Comparative Study between Simplex and Graphical Methods in a FLPP

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Abstract— This paper portrays the recent techniques that have been developed for optimization of linear programming problems. This paper aims to solve the fuzzy linear programming problem using ranking function and here the fuzzy numbers which are involved are trapezoidal fuzzy numbers.

Key words: Trapezoidal Fuzzy Number, Ranking Function, Simplex, Graphical Methods

I. INTRODUCTION

Linear programming problem is one of the optimization problems. Due to the availability of limited resources and other restrictions, we have to find out an optimal solution for the problem. LPP has a very wide variety of application in our daily problems. However, in practice, these LPP often fail to represent the real solutions. Such failures can be caused by some tight modeling assumptions. One attempt to address this failure is to replace the classical set into fuzzy sets. In this case, we call it as fuzzy Linear Programming. There are various types of fuzzy linear programming problems. One type is the right hand side of the constraints is fuzzy numbers. The other type is the coefficients of the objective functions are the fuzzy numbers. The most complicated type is the right hand side where the coefficients of the variables and the coefficients of the objective functions are fuzzy numbers. Gani et. al., (2009) [1] introduced fuzzy linear programming problem based on L-R fuzzy number. Jimenez et .al., (2005) [5] proposed a method for solving linear programming problems where all the coefficients are, in general, fuzzy numbers by using linear ranking technique. Bazaar et. al,. (1990) [2] and Nasseri et. al., (2005) [7] defined the linear programming problems with fuzzy numbers and simplex and graphical method to find out the optimal solution of the fuzzy problem. In this paper we will exploit the ranking function introduced by Yager's and construct an algorithm to solve the trapezoidal fuzzy numbers.

II. PRELIMINARIES

A. Definition 2.1.1

A Crisp set is defined in such a way as to divide the individuals in some given universe into two groups.

1) members that certainly belong to the set.
2) members that certainly do not belong to the set.

B. Definition 2.1.2

If X is a collection of objects denoted generally by x, then a fuzzy set $\bar{A}$ in X is defined as a set of ordered pairs $\bar{A}$=(x; $\mu_{\bar{A}}(x)$)/x ∈ (X),where $\mu_{\bar{A}}(x)$ is called the membership function for the fuzzy set $\bar{A}$.

C. Definition 2.1.3

A fuzzy number is a generalization of a regular, real number in the sense that it does not refer to one single value but rather to a connected set of possible values, where each possible value has its own weight between 0 and 1. This weight is called the membership function.

D. Definition 2.1.4

A fuzzy number $\bar{A} = (a^L, a^U, \alpha_1, \alpha_2)$ is a Trapezoidal fuzzy number if the membership function is defined as below:

$$f(x) = \begin{cases} \frac{x-(a^L-\alpha_1)}{\alpha_1}, & a^L - \alpha_1 \le x \le a^L \\ 1, & a^L \le x \le a^U \\ \frac{(a^U+\alpha_2)-x}{\alpha_2}, & a^U \le x \le a^U + \alpha_2 \\ 0, & \text{others} \end{cases}$$

Any trapezoidal fuzzy number $\bar{A} = (a^L, a^U, \alpha_1, \alpha_2)$ where the support of $\bar{a}$ is $(a^L - \alpha_1, a^U - \alpha_2)$ and the model set of $\bar{a}$ is $[a^L, a^U]$.

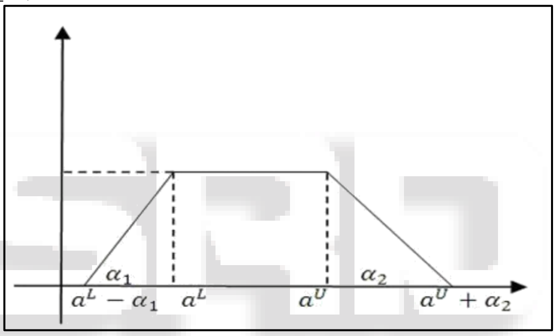


Fig. 1.1: Trapezoidal fuzzy number

E. Definition 2.1.5

Ranking function is a mapping R : F(R) → R, from the set of fuzzy number into real number. Forevery two fuzzy numbers $\bar{A}_1$ and $\bar{A}_2$ in F(R), we are define the relations as follows:

1) $\bar{A}_1 \le \bar{A}_2$ if and only if $R(\bar{A}_1) \le R(\bar{A}_2)$
2) $\bar{A}_1 \ge \bar{A}_2$ if and only if $R(\bar{A}_1) \ge R(\bar{A}_2)$
3) $\bar{A}_1 = \bar{A}_2$ if and only if $R(\bar{A}_1) = R(\bar{A}_2)$

Ranking function R which meets $R(k\bar{A}_1+L\bar{A}_2) = KR(\bar{A}_1)+LR(\bar{A}_2)$ for every $\bar{A}_1$, $\bar{A}_2$ belongs to F(R).For trapezoidal fuzzy number $\bar{A} = (a^L, a^U, \alpha_1, \alpha_2)$, then according to yager's ranking

$$\Re(\bar{A}) = \frac{1}{2}\left(a^L + a^U + \frac{1}{2}(\alpha_2 - \alpha_1)\right)$$

III. SIMPLEX METHOD

Linear programming problem by simplex method is a technique used by the various business organizations for their day to day activities to arrive at a correct solution to their problems. There is a situation where a business man has to take various decisions out of different alternatives but which has certain limitations and so this method is used.

A. Algorithm

1) Step 1:

Maximize $\bar{z}$ = cx
subject to the constraints: Ax = b; x≥0where b ∈ R^m, x ∈ R^n, A ∈ R^{m*n}, c^{-T}∈$(F(R))^n$ We will develop the crisp simplex method into fuzzy simplex method. We propose the fuzzy simplex method which is based on the arithmetic operation of the trapezoidal fuzzy numbers, the properties of ranking functions and the classic simplex method.

2) Step 2: Transform the fuzzy Programming problem into a canonical form(theconstraint must be positive, if necessary, change it into "=" relation by adding a slack variable).
3) Step 3: Create an initial fuzzy simplex table, same as the crisp simplex table, only on its coefficient row, the objective function is a trapezoidal fuzzy numbers.
4) Step 4: To test the optimality, we can see the value on the z_j-c_j row. If there is still a z_j-c_j<0 then is not optimal yet. We use the same way with R(z_j-c_j) <0 to determine if z_j-c_j<0.
5) Step 5: To fix the solution, choose an entering variable with the most negative R(z_j-c_j), and choose a leaving variable by choosing the most positive smallest (R_j).
6) Step 6: On the new table, the element pivot must be transformed into 1, and the other elements on the same row must be transformed into 0 using elementary row operations. Repeat step 3 till the optimal solution is obtained.

B. *Graphical Method*

Graphs are powerful data evaluation tools. They provide quick, visual summaries of essential data characteristics. A few simple plots can replace complex statistical equations or tests to interpret environmental data. Graphical Methods are typically used with quantitative statistical evaluations.

1) *Algorithm*

1) Step 1: The chosen problem is formulated as the following fuzzy linear Programming Problem,

$$\text{Max } z= \sum_{j=1}^{n} c_j x_j$$

subject to the constraints :

$$\sum_{j=1}^{n} a_{ij} x_{ij} \le b_i \quad , \quad i=1,2,3......m_0 \text{ and } x_j \geq 0 \text{ for all } j=1,2,3....m_0.$$

2) Step 2: Substituting the value of b_i = [b_1,b_2, b_3, b_4] and c_j = [c_1, c_2, c_3, c_4] in the

fuzzy linear programming problem obtained in step 1, we get

$$\text{Max } z = \sum_{j=1}^{n} [c_1, c_2, c_3, c_4] x_j$$

subject to the constraints $\sum_{j=1}^{n} a_{ij}$ x_j≤ b_i where b_i =[b_1, b_2, b_3, b_4]and xj ≥ 0 for all j=1,2,3....n.

3) Step 3: Using the ranking function the fuzzy linear programming problem is formulated as,

$$\text{Max } z = \sum_{j=1}^{n} [c_1, c_2, c_3, c_4] x_j$$

subject to the constraints

$\sum_{j=1}^{n} a_{ij}$ x_j ≤ R[b_1, b_2, b_3, b_4]and xj≥ 0 for all j=1,2,3....n

4) Step 4: The optimal solution is obtained by solving the above linear Programming Problem.

IV. Numerical Example

Maximize z=(4, 6, 2, 3)x_1 + (5, 8, 2, 4)x_2
subject to the constraints
$3x_1 + 6x_2 \leq 18$
$5x_1 + 4x_2 \leq 20$
where $x_1, x_2 \geq 0$

1) Step1: Based on the standard maximum problem, the canonical form is as follows:

Maximize z=(4,6, 2, 3)x_1 + (5, 8, 2, 4)x_2 +0x_3 +0x_4
Subject to the constraints:
$3x_1 + 6x_2 + x_3 = 18$
$5x_1 + 4x_2 + x_4 = 20$
$x_1, x_2, x_3, x_4 \geq 0$

Iteration:1	c_j	(4,6,2,3)	(5,8,2,4)	0	0		
	x_i/x_j	X_1	X_2	X_3	X_4	b_i	R_i
0	X_3	3	6	1	0	18	3
0	X_4	5	4	0	1	20	5
	z_j	0	0	0	0	0	
		Z_j-c_j	(-6,-4,3,2)	0	0		
		R(z_j-c_j)	-19/4	-7	0	0	

Table 1.1: Initial table

From the initial table above, the problem has a solution
Max z = 0, for x_1= 0,x_2= 0,x_3= 18,x_4 = 20,
(i.e)Max z = (4, 6, 2, 3).0 + (5, 8, 2, 4).0 +0.18 +0.20 = 0.

2) Step 2: The table above shows that the solution has not optimum yet, because there is still R(z_j-c_j) <0. It means z_j-c_j<0.

To fix the solution, we have to choose the entering variable by choosing the smallest negative value R(z_j-c_j) < 0,in this case it is x_2.

3) Step 3: Then we determine the value of R_i= bi/a_{i2}, and choose the smallest positive value, in this case we choose R_1 = 3. The new basic variable x_2 replace the old basic variable, X_3. Element $a_{2\,1}$ is called a pivot element.
4) Step 4: This element is located at the intersection of the row where leaving variable is located and the column of where the entering variable exists.

The element pivot then turns into "1" by using elementary row operation. The revised tables as follows.

Iteration:2	c_j	(4,6,2,3)	(5,8,2,4)	0	0		
	X_i/X_j	X_1	X_2	X_3	X_4	b_i	R_i
(5,8,2,4)	X_2	1/2	1	1/6	0	3	6

0	X_4	3	0	2/3	1	8	8/3
	z_j	(5/2,4,1,2)	(5,8,2,4)	(5/6,4/3,1/3,2/3)	0	(15,24,6,12)	
	Z_j-c_j	(-7/4,0,4,4)	(-3,-3,6,-6)	(5/6,4/3,1/3,2/3)	0		
	R(z_j-c_j)	-7/4	0	7/6	0		

Table 1.2: Revised simplex table

5) Step 5: Analogously, we obtain the new entering variable, x_1 and the leaving variable x_4. Furthermore, we get the next revised table as follows

Iteration:3	c_j	(4,6,2,3)	(5,8,2,4)	0	0	
C_j	Xi/xj	X_1	X_2	X_3	X_4	b_i
(5,8,2,4)	X_2	0	1	5/18	-1/6	5/3
(4,6,2,3)	X_1	1	0	-2/9	1/3	8/3
	Z_j	(4,6,2,3)	(5,8,2,4)	(-1/18,4/3,11/9,14/9)	(0,7/6,4/3,4/3)	(57/3,88/,26/3,44/3)
	Z_j-c_j	(-2,2,5,5)	(-3,3,6,6)	(-11/18,4/3,11/9,14/9)	(0,7/6,4/3,4/3)	
	R(z_j-c_j)	0	0	4/9	7/12	

Table 1.3:

6) Step 6: From the below table, it shows that (z_J-c_j) $\geq$ 0, thus we obtain an optimal solution.

Using the Yager's ranking the optimum solution is Maximum Z =25.2 , X_1 =2.6 , X_2 =1.6

The above problem is also solved by Graphical Method

1) Step 1:

Maximize z=(4, 6, 2, 3) x_1 + (5, 8, 2, 4) x_2

subject to the constraints:

$3x_1 + 6x_2 = 18$.........(1)

$5x_1 + 4x_2 = 20$.........(2)

2) Step 2: put $x_1 = 0$ in equation (1) $6x_2=18$ $x_2 = 3$.

Therefore the point A is (0,3)

put $x_2 = 0$ in equation (1) $3x_1=18$ $x_1 = 6$.

Therefore the point B is (6,0)

3) Step 3: put $x_1 = 0$ in equation (2) $4x_2 = 20$ $x_2 = 5$.

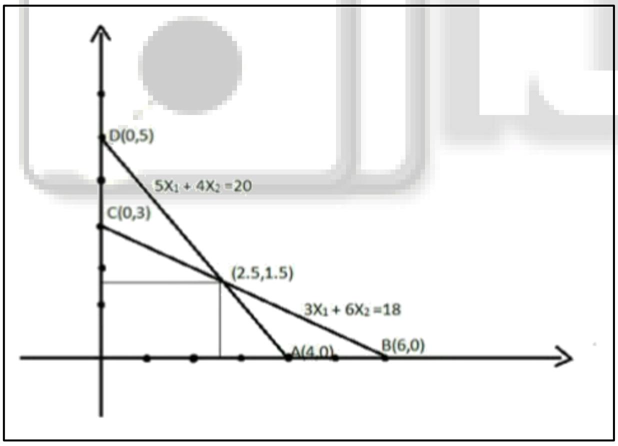
Therefore the point c is (0,5)

put $x_2 = 0$ in equation (2) $5x_1 = 20$ $x_1 = 4$.

Therefore the point D is (4,0)

Extreme points	O	A	B	C
Co-ordinate points	0	(3,0)	(2.5,1.5)	(4,0)
Objective function	0	(15,24,6,8)	(17.5,17,8,13.5)	(16,24,18,12)

Table 1.4:



Maximum Z=14, X_1 =2.5, X_2 =1.5

Simplex method	Graphical method
Max Z=25.2	Max Z=14
X_1=2.6	X_1=2.5
X_2=1.6	X_2=1.5

Table 1.5:

V. Conclusion

In this paper, we have found that a method for Trapezoidal Fuzzy Linear Programming problem, by converting them into a crisp value using yager's ranking technique to derive a fuzzy optimal solution. On comparison between Simplex and Graphical Method, we found that Simplex Method gives a better value than the Graphical Method.

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