Design and Analysis of Rear Drive Axle of Go-Kart

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Abstract— Go-Kart is a light weight, single seater, low cost, four wheeler, and open cabin vehicle without any suspension and differential gear box. It is generally powered with single cylinder IC engine of maximum capacity of 135 CC. It is very simple in construction and equipped with semi floating, solid rear axle that subjected to the static and dynamic loading. This paper is mainly focus on the design of rear drive axle of the go kart and its various possible load conditions. The rear axle of go-kart is very crucial part because it is responsible for the kart motion and also bear a large fraction of total load. This study helps to design the rear solid axle of the go-kart for the safe functioning during operation and also a deep understanding about various loading scenarios.

Keywords: Go-Kart

I. Introduction

In Go-Kart competition, team of B-tech or diploma students design and manufacture their kart for participating in this championship. The mission of this championship is to encrypting students with technical as well as practical aspects of design and manufacturing a go-kart. This challenge is on global platform where the various national and international college teams are participate and show their best of the talent thought design and manufacturing on international level. Reason of this publication is to study about the various possible loading condition and its combined effect on axle during accelerating, climbing, turning and braking. So that to design our solid rear axle of go-kart. Here we prefer to use of solid shaft instead of hollow, because solid shaft has more torsional rigidity as compare to the hollow and hollow shaft is larger in diameter as compare to solid shaft for same strength, thus hollow shaft is required more bigger size of bearings and also required more space.

The following are the conditions of various loading on rear axle:-

- Bending moment due to weight shift of kart during accelerating, climbing and turning.
- Bending moment due to torque transmission through chain and sprocket.
- Bending moment due to resisting torque exerted by the caliper on disc brake.
- Twisting moment due to engine torque and braking
- Here we are using double bearings at each side of the shaft that increase the rigidity at some extent. And all the shear force and bending moment are transfer to the chassis through the bearings. Thus due to vertical load at the extreme ends of the axle (wheel hubs), the inside portion of the axle (between the bearings) doesn't deflect.

Dimensional Data	
Wheelbase	1100mm
Track Width	880mm
Ground Clearance	1.25inch

Length/Width/Height	1925.4mm/985.4mm/845.1mm
Kerb Weight	95 kg
Fuel Tank Capacity	5 litres

Table 1: Kart specification [1]

Cooling Type	Air cooled
No of Stroke	4 Stroke
No of cylinder	1
Displacement	124.7 cc
Maximum power	11.6 bhp @ 8000rpm
Maximum torque	11Nm @ 6500 rpm
Bore	52.4mm
Stroke	57.8mm
Compression ratio	9.2 : 1

Table 2: Engine Specifications [2]

Primary reduction	3.35 : 1
1st Gear	3.076 : 1
2nd Gear	1.944 : 1
3rd Gear	1.473 : 1
4th Gear	1.19 : 1
5th Gear	1.038:1
Final Reduction Ratio	3.071:1

Table 3: Gear Ratios [2]

Tires	Dimensions (in inches)
Front	10x4.5-5
Rear	11x7.1-5

Table 4: wheel size [3]

$$Number\ of\ teeth\ on\ driver\ sprocket = 14$$
$$Number\ of\ teeth\ on\ driven\ sprocket \cong 14 \times 3.071$$
$$Number\ of\ teeth\ on\ driven\ sprocket \cong 42$$

II. Bending Load of Outer End of Rear Axle Due to Road Gradient

Assuming the road gradient of 4m raise in a height of 100m in length.

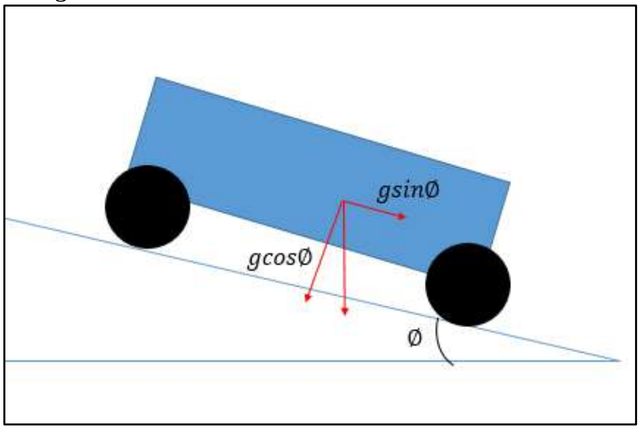

Fig. 1: Road gradient

$$\emptyset = tan^{-1}\frac{4}{100} \cong 2.3°$$

$$R1 = R + \frac{m \times H \times g \times sin\emptyset}{L}$$

$$R1 = 509.6 + \frac{160 \times 0.2 \times 9.8 \times sin\emptyset}{1.1} = 521.04N$$

III. BENDING LOAD ON OUTER END OF REAR AXLE DUE TO THE KART ACCELERATION

Here the kart acceleration is maximum at first gear. So the weight shift on rear axle is also maximum at first gear.

By considering the efficiency of the transmission system is 100%

$$T = Primary\ Reduction \times 1st\ Gear\ ration \times Final\ Reduction \times Engine\ Torque$$

$$T = 3.35 \times 3.076 \times 3.071 \times 11\ Nm$$

$$T = 348.1Nm$$

Driving force developed at the circumference of tire due to engine torque is,

$$Fr = \frac{T}{r} = \frac{348.1}{0.1397} = 2491.76N$$

By Assuming the coefficient of friction between tires and ground is 0.85, then available traction force during climbing is $2 \times R2 \times \mu = 2 \times 521.04 \times 0.85 = 885.76N$

Maximum possible acceleration, A.

$$A = \frac{2 \times R1 \times \mu}{m} = \frac{885.76}{160} = 5.53m/s^2$$

When kart is accelerating with maximum acceleration, A, then due to its inertial force that increases the loading on rear axle due to weight on rear side.

$$2 \times R2 = 2 \times R + \frac{mAH}{L}$$

$$R2 = 509.6 + \frac{160 \times 5.53 \times 0.2}{2 \times 1.1} = 590.03N$$

IV. BENDING LOAD ON OUTER END OF REAR AXLE DUE TO TURNING

When kart is turning then, it experiences a centrifugal force having the direction radially outside from the center of turn circle. And centrifugal force is equal to the centripetal force. Here the centripetal force is exerted due to the friction force between tires and ground surface. Hence the maximum possible value of centrifugal force is totally depends on the maximum available friction force between tires and ground. Weight shift on outer wheel is maximum, if centripetal force is equals to maximum static friction then.

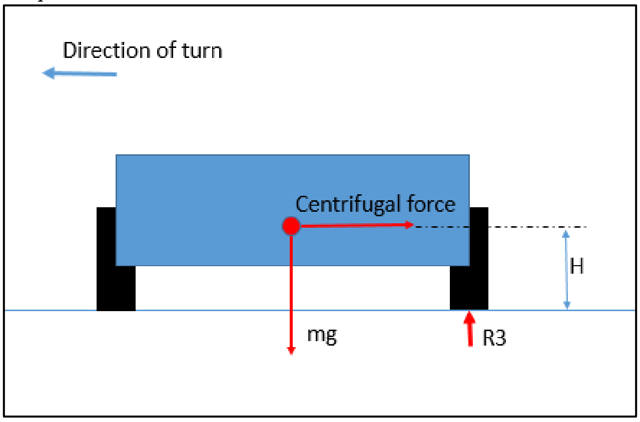

Fig. 2: Turning

$$Centripetal\ force = Maximum\ static\ friction$$

Assuming the coefficient of friction between tires and ground is 0.85, then, $Centripetal\ forcce = \mu \times m \times 9.8 = 0.85 \times 160 \times 9.8$

$$Centripetal\ forcce = Centrifugal\ force = 1332.8N$$

Taking moment about point of contact of inner wheel and ground

$P \times B = mg \times \frac{B}{2} + Centripetal\ force \times H$(Here P is the total loading on outer front and rear wheels during turning)

$$P = \frac{mg}{2} + \frac{Centripetal\ force \times H}{B}$$

$$P = \frac{160 \times 9.8}{2} + \frac{1332.8 \times 0.2}{0.88} = 1086.9N$$

$$R3 = \frac{P \times j}{L} = \frac{1086.9 \times 0.715}{1.1} = 706.5N$$

Here j is the distance of C.G of the kart from the from wheel axis

By superimposing the reaction forces R1, R2 and R3, we can get the combined resultant of maximum reaction force on rear axle. It is equivalent to that case, in which the vehicle is moving with acceleration A, climbing on road gradient of 4m raise in 100m length and simultaneously taking a turn. For design the rear axle to withstand in possible extreme scenario against bending.

$R4 = R1 + R2 + R3$

$R4 = 521.04 + 590.03 + 706.5 = 1817.57N$

V. BENDING LOAD ON REAR AXLE DUE TO TRACTION FORCE TO PROPEL THE VEHICLE

Driven force is developed at the circumference of the tire due to engine torque, in case of pure rolling, there is always equal and opposite friction force is acting at the point of contact between tire and ground to propel the vehicle. The effect of this driving friction force, causes the bending of rear axle shaft in a plane of parallel to the ground.

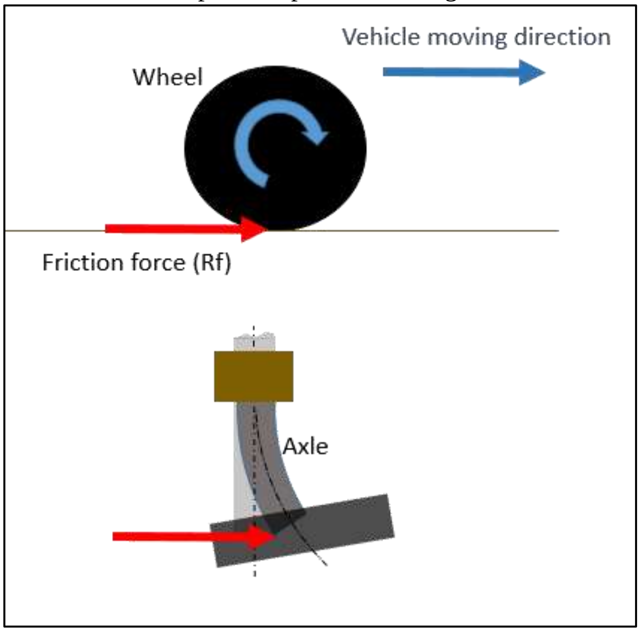

Fig. 3: Propulsion

While considering the maximum reaction force, R4 getting higher value of the available traction as compare to the maximum circumferential force, Fr develop due to engine torque.

$$Rf = \mu \times R4 = 0.85 \times 1817.57 = 1545N$$

Thus Fr load is to be considered for maximum bending load parallel to the ground on the axle shaft.

$$Fr = 1248.1N.$$

VI. RESULTANT OF REACTION FORCES

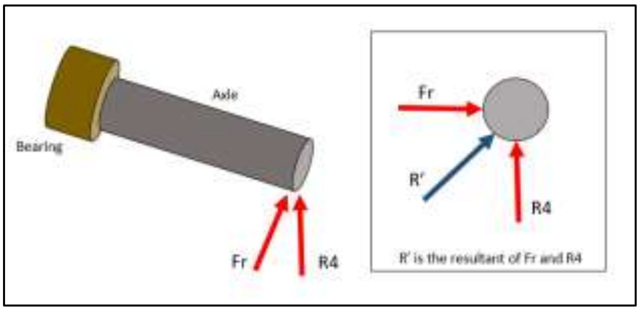

Fig. 4: Resultant Reaction R'

$$R' = \sqrt{Fr^2 + R4^2} = \sqrt{1248.1^2 + 1817.57^2} = 2204.83N$$

VII. BENDING STRESS ANALYSIS ($\sigma 1$) DUE TO THE RESULTANT LOAD (R') AT WHEEL HUB CENTER

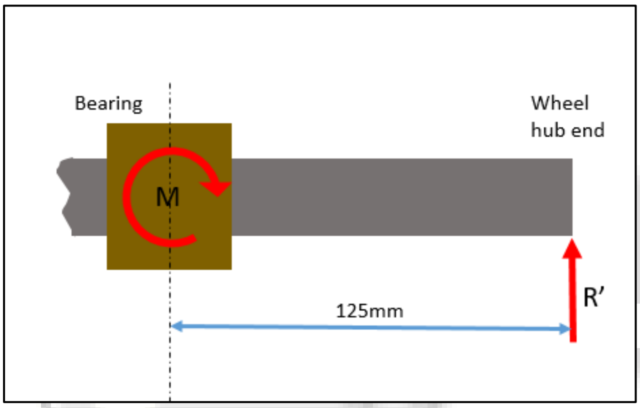

Fig. 5: Bending moment M1

$$M1 = R' \times 0.125 = 2204.83 \times 0.125 = 275.6Nm$$

$$\sigma 1 = \frac{M1}{Z} = \frac{275.6}{\frac{\pi}{32} \times D^3} N/m^2$$

VIII. MAXIMUM TWISTING TORQUE DURING ACCELERATING AND BRAKING

In this case, force at the circumference of tire due to the engine torque is less than the available traction force on rear wheel during the maximum possible weight shift. Thus the possible max twisting moment that axle may have subjected is equal to engine torque. $Te = T = 348.1Nm$.

$$\tau e = \frac{Te}{Zp} = \frac{Te}{\frac{\pi}{16} \times D^3} = \frac{348.1}{\frac{\pi}{16} \times D^3} N/m^2$$

Maximum possible twisting moment during braking by considering 100% effective braking is due to maximum available traction, Rf on rear wheel. During braking some weight of the kart is shift towards the front end. Hence during this the reaction force at the rear end is always smaller than the reaction force R, at stationary condition. For calculating the twisting moment Tb during braking reaction force R is considered for conservative approach.

$$Tb = R \cdot \mu \cdot r = 509.6 \times 0.85 \times 0.3095 = 134.06Nm$$

$$\tau b = \frac{Tb}{Zp} = \frac{134.06}{\frac{\pi}{16} \times D^3} N/m^2$$

IX. BENDING ANALYSIS OF THE INSIDE PORTION OF THE AXLE SHAFT BETWEEN BEARINGS DUE TO ENGINE TORQUE

For go-kart chain and sprocket mechanism is very commonly used to transmit the power from engine to the rear axle. Bending load due to engine torque on axle can be calculated by finding the resultant of all the forces exerted by chain on each engaged teeth of the sprocket of axle. And the magnitude of this resultant force is maximum when the angle of contact between chain and sprocket is 180+2α°, here α is the pressure angle of chain and sprocket.

Assuming 17° pressure angle for calculation of resultant force P1 on axle.

Here 25 teeth are in contact with the chain, then the tangential force at each teeth of the sprocket during transmitting the maximum torque Te at first gear is

$$\frac{Te}{25 \times radius\ of\ sprocket} = \frac{348.1}{25 \times 0.0645} = 215.87N.$$

And the resultant of these forces is calculated graphically by using AutoCAD.

Resultant bending load P1 is 2760N.

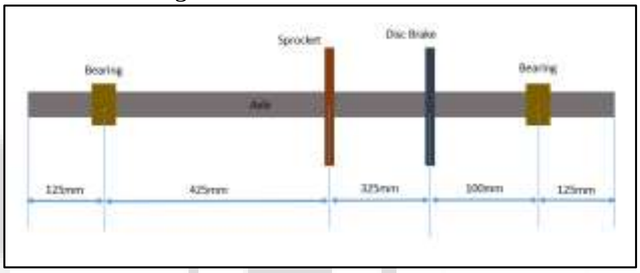

Fig. 6: Position of Sprocket, disc and Bearing center

Position of the sprocket is at the center of the rear axle.

$$M2 = \frac{2760}{2} \times 0.55 = 759Nm$$

$$\sigma 2 = \frac{M2}{Z} = \frac{759}{\frac{\pi}{32} \times D^3} N/m^2$$

X. BENDING LOAD ON REAR AXLE SHAFT DURING BREAKING

The more appropriate size of disc brake is of 200mm in diameter because it provides enough clear distance of the brake disc from the ground. So that the rear disc of Apache RTR 180 is selecting for this go kart that is 200mm in diameter. And the effective radius of the disc is 90mm. For designing the rear axle during effective barking, by considering that the brake caliper exert the force on disc that develop resisting torque equal to Tb.

$$P2 = \frac{Tb}{0.09} = \frac{134.06}{0.09} = 1489.55N$$

I have fixed the position of disc at the distance of 750mm from left rear wheel bearing.

$$Reaction\ force\ at\ the\ right\ bearing, (Rb') = \frac{P2 \times 0.75}{0.85}$$

$$= \frac{1489.55 \times 0.75}{0.85} = 1314.30N$$

$$M3 = Rb' \times (0.85 - 0.75) = 1314.30 \times 0.1 = 131.43Nm$$

$$\sigma 3 = \frac{M3}{Z} = \frac{M3}{\frac{\pi}{32} \times D^3} = \frac{131.43}{\frac{\pi}{32} \times D^3} N/m^2$$

XI. MATERIAL SELECTION

AISI 1020 is selected for rear axle of kart due to its availability and cheap as compare to AISI 4130. Lower yield strength and machinability is compromised with their cost.

Material	Tensile strength	Yield strength	Machinability
AISI 1020	420 MPa	350 MPa	65%

Table 5: Material Properties [6]

XII. COMBINED STRESS ANALYSIS ON THE REAR AXLE FOR VARIOUS FOLLOWING CASES

A. Case 1:- Analysis of the Outer Portion of Axle, i.e from Bearing to Hub Center.

When kart is accelerating with acceleration A, then the rear axle shaft may subjected to twisting moment due to engine torque and simultaneously to bending load due to weight shift during accelerating, climbing and turning. Considering a small element at the surface of the shaft where tensile stress due to bending and shear stress due to twisting moment are maximum.

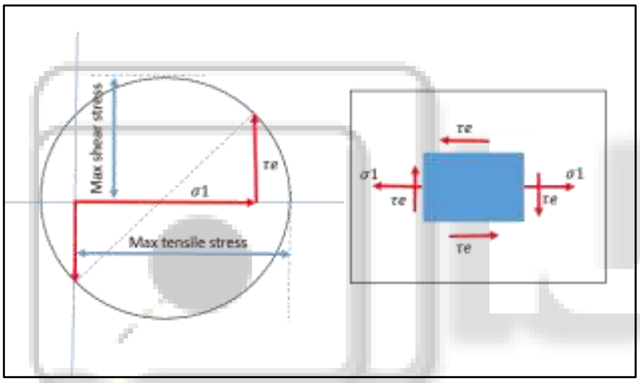

Fig. 7: Mohr's Circle

$$\sigma max = \frac{\sigma 1}{2} + \sqrt{\left(\frac{\sigma 1}{2}\right)^2 + \tau e^2} = \frac{3664.85}{D^3} N/m^2$$

$$\sigma min = \frac{\sigma 1}{2} - \sqrt{\left(\frac{\sigma 1}{2}\right)^2 + \tau e^2} = -\frac{857.61}{D^3} N/m^2$$

By using the shear strain energy theory

Taking factor of safety is 1.5 over yield strength 350(MPa).

Getting diameter of shaft is 26.12(mm)

B. Case 2:- Analysis of the Inner Part of Axle, I.E between the Bearings during Torque Transmission.

This possible cases of loading, during maximum torque transmission through chain and sprocket.

Considering a small element at the surface of the shaft where bending stress due to the resultant of forces act on each teeth of the sprocket by chain and shear stress due to twisting moment, Te are maximum.

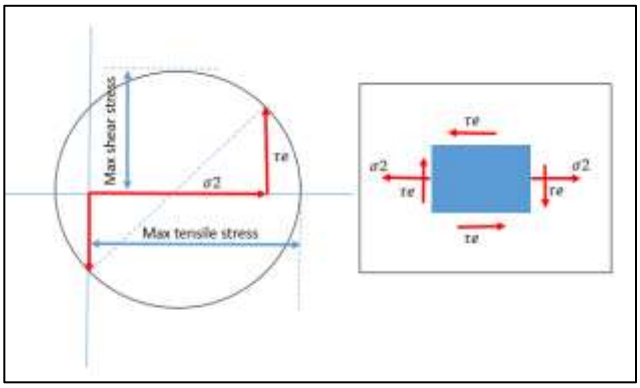

Fig. 8: Mohr's Circle

$$\sigma max = \frac{\sigma 2}{2} + \sqrt{\left(\frac{\sigma 2}{2}\right)^2 + \tau e^2} = \frac{8118.27}{D^3} N/m^2$$

$$\sigma min = \frac{\sigma 2}{2} - \sqrt{\left(\frac{\sigma 2}{2}\right)^2 + \tau e^2} = -\frac{387.15}{D^3} N/m^2$$

By using the shear strain energy theory

Taking factor of safety is 1.5 over yield strength 350(MPa).

Getting diameter of shaft is 32.91(mm)

C. Case 3:- Analysis of the Inner Part of Axle, I.E between the Bearings during Braking.

When brakes will apply, the shaft will experience a bending load, P1 and a twisting moment, Tb. Considering a small element at surface of the shaft where both bending stress and shear stress are maximum.

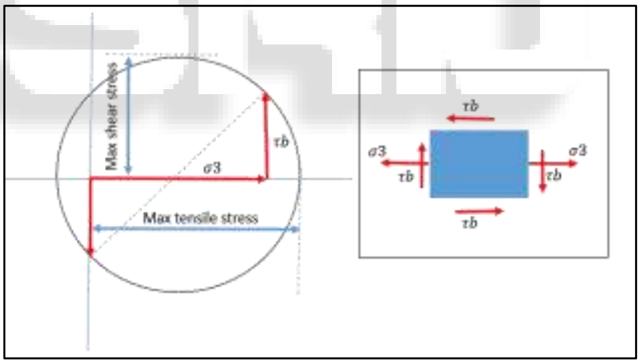

Fig. 9: Mohr's Circle

$$\sigma max = \frac{\sigma 3}{2} + \sqrt{\left(\frac{\sigma 3}{2}\right)^2 + \tau b^2} = \frac{1625.50}{D^3} N/m^2$$

$$\sigma min = \frac{\sigma 3}{2} - \sqrt{\left(\frac{\sigma 3}{2}\right)^2 + \tau b^2} = -\frac{286.78}{D^3} N/m^2$$

By using the shear strain energy theory

Taking factor of safety is 1.5 over yield strength 350(MPa).

Getting diameter of shaft is 19.70(mm)

D. Case 4:- Analysis of the Rear Axle for the Direct Shear Stress Caused by Various Lateral Loading on the Axle.

$$Direct\ shear\ stress = \frac{max(R', P1, P2)}{\frac{\pi}{4} \times D^2}$$

Shear yield strength = 0.577*tensile yield strength

But the case of direct shear is very less significant as compare to other cases.

XIII. CONCLUSION

I studied various possible load conditions, as the rear axle might be subjected during operation. And calculated the appropriate diameter of the axle shaft by superimposing the possible load conditions in different scenario and selecting the maximum value of diameter 32.91(mm) say 33(mm) of the axle for safe function of the shaft in all cases during operation.

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I hope this research work helps to all those student who are going to participate in Go kart Championship.

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ABBREVIATION

T	Maximum torque transmitted through axle, Nm
r	Radius of tyre, (m)
Fr	Force developed at circumference of wheel due to engine torque, (N)
μ	Coefficient of friction
m	Mass of kart with operator, (kg)
g	Gravity (9.8m/s^2)
H	Height of the C.G of vehicle, (0.2m)
j	Horizontal distance between front axle and C.G of vehicle. (0.715m)
A	Max acceleration at 1st Gear
L	Wheel base, (m)
B	Track width, (m)
R	Normal reaction force at each wheel of rear at static condition, (N)
R1	Normal reaction force on rear wheel due to road gradient, (N)
R2	Normal reaction force on rear wheel due to kart acceleration A, (N)
R3	Normal reaction force on rear wheel during turning of kart, N
R4	Resultant of all reaction forces (R1, R2, R3), N
Rf	Traction force
R'	Resultant of all reaction forces R1, R2, R3, Rf,, (N)
Te	Maximum twisting moment due to engine torque, (Nm)
Tb	Maximum twisting moment due to available traction force during braking, (Nm)
P1	Bending load on axle due to engine torque, (N)
P2	Bending load on axle during applying brake, (N)
M1	Bending moment due to load R', (Nm)
M2	Bending moment due to engine torque, (Nm)
M3	Bending moment due to applying brakes, (Nm)
τe	Maximum shear stress due to twisting moment Te, $(^{N}/_{m^2})$
τb	Maximum shear stress due to twisting moment Tb,($^{N}/_{m^2})$
E	Young's modulus of Elasticity of material
D	Outer diameter of axle, (m)
I	Second moment of area,(m^4)
Ip	Polar moment of inertia, (m^4)
Z	Section modulus, (m^3)
Zp	Polar modulus, (m^3)
$\sigma 1$	Maximum bending tensile stress at extended portion of the axle
$\sigma 2$	Maximum bending tensile stress due to engine torque
$\sigma 3$	Maximum bending tensile stress due to applying brakes