Application of I-Function in Boundary Value Problems

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Abstract— In the paper first we evaluate an integral involving I-function of one variable and then we make its application to solve two boundary value problem on (1)heat conduction in bar (2)deflection of vibrating string under certain conditions.

Key words: Hacksaw Machine, Motor, Blades

I. Introduction

The I-function of one variable is defined by Saxena [3,p.366-375] and we will present here in the following manner

$$I^{m,n}_{p_iq_i}[x|^{[(a_j\alpha_j)_{1n}],[(a_{j1}\alpha_{j1})_{n+1}p_i]}_{[(b_j\beta_j)_{1m}],[(b_{j1}\beta_{j1})_{n+1}q_i]}]=1/2\pi\omega\int_L\theta(s)x^2ds \quad (1.1)$$

Where $\omega=\sqrt{-1}$

$$\theta(s)=\frac{\prod_{j=1}^{m}\Gamma(b_j-\beta_j s)\prod_{j=1}^{n}\Gamma(1-a_j-\alpha_j s)}{\sum_{i=1}^{r}(\prod_{j=m+1}^{qi}\Gamma(1-b_{ji}-\beta_{ji} s)\prod_{j=n+1}^{pi}\Gamma(a_{ji}-\alpha_{ji} s))}$$

Integral is convergent, when (b > 0 ,A ≤0) ,where

$$B=\sum_{j=1}^{n}a_j-\sum_{j=n+1}^{pi}a_{ji}+\sum_{j=1}^{m}\beta_j-\sum_{j=m+1}^{qi}\beta_{ji}, \quad (1.2)$$

$$A=\sum_{j=n}^{pi}a_{ji}-\sum_{j=1}^{qi}\beta_{ji} \quad (1.3)$$

$$|argX| < ½ B \pi , \forall i\epsilon (1,2,3….r). \quad (1.4)$$

In this paper, we shall make application of following modified

From of the integral [1,p.372,(1)]:

$$\int_0^L(\sin\pi x/L)^{\omega-1}\sin\ n\pi x/L\,dx=\frac{L\sin 1/2n\pi\Gamma(\omega)}{2^{\omega-1}\Gamma\{\frac{1}{2}(\omega+n+1)\Gamma\{\frac{1}{2}(\omega-n+1)\}} \quad (1.5)$$

Where n is any integral.

II. Integral

The integral involving the I-function of one variable to be established is

$$\int_0^L(\sin\pi x/L)^{\omega-1}\sin\ n\pi x/L = I^{m.l}_{p_iq_{i:r}}[z(\sin n\pi/L)^{\lambda}|^{...,\ ......}_{.......,.....}]dx$$

$$=2^{\omega-1}\sin\frac{n\pi}{2}I^{m,l+1}_{p_{i+1}q_{i+2:r}}[z2^{-\lambda}|^{(1-\omega,\lambda)..............,..............}_{..............,........(\frac{1}{2}-\frac{\omega}{2}\pm n/2,\lambda/2}]dx \quad (2.1)$$

Where

$B=\sum_{j=1}^{n}a_j-\sum_{j=n+1}^{pi}a_{ji}+\sum_{j=1}^{m}\beta_j-\sum_{j=m+1}^{qi}\beta_{ji},\equiv B>0,$

$|argX| < ½ B \pi , \forall i\epsilon (1,2,3….r).$

A. Proof of (2.1)

Replace the I-Function by its equivalent contour integral As given in (1.1), change the order of integration evaluate the nnerntegral with the help of (1.5)and finally interpret it with (1.1), to get(2.1)

III. Application to Heat Conduction in a Bar

In the section, we consider a problem on heat conduction in a bar under certain Boundary condition If the sides of the bar are insulated and the loss of heat From the side by condition or radiation negligible, then the temperature U(x,t)in a uniform bar $0 \le x \le L$ satisfy the heat equation.

$$\frac{\partial u}{\partial t}=c(\frac{\partial^2 u}{\partial xt^2}),\ t\ge 0 \quad (3.1)$$

If the take following boundary conditions

$$U(0,t),(L,t)=0 \quad (3.2)$$

And the initial condition

$$U(x.0)=f(x) \quad (3.3)$$

Then the solution of partial direction equation(3.1) given by [2,p.143,((9)]:

$$U(x,t)=\sum_{n=1}^{\infty}B_n\sin nL\pi x\ \exp[-t(nc\pi)^2]/L \quad (3.4)$$

$$B_n=(2/L)\int_0^L f(x)\sin nL\pi x\,dx \quad (3.5)$$

Now we shall onside the problem of determine(x,t), where

$$U(x,o)=f(x)=(\sin n\pi/L)^{\omega-1}$$

$$I^{m,n}_{p_iq_{i:r}}[z(\sin\frac{\pi x}{L})^{\lambda}|^{..................,..................}_{...............,...................}] \quad (3.6)$$

IV. Solution to the Problem

Combining (3.5) and (3.6) and making the use of integral (2.1) we drive

$$B_n=2^{2-\omega}\sin n\pi I^{m,l+1}_{p_{i+1}q_{i+2:r}}[z2^{-\lambda}|^{(1-\omega,\lambda)..............,..............}_{..............,........(\frac{1}{2}-\frac{\omega}{2}\pm n/2,\lambda/2}] \quad (4.1)$$

Putting the value of B_n from (4.1) in (3.4) we get following rewuired solution Of the problem u(x,t) = $2^{2-\omega}\sum_{n=1}^{\infty}\sin nL\ \exp[-t(nc\pi)^2]/L\ \sin½n\pi$

$$X\ I^{m,l+1}_{p_{i+1}q_{i+1:r}}[z2^{-\lambda}|^{(1-\omega,\lambda)..............,..............}_{..............,........(\frac{1}{2}-\frac{\omega}{2}\pm n/2,\lambda/2}] \quad (4.2)$$

V. Application to Homogenous Wave Problem

In this sector, we shall determine the deflectionu (x,t) of the vibrating String, if the weight of string due to tension is the negligible then deflection u(x,t) Satisfies the partial deferential equation

$$\frac{\partial^2 u}{\partial t^2}=c^2(\frac{\partial^2 u}{\partial t^2}),\ 0< x < L \quad (5.1)$$

Now we assume the boundary condition

$$U(0,t),u(L,t)=0, \quad (5.2)$$

And initial conditions

$$\frac{\partial u}{\partial t}(x,0)=g(x),(\text{initial velocity}) \quad (5.3)$$

And (x,0)=f(x)

Then the solution of partial equation (5.1) is given by [2,p.136-(5)]:

$$\text{And } (x,0)=f(x) \quad (5.4)$$

Then the solution of partial equation (5.1) is given by [2,p.136-(5)]:

$$U(x,t)=\sum_{n=0}^{\infty}B_n[\cos n\pi ct/L+C_n\sin n\pi ct/L] \quad (5.5)$$

Where Bn is given by (3.5) and

$$c_n=(2/n\pi c)\int_0^L g(X)\sin n\pi x/L \quad (5.6)$$

Now we consider the problem f the determining U(x,t) where U(x,0)f(x) is given by (3.6),while

$$g(x)=(\sin\pi x/L)^{\omega-1}I^{M,N}_{P_iQ_{i:R}}[z(\frac{\sin n\pi}{L})^{\eta}|^{(1-\omega,\lambda)...............,..................}_{...............,...............................}] \quad (5.7)$$

after the combining (5.6)and(5.7) and making the use of integral (2.1), we arrive at

$$C_n = 2^{2-\acute{\omega}} (L/n\pi c) \sin \tfrac{1}{2} n\pi \, I^{M,N}_{P_{i+1} Q_{i+1:R}} \left[z \, 2^{-\acute{\lambda}} \Big|^{(1-\acute{\omega},\eta) \ldots\ldots,\ldots\ldots}_{\ldots\ldots,\ldots\ldots (\frac{1}{2}-\frac{\acute{\omega}}{2} \pm \frac{n}{2}, \frac{\eta}{2})} \right] \quad (5.8)$$

Putting the value of B_n and c_n in (5.5) to ger required solution of the problem in the following form

$$U(x,t) = 2^{2-\omega} \sum_{n=1}^{\infty} \sin n\pi ct/L \cos n\pi ct/L \sin \tfrac{1}{2} n\pi$$

$$X \, I^{m,l+1}_{P_{i+1} Q_{i+2:r}} \left[z2^{-\acute{\lambda}} \Big|^{(1-\omega,\lambda) \ldots\ldots,\ldots\ldots}_{\ldots\ldots,\ldots\ldots (\frac{1}{2}-\frac{\acute{\omega}}{2} \pm \frac{n}{2}, \frac{\lambda}{2})} \right]$$

$$U(x,t) = 2^{2-\omega} \sum_{n=1}^{\infty} \left(\frac{L}{n\pi c}\right) \sin n\pi x/L \sin n\pi ct/L \sin \tfrac{1}{2} n\pi$$

$$X \, I^{M,N+1}_{P_{i+1} Q_{i+2:R}} \left[z2^{-\acute{\lambda}} \Big|^{(1-\acute{\omega},\eta) \ldots\ldots,\ldots\ldots}_{\ldots\ldots,\ldots\ldots (\frac{1}{2}-\frac{\acute{\omega}}{2} \pm \frac{n}{2}, \frac{\lambda\eta}{2})} \right] \quad (5.9)$$

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