Analysis of Plates for Building Construction with Different Shapes by Classical Plate Theory and Navier's Theory

Rajesh Kumar Jha[1] Prof. Thakur Ramji Ram[2]
[2]Professor & Head of Department
[1,2]Department of Civil Engineering
[1,2]NRI Institute of Research & Technology, Bhopal, India

Abstract— The analysis of isotropic rectangular and square plates using Classical Plate Theory and Navier's Theory. In case of rectangular plates variation is done for length width ratio and thickness. And in case of square plate variation is done for thickness. The symmetric bending of Rectangular and Square plate is considered in the present study. Plates are analyzed for uniformly distributed load and concentrated point load in the present study. In this research work, the effect of varying thickness of the plate on its deflection and bending stress is studied. So, the key point of research work is thickness variation of plate. Once deflection is obtained by using Classical Plate Theory, bending moments and bending stresses are easily calculated by usual relations. This study discusses the elastic equilibrium problems of rectangular and square thin plates of varying thickness and simply supported on all four sides by linear and nonlinear theory. Using the Classical and Navier's method to seek an approach to the problem, and illustrates the solution with two examples. In contusion, mention is made of scope of application and the convergences of the solution. In this method, the deflection was split into x and y components of deflection. It was assumed that the deflection of the rectangular plate is the product of these two components. With this assumption, total potential energy functional was derived from principles of theory of elasticity.

Keywords: Plate Theories, High Order Theory, Laminate, Composite, FGM, laminated composites, shear deformation plate theory, bending, vibration, buckling

I. INTRODUCTION

Plates are extensively used in many engineering applications like roof and floor of building, deck slab of bridge, foundation of footing, water tanks, turbine disks etc. Plates used in such applications are normally subjected to lateral loads, causing bending of the plate. Bending of plates or plate bending refers to the deflection of a plate perpendicular to the plane of plate under the action of external forces and moments. Hence bending analysis of plate is all most importance. The geometry of the plate normally defined by middle plane which is plane equidistance from the top and bottom faces of the plate. The flexural properties of plate largely depend on its thickness rather than its two dimensions (length and width). The amount of deflection can be determined by solving the differential equations of an appropriate plate theory. The stresses in the plate can be calculated from these deflections. Once the stresses are known, failure theories can be used to determine whether a plate will fail under a given load. Plates are usually subdivided based on their structural action as:

1) Stiff Plates, which are thin plates with flexural rigidity and carry the loads two dimensionally. In engineering practice, a plate is understood as a stiff plate unless specified.
2) Membranes, which are thin plates without flexural rigidity and carry the lateral loads by axial shear forces. This load carrying action is approximated by a network of stressed cables since their moment resistance is of a negligible order of magnitude.
3) Flexible Plates, which represent a combination of stiff plates and membranes. They carry external loads by the combined action of internal moments and transverse shear forces.
4) Thick Plates, whose internal stress condition resembles that of three dimensional structures. A plate is a planer structure with a very small thickness in comparison to the planer dimensions. A plate theory takes advantage of this disparity in length scale to reduce the full three-dimensional solid mechanics problem to a two dimensional problem.

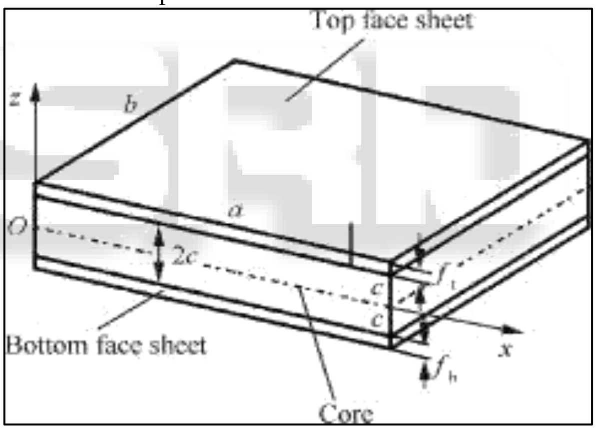


Fig. 1: Shapes of Classical Plate

II. ASSUMPTIONS IN CLASSICAL PLATE THEORY

The following fundamental assumptions are made in the classical small deflection theory of thin homogenous elastic plates.

1) Straight line initially normal to the middle surface to the plate remains straight and normal to the deformed middle surface of the plate and unchanged in length.
2) Displacement w is assumed to be very small. This means the slope of the deflected surface is small and hence square of the slope would be negligible in comparison with unity.
3) The normal stresses σ_x and σ_y in plane shear stress τ_{xy} are assumed to be zero at middle surface of the plate. i.e. ($w \gg h$)
4) The mid plane remains unstrained after bending.

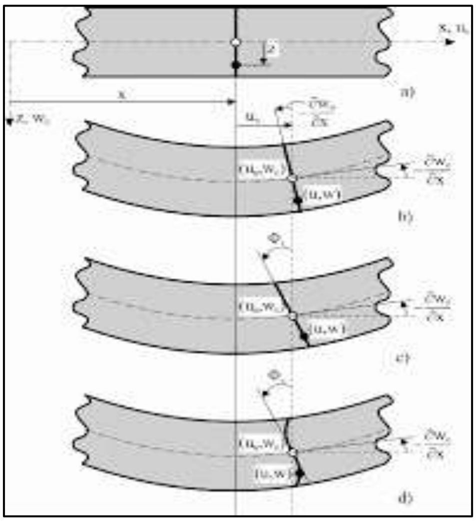


Fig. 2: Classical Plate Theory

The use of Fly-Ash silica in concrete mix has shown results of increase in the compressive, tensile and flexural strength of concrete. It sets early and hence generally requires admixtures during mix design. Fly-Ash-silica mixed cement can generate Fly-Ash-crystals of C-S-H gel after hydration. These Fly-Ash-crystals accommodate in the micro pores of the cement concrete, hence improving the permeability and strength of concrete.

III. INDENTATIONS & EQUATIONS

the coordinate axes as Fig. 3.1 We assume the load distributed over the surface of the plate is given by the expression

$$q = q_o \sin\frac{\pi x}{a} . \sin\frac{\pi y}{b}$$

in which q_o represents the intensity of the load of the plate.

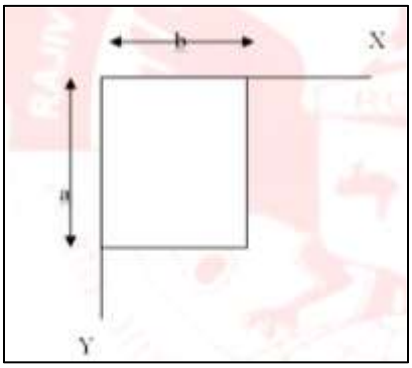
The differential equation for the deflection surface in this surface in this case becomes

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial w}{\partial x^2\,\partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{q_o}{D}\sin\frac{\pi x}{a} . \sin\frac{\pi y}{b}$$

The boundary conditions for simply supported edges are

$w = 0 \quad M_x = 0$ for x =0 and x = a

$w = 0 \quad M_y = 0$ for y =0 and y = b



Using expression for bending moment and observing that, sense w= 0 at the edges $\frac{\partial^2 \omega}{\partial x^2} = 0$ for the edges parallel to the x and y axes, respectively; we can represent the boundary conditions in following form:

1) $w = 0$
2) $\frac{\partial^2 w}{\partial x^2} = 0$ for x =0 and x = a
3) $w = 0$
4) $\frac{\partial^2 w}{\partial y^2} = 0$ for y =0 and y = b

IV. NAVIER'S SOLUTION FOR SIMPLY SUPPORTED PLATES.

The solution of the preceding article can be used in calculating deflections produced in a simply supported rectangular plate by any kind of loading given by the equation.

$$q = f(x,y)$$

For this purpose we represent the function f(x,y) in the form of a double trigonometric series:

$$f(x,y) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} a_{mn} \sin\frac{m\pi x}{a} . \sin\frac{n\pi y}{b}$$

To calculate any particular coefficient $a_{m'n'}$ of this series we multiply both sides sin (n'lry/b) dyand integrate from 0 to b. observing that:

$$\int_0^b \sin\frac{n\pi y}{b} . \sin\frac{n'\pi y}{b} dy = 0 \text{when } n \neq n'$$

$$\int_0^b \sin\frac{n\pi y}{b} . \sin\frac{n'\pi y}{b} dy = \frac{b}{2} \text{when } n = n$$

V. CLASSICAL PLATE THEORY (CPT)

Classical solution for simply supported rectangular plat subjected to UDL with thickness is done as follows:-
Deflection of a simply supported rectangular plate is given as follows

$$\text{Deflection } (w)\text{max} = \frac{q_0}{\pi^4 D(\frac{1}{a^2}+\frac{1}{b^2})^2}$$

Flexural rigidity is given by following equation

$$D = \frac{Eh^3}{12\,(1-\mu^2)}$$

VI. NAVIER'S THEORY (NT)

Navier's solution for simply supported rectangular plat subjected to UDL with thickness is done as follows:-
Deflection of a simply supported rectangular is given as follows:

$$\text{Deflection}(W)_{max} = \frac{16\,q_0}{\pi^6 D}\left[\frac{1}{\left(\frac{1}{a^2}+\frac{1}{b^2}\right)^2} + \frac{1}{\left(\frac{9}{a^2}+\frac{9}{b^2}\right)^2} + \frac{1}{\left(\frac{25}{a^2}+\frac{25}{b^2}\right)^2}\right]$$

Flexural rigidity is given by following equation

$$D = \frac{Eh^3}{12\,(1-\mu^2)}$$

VII. NUMERICAL RESULT

Flexural parameters like deflections and bending stresses for various cases of rectangular and square plate are obtained from Classical method and Navier'solution are compared as follows:

Comparison of bending moment for simply supported Square plate subjected to UDL obtained from classical method and Navier's theory

a/b a=30to300mm b=300mm		Max.Moments (N-m)	
		Classical	Navier's
0.1	M_x	26.8981	50.1949
	M_y	8.3134	15.5138
0.2	M_x	102.385	191.062
	M_y	34.3982	64.190
0.3	M_x	212.8256	397.156
	M_y	80.8198	150.818

0.4	M_x	304.901	636.160
	M_y	149.632	279.230
0.5	M_x	470.533	878.071
	M_y	340.739	449.245
0.6	M_x	589.968	1100.945
	M_y	351.424	655.798
0.7	M_x	692.549	1292.37
	M_y	476.995	917.5346
0.8	M_x	775.9478	1448.00
	M_y	611.905	1141.88
0.9	M_x	840.742	1568.91
	M_y	750.783	1401.04
1	M_x	478.78	1441.3439
	M_y	478.78	1441.3439

VIII. Conclusion

the parametric study of deflection analysis of simply supported Rectangular and Square plate by using classical plate theory and Navier's theory following conclusions are shown:-

1) Within creasing in thickness deflection of thin Rectangular and Square plate are decreases.
2) The results obtained from Rectangular and Square plates for deflection (Both for simply supported and UDL) shows more variation in deflection because of the thickness of plate.
3) The results obtained from classical method and Navier's for deflection for 10mm and 20 mm thickness of plate (Both for simply supported and UDL)shows greater variation because of the thickness of the plate i.e. plate become more thick for 10mm and 20mm thickness
4) Results for deflection and bending stress obtained from Navier's theory method does not gives good agreement with the results obtained from classical plate theory for both simply supported rectangular plate

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