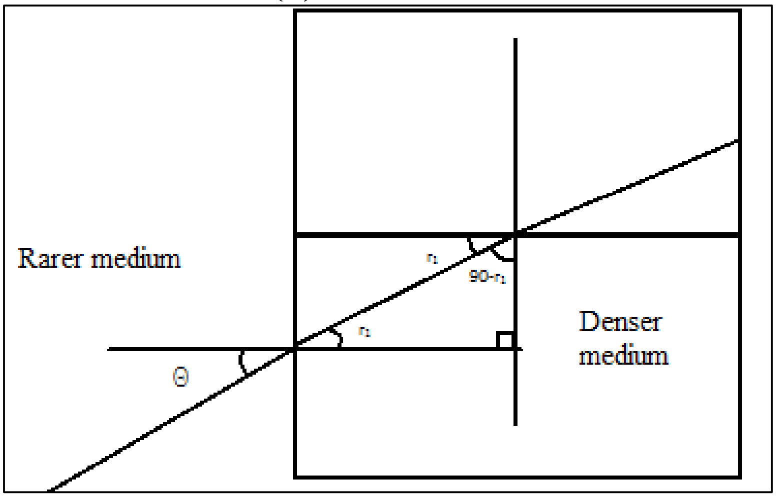

Fig. 2: Ray of light entering in first medium.

As shown above the incident angle when the ray travels from first medium to second medium is $(90\text{-}r_1)$
Therefore ,by Snell's law

$$\frac{sin(90-r1)}{sin(r2)} = \frac{n2}{n1}\ldots\ldots(2)$$

Also,

$$\frac{\sqrt{1-sin^2(r1)}}{sin(r2)} = \frac{n2}{n1},$$

Now squaring on both sides, we get,

$$\frac{1-sin^2(r1)}{sin^2(r2)} = \frac{(n2)^2}{(n1)^2},$$

Now on solving for r_2 we get the following equation;

$$sin(r2) = \frac{n1}{n2}\sqrt{1 - sin^2(r1)}\ldots\ldots\ldots\ldots(3)$$

Now from equation (1) we get,

$$sin(r1) = \frac{sin(\Theta)}{n1}\ldots\ldots\ldots\text{(since } n_0\approx 1)$$

Substitute the following equation in (3) & on simplification, we get;

$$sin(r2)=\frac{\sqrt{(n1)^2-sin^2(\Theta)}}{n2}\ldots\ldots\ldots\ldots\ldots(4)$$

Now similarly applying Snell's law between layers 2&3, we get;

$$\frac{sin(r2)}{sin(r3)} = \frac{n3}{n2},$$

On substituting equation (4) in above equation & on simplification, we get;

$$sin(r3)=\frac{\sqrt{(n1)^2-sin^2(\Theta)}}{n3}\ldots\ldots\ldots\ldots(5)$$

so we can generalize that

$$sin(rt)=\frac{\sqrt{(n1)^2-sin^2(\Theta)}}{nt};$$

where t ranges from 1 to z-1;
Now, n_{z-1}& n_z are refractive indexes of those layer between whom total internal reflection occur;
Therefore,

$$sin(ic)=\frac{nz}{n(z-1)};$$

Where $n(z-1)=n_{z-1}$
Also, we can conclude that $ic=r_{z-1}$

Therefore by applying Snell's law between layers with refractive index n_{z-1}&n_{z-2} & on simplifying, we get;

$$\sin(ic)=\frac{\sqrt{(n1)^2-\sin^2(\Theta)}}{n(z-1)};$$

Therefore by the above process we get the relation between critical angle (ic) & maximum acceptance angle (Θ).

The above process is a model on which the following program has been written to find the critical angle (ic), maximum acceptance angle & angle of refraction in each layer of ray of light which is entered in the graded index fibre.

III. PROGRAM

```
#include <iostream>

#include <vector>

#include <cmath>

#include <cstring>

#include <algorithm>


using namespace std;


float degrees(float rad)

{

   return (rad*180)/M_PI;

}


int main()

{

   bool flag =false ;

   int n;

   cout<<"Enter number of layers : ";

   cin>>n;

   float mu[n];

   cout<<"enter the value of refractive index from core to cladding"<<endl;

   for(int i=0; i<n; ++i)

   {

      cout<<"refractive index of layer#"<<i+1<<"="<<endl;

   cin>>mu[i];

   }

   vector<float> r(1);
```

```
r[0] = asin( 1/mu[0] );
for(int i=1; i<n; ++i)
{
   float temp = sqrt(pow(mu[0], 2) - 1)/mu[i];
   if(temp >= 1)
   {
      cout<<"Total Internal Reflection achieved between "<<i<<" and "<<i+1<<"\n";
      flag=true;
      break;
   }
   else
      r.push_back(asin(temp));
}
if(!flag)
{
   cout<<"there is a data loss"<<endl;

}
for(int i=0; i<r.size(); ++i)
{
   if(isnan(r[i]))
      r[i] = -1;
   else
      r[i] = degrees(r[i]);
   cout<<"refraction in Layer#"<<i+1<<"="<<r[i]<<endl;
}
vector<float>::iterator it = find(r.begin(), r.end(), -1);
```

```
    r.erase(it, r.end());

    cout<<"Enter the layer number for which maximum acceptance angle (theta) is required : ";

    int indx;

    cin>>indx;

    int layer1 = indx-1, layer2 = indx;

    float ic = asin(mu[layer2]/mu[layer1]);

    cout<<"IC is : "<<degrees(ic)<<endl;

    float theta = asin( sqrt( mu[0]*mu[0] - pow( mu[layer1]*sin(ic), 2 ) ) );

    cout<<"Theta is : "<<degrees(theta)<<endl;

    vector<float> rf(indx);

    for(int i=indx-1; i>=1; --i)

      rf[i] = asin( sqrt( mu[0]*mu[0] - sin(theta)*sin(theta) )/mu[i] );

    rf[0] = asin( sin(theta)/mu[0] );

    cout<<"Refraction in each medium:\n";

    for(int i=0; i<rf.size(); ++i)

      cout<<"Layer#"<<i+1<<" : "<<degrees(rf[i])<<endl;

    return 0;

}
```

IV. DISCUSSION

The following program gives precise information about maximum acceptance angle, critical angle for the following acceptance angle & also the angle of refraction in each layer. The following data can be useful to roughly identify the path followed by the ray of light after entering in the graded index fibre.

Firstly the manufacturer is asked to input the number of layers the graded optic fibre is to be made. Then he is asked to enter the values of refractive index of the material which is made from the graded index fibre from core to cladding. Note: refractive indexes should be inputted in the decreasing order or there will be an error shown. The reason for this error is that the value of sine function in the construction could generate a greater than 1 which is not possible and hence will show an error. Then the program starts performing an algorithm which will work by considering the maximum acceptance angle ‘90’degree and finding the maximum limit where the total internal reflection will occur in the system. The maximum acceptance angle is considered to be ‘90’ degree for the only reason that it covers all the possible angles a ray of light can be incident on an optical fibre. If the ray of light could not reach the maximum limit of the optic fibre i.e if the ray could not achieve the condition of total internal reflection due to less variation in refractive index and reaches the end of the optic fibre, there is an output observed saying “there is a data loss in the fibre”. Now to the second part of the program the user is ask to input the layer number for which he wants the value of maximum acceptance angle (ϴ), critical angle (ic) & also angle of refraction in each layer up to the layer which the user inputted. If the user inputted the layer number greater than the maximum limit of the layer then there will be an error shown. This error is shown because the ray of light which enters the optical fibre changes its path from the maximum limit, if the user input the greater value then the maximum value, since the ray of light is not able to reach that layer it shows an error.

V. CONCLUSION

The program is user friendly, easy to use & gives basic knowledge to a manufacturer for manufacturing a graded index fibre. Basic information contains the maximum value for angle of acceptance for the considered optical fibre.

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