Analysis of Numerical Methods to Solve Differential Equations

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Abstract— Mathematics Introduces all the rules and problems as formulas, and searches for a solution. A part of the mathematics that is widely used in all sciences is the differential equation. Each parts of these equations has its own method for solving, and we have generally studied the analytic methods in the calculus, and here we will introduce the numerical solutions. It is worth noting that in analytic methods cannot gives solution, for all equations this is where scientists have discovered the numerical methods that can be solved by those methods for those equations that are not solved in an analytical method.

Keywords: Economic Dispatch, Economic Load Dispatch, MFO, Chaotic Maps, Optimal Power Flow

I. INTRODUCTION

Mathematical modelling of most physical systems leads to linear/nonlinear partial differential equations (PDEs) in various fields of science. PDEs have enormous applications compared to ordinary differential equations (ODEs) such as in dynamics, electricity, heat transfer, electromagnetic theory, quantum mechanics.

Numerical analysis is a technique of doing high mathematical problems with a numerical process that is commonly used by scientists and engineers to resolve their difficulties. One of the main gains of numerical analysis.

The outcome from numerical analysis is an approximation, which can be made closely as wanted. Analysis of errors in numerical methods is an essential part of the learning of numerical analysis [7]. Therefore, calculation of error is a requirement because it is a way of computing the efficiency of methods. Numerical methods require highly exhausting and monotonous calculate, which can be prepared only by using a computer. Numerical methods for ODEs are the procedures used to make numerical approximations for the solution of ODE.Since this paper is about solution of differential equation, so there is needed to have some basic concepts [9]. Differential equation is one of the essential, important parts of mathematics, which has significant applications.

Ordinary differential equation is an equation in which unknown function depends only on one independent variable [5]. PDE: If the unknown function depends on two or more independent variables, then the differential equation is a partial differential equation. Numerical Methods It is mathematical methods that are used to approximate the solution of complicated problems, and it can solve issues that cannot be solvable in the analysis method, and it is very useful because they are suitable for the use with computers [4]

Numerical method is the rules of mathematics, which will find an approximate solution for a problem, so in this section we will introduce some Numerical method which is solved initial value problem.

Theorems giving the conditions for existence and uniqueness of solutions of highorder boundary-value problems are contained in the papers and book of Agarwal and Krishnamoorthy [13],Agarwal [14, 16] and Agarwal and Akrivis [15], and non-numerical techniques for solving such problems are contained in the papers of Baldwin [11, 12], Davis and Henderson [17], Granhoff and Bleistein [18] and Sloan and Huang [19]. In this paper we give a non-exhaustive review of the numerical methods most commonly used during the last two decades to solve high-order ordinary differential equations, including initial-value,

II. FINITE-DIFFERENCE METHODS:

For finite-difference methods (all orders), the interval $a \le x \le b$ is generally subdivided into N + 1 subintervals each of width $h = (b - a)/(N + 1)$, the solution w(x) is computed at the points $x_i = a + ih$ (i = 1, 2,...,N) and the notation w_i is used to denote the solution of an approximating difference scheme at the grid point x_i.

III. FOURTH-ORDER METHODS:

As indicated in the Introduction, the static deflection w(x) of a uniform beam of length L carrying a load r(x) per unit length is given by the fourth-order differential equation

$$EID^4 w(x) = r(x, w),\ D^4 = d^4/dx^4, \qquad (2)$$

where E is the Young's modulus of elasticity of the material of the beam and I is the moment of inertia of a cross-section of the beam. The product EI is called the flexural rigidity of the beam. Equation (2) is a particular case of the general fourth-order boundary-value problem

$$D^4 w(x) = f(x, w, w, w, w), \qquad (3)$$

with appropriate initial and boundary conditions. Many authors have dealt with the numerical solutions of fourth-order boundary-value problems. Special and general nonlinear fourth-order boundary-value problems were considered by Twizell [28, 29]. Twizell used finite-difference methods with extrapolation to solve numerically the problem D4 w(x) = f (x, w), a < x.

IV. VARIATIONAL APPROACH:

A variational approach to sixth-order boundary-value problems was proposed by He [91] as follows. Considering the sixth-order boundary-value problem D6\ w(x) = f (x, w), 0 < x < b, with reference to the decomposition method used by Wazwaz, he noted that a much simpler solution is obtained using the Ritz method, as follows: J (w) = b 0 1 2 (w) 2 + F (x, w)/ dx, where F is the potential function. Other variational methods were considered.

V. FDMS:

A widely and frequently used numerical technique for Fisher's equation is the FDM. However, the usual FDM shows shortcomings in computational accuracy. The two widely known methods used to improve such shortcomings are the application of the compact finite difference method and Richardson extrapolation technique [15].

Due to its high accuracy and stability property, the compact finite difference method attracts much attention from many scholars for finding approximate solutions of various kinds of equations [16], and it is much more accurate than the corresponding explicit scheme of the same order [17]. A high-order compact finite difference method was applied for systems of reaction-diffusion equations in [18], and the Helmholtz equation was approximated by a sixth-order compact finite difference (CFD6) method in [19]. Dennis et al. [20] proposed the fourth-order CFDS for convection-diffusion problems. Bastani and Salkuyeh [14] had combined a CFD6 scheme for second derivative in space and a third-order total variation diminishing the Runge–Kutta (TVD-RK3) scheme in time to approximate Fisher's equation.

Another way for improving the accuracy and rate of convergence of the FDM is through the application of Richardson's extrapolation (RE) provided that their error term is expressible as a polynomial or power series in h [21, 22]. Furthermore, RE does not require any knowledge of the underlying methodology except the order of accuracy, which guarantees the minimal intervention to the existing computational tools [23]. Gordin [24] applied the RE method to improve a fourth-order CFDS to sixth-order in 1D parabolic equations and Schrödinger-type equations. Compared to low-order methods, high-order methods can achieve satisfactory errors on much coarser grids and thus greatly curtail the computational cost

VI. Conclusion

In this report single step methods of IVP from ODE has been studied. Various methods such as Taylor's series method, Euler's method, Runge Kutta first, second, third, fourth order has been described. These methods are critically studied and differentiated. Comparative study of these methods has been done using different step size with use of MATLAB. Various table, graphs and bar graph depicts the more convergent methods among all off these mentioned methods. As we know that, for small step size, number of iteration are large that gives more accurate result by this, we concluded that R-K method of fourth order is the best method, to give accurate result, and Euler method gives least accurate result. R-K method of fourth order gives less error while Euler's method gives more error which is cleared in the comparative table, graph and bar graph in the report.

The spline methods have the advantage of yielding continuous approximations over each subinterval. In the case of high-order problems, however, most of the papers cited in this review used matrices with huge numbers, making the methods difficult to use. The relatively recent decomposition methods seem more elegant but, on the basis of the papers reviewed, there is no evidence showing their supremacy. Collocation methods are also known to be attractive because of their simplicity and convenience, but they may lack accuracy. Finally, some authors have shown that variational methods are very efficient in dealing with high-order problems

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