Graceful Labelling of Some Graphs & Their Subgraphs

Swedha A.[1] Karthika N. P.[2]
[1,2]UG Scholar
[1,2]Department of Mathematics
[1,2]Sri Krishna Adithya College of Arts & Science, India

Abstract— In this paper, the topic discussed is about graph, graceful graphs, labelling to such graphs. Graceful labelling of some simple graphs and their sub graphs. Graph is a mathematical representation of a network and it describes the relation between lines and points. A graph consists of some points and lines between them. The length of the lines and position of the points do not matter. Every object in a graph is called a node.

Key words: Graceful, Labelling, Subgraph, Fan Graph, Flower Graph

I. INTRODUCTION

Labelling of vertices and edges play a vital role in graph theory. Graceful numberings are very much useful in building models of real life situation amenable for structural modelling .Construction of sub graceful graph are useful in addressing and identification system on a communication network to larger network obtained by introducing new user terminals and new communication links between the new terminals of the original network. Also labelled graph serve as a useful models for coding theory, X-ray, radar, astronomy, circuit design. Graph labelling were first introduced in the late 1960's. Rosa [1967] defined a β-valuation of a graph G with e edges as an injection from the vertices of G to the set{0, 1, 2, …, e} such that when each edge xy is assigned the label│f(x) - f(y)│the resulting edge labels are distinct. β-valuations are functions that produce graceful labelling. However, the term graceful labelling was not used until Golomb studied such labelling several years later [1977].

Acharya [1982] obtained that every graph can be implanted as an Induced sub graph of a graceful graph and a connected graph can be implanted as an induced subgraph of a graceful connected graph.

Acharya, Rao, and Arumugam [2008] found: every triangle - free graph can be implanted as an induced sub graph of a triangle - free graceful graph; every planar graph can be implanted as an induced sub graph of a planar graceful graph; and every tree can be implanted as an induced subgraph of a graceful tree.

A. *Definition (Graph)*

A Graph G is a finite non-empty set V(G) of elements called vertices or point. And let E(G) of unordered pair of distinct elements of V(G) called edges or lines .V(G) and E(G) are called vertex set and edges set respectively.

Let e={u,v}belongs to E(G). Then e is an edge. Joining the vertices u and v of V(G) and e is also written as e=uv.

– The no of elements in vertex set-ordered of G.
– The no of elements in edge set-size of G.
– If G is a graph of order P and size Q, Then G is a (p , q) graph.

Every graph can be represented by a diagram where the vertices of the graph .Corresponds to the dots and small circles and each edge corresponds to a line joining the vertices associated to it.

II. GRACEFUL GRAPH

A Graceful labelling of a graph with edge in a labelling of its vertices with some subsets of the integer between o and m inclusive, such that no two vertices share a label, and each edge is uniquely identifiers by the absolute difference between 1 and m inclusive. A graph which admits a graceful labelling is called a graceful graph.

A. *Example 1*

Let G ={V, E, f}be a graph with V = {0,1,3};E = {1,2,3}and function (f) be defined as f(1,3) = 2; f(3,0) = 3; f(0,1) =1.Then G is graceful. Moreover, it is a graceful graph with order 3 and size 3.

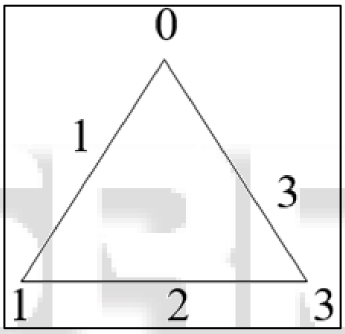

Fig. 1: A Gracefull Graph

B. *Example 2*

Let G = {V, E, f} is a graph with V={0,1,4,6},E={1,2,3,4,5,6} and edges f(0,1) =1; f(1,4) = 3; f(1,6) = 5; f(4,6) = 2; f(0,4) = 4; f(0,6) = 6.Then G is graceful graph with order 4 and size 5.

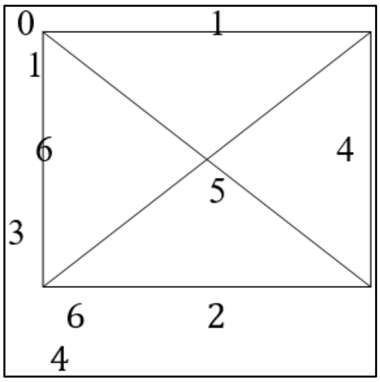

Fig. 2: A Graceful Graph

In labelling the graphs, a pair of vertices is associated with an edge connecting them. So it is natural to search a relation between the order and size of the graph so that desired labelling can be done. The solution set of recurrence relation a1 and $an = an-1 + (n - 1)$ enables to enumerate the edges of the given graph, when the adjacent vertices are labelled as $an-1$, an. Suppose, if there are 'm' edges, the first m solutions of the above recurrence solution are labelled at adjacent vertices. Usually, the edges are enumerated only for the graphs which have degree two for all vertices except the initial and terminal vertices. If the initial and terminal vertices coincide each other with degree n-1 and the degree of remaining vertices is exactly one, then the graph becomes graceful graph.

III. FLOWER GRAPH

A. Definition

Let G be a graph with order (n) and size (n-1) such that exactly one node is adjacent to every other (n-1) nodes. This is called flower graph with (n-1) petals.

B. Theorem

Every flower graph is graceful.

1) Proof

Let G denote a flower graph with (n) vertices and (n-1) edges. Therefore,

$V=\{v_1,v_2,\ldots v_n\}$

$E=\{e_1,e_2,\ldots e_{(n-1)}\}$

- In a flower graph exactly only one vertex is adjacent with all the remaining (n-1) vertices.
- Let it be v_1.
- Assume that vi is labelled with integer i.
- Then the mapping defined by $f(v_i, v_j) = |i-j|$ is a graceful mapping and G is graceful graph.
- Hence the proof.

C. Example 1

- If we define function as f(1,7) = |1-7| = 6; f(1,6) = |1-6|=5…………
- f (1,2) = |1-2|=1, then G becomes graceful graph.

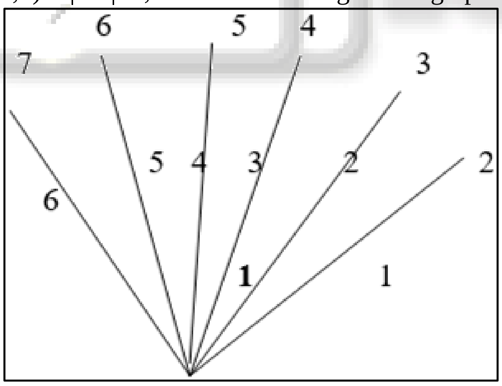

Fig. 3: A Graceful Flower Graph

D. Example 2

A flower graph with 6 petals is given below.

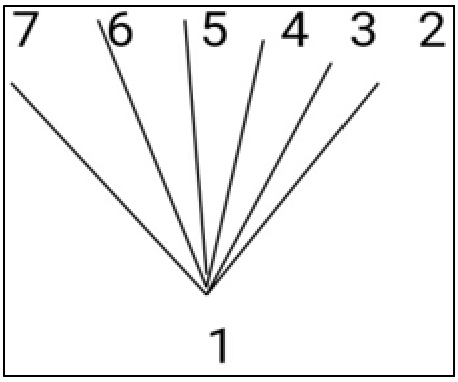

Fig. 4: A Flower Graph with 6 Petals

IV. SUB GRACEFUL GRAPH

A sub graph H of a graceful graph G is said to be graceful (G). If the graceful map f (G) is also graceful on the set of labels of V (H).

A. Theorem

A graceful graph has at least one sub graceful graph other than itself.

1) Proof

- Let G be a graph with n vertices and m edges.
- Let v_i and v_j are the vertices in G with labels x_i and x_j respectively such that $|x_i-x_j| = m$.
- By deleting the edge labelled m, we get a sub graceful graph H of G with order n and size (m-1).

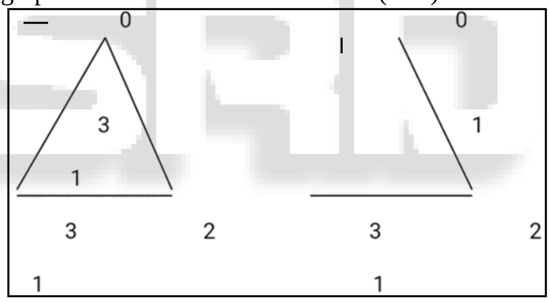

Fig. 5: A graceful graph G and its sub graceful graph H

- H is the required sub graceful graph.
- Hence the theorem proved.

2) Note

The order and size are different in graceful graphs.

B. Example

The graph G and its sub graph H are graceful. Moreover H is a sub graceful graph of G with size 5 while G is of size 6. In sub graceful graphs not only the edge-vertex incidence but also the edge-vertex labels are preserved

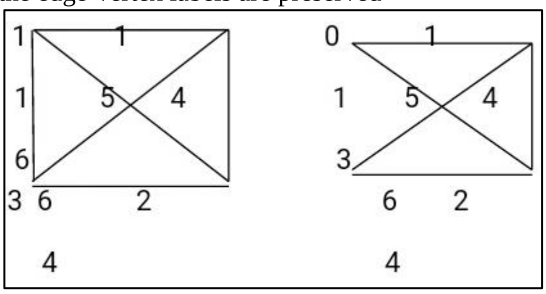

Fig. 6: A Graceful Graph G and Its Sub Graceful Graph H

C. Edge Graceful Graph Labeling (EGL)

1) Definition

A graph G with q edges and p vertices is said to have an edge - graceful labeling (EGL) if there exists a bijection f from the edge set to the set {1, 2, ..., q} so that the induced mapping f + from the vertex set to the set {0, 1, 2,..., p – 1} given by f + (x) = Σ { f (xy) | xyÎE(G)} (mod p) is a bijection. A graph G isedge - graceful graph (EGG) if it admits an edge - graceful labeling.

Fan Graph (Fn)

2) Definition

Let P_n denote the path on n vertices. Then the join of K_1 with P_n is defined as fan and is denoted by F_n (i.e) $F_n = K_1 + P_n$.

Theorem

The fan $F4n–2$ ($n \geq 2$) is strong edge – graceful for all n.

3) Proof

Let {v, $v1$, $v2$, $v3$..., vt} be the vertices of F_t and {vv_i: $1 \leq i \leq t$} ∪ {$e_i = (v_i, v_{i+1}) : 1 \leq i \leq t - 1$} be the edges of F_t.

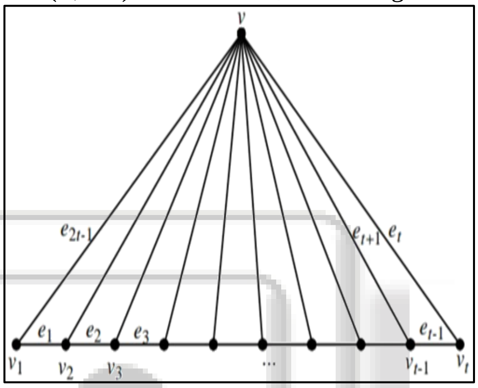

Fig. 7: Ft with Ordinary Labelling

Take $t = 4n – 2$.

We first label the edges of F_{4n-2} as follows:

$f(e_i) = i \qquad 1 \leq i \leq 8n – 5$

Then the induced vertex labels are

$f^+(v_i) = 2(4n – 2) + i – 1 \qquad 1 \leq i \leq 2$

$f^+(v_3) = 0$

$f^+(v_i) = i – 3 \qquad 4 \leq i \leq 4n – 3$

$(f^+(v_{4n-2}) = 8n – 5$; $\quad f^s(v) = 4n+1$.

Clearly, the vertex labels are all distinct. Hence, the fan F_{4n-2} ($n \geq 2$) is strong edge – graceful for all n.

The strong edge- Graceful labelling (SEGL) of F_6, F_{10}.

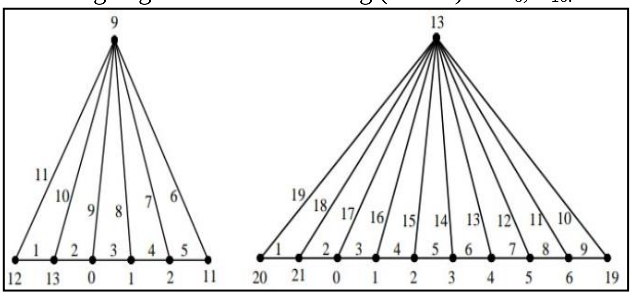

Fig. 8: F_6 with SEGL $\qquad$ $F_{10}$with SEGL

V. Conclusion

In this paper, In Graph theory that every flower graph is graceful. Every graceful graph has at least one sub graceful graph and the existence of connected sub graceful graphs depends on the incidence of the edges and the number of vertices of a given graceful graph. It will be interesting to find a graceful graph for which all the existing sub graphs are graceful.

Reference

[1] Acharya B.D, Construction of certain infinite families of graceful graphs from a given graceful graph, Def Sci J, Vol 32, No 3,July 1982, PP 231-236.
[2] Joseph A. Gallian, a Dynamic Survey of Graph Labelling.
[3] "A conjecture on amalgamation of graceful graphs with star graphs" International Journal of Contemporary Mathematical Sciences (Accepted).
[4] G.S.Bloom and S.W.Golomb, Applications of numbered undirected graphs, Proc. IEEE, 65 (1977) 562-570.
[5] Murugesan. N, Uma. R, Super vertex gracefulness of complete Bipartite graphs, International journal of Math. Sci & engg. Applications, Vol 8, No.VI, (Nov 2011).
[6] Solomon W. Golomb, The largest graceful sub graph of the complete graph, The American Mathematical monthly, Vol.81, No.5 (May, 1974), pp.459-501.
[7] Vaidya S.K.,.Vihol P.L, Fibonacci and Super Fibonacci Graceful Labeling of some graphs, Studies in Mathematical Sciences, Vol. 2, No. 2, 2011, pp. 24- 35.

