Split Middle Domination in Fuzzy Graphs

Dr. C. V. R. Harinarayanan[1] Dr. R. Muthuraj[2] S. Geetha[3]
[1,2,3]Assistant Professor
[1,2,3]Department of Mathematics
[1]Government Arts College, Paramakudi [2]H.H. The Rajah's College Pudukkottai [3]Kings College of Engineering, Punalkulam

Abstract— In this paper we introduce the concept of split middle domination in fuzzy graphs. We determine the domination number $\gamma_{SM}(G)$ for fuzzy graphs. And many bounds on $\gamma_{SM}(G)$ were obtained in terms of the vertices, edges and many other different parameters of G. Also its relation with other different parameters also developed.

Key words: Middle Fuzzy Graph, Domination Number, Edge Domination, Connected Domination Number

I. Introduction

The concept of fuzzy graph was proposed by Kaufmann, from the fuzzy relations introduced by Zadeh[10]. Rosenfeld (1975)[8] introduced the notion of fuzzy graphs and several fuzzy analogs of graph theoretic concepts such as paths, cycles, and connectedness. In the year 1998, the concept of domination in fuzzy graphs was investigated by A. Somasundaram,and S.Somasundaram[9]. In the year, 2004 A.Somasundaram investigated the concepts of domination in fuzzy graph – II. In the year 2003, A.Nagoor Gani and M. Basheer Ahamed investigated order and size in fuzzy graph. Veena Mathad and B. Sharada (2012)[3] introduced the concept of fuzzy middle graph and its properties. In this paper we introduce the concept of split middle domination in fuzzy graphs.

II. Preliminaries

A. Definition

A fuzzy graphG=(σ,μ) is a set with a pair of relations σ:V→[0,1] and μ:VxV→[0,1] such that μ(u,v)≤ σ(u) σ(u) for all u,v ∈V

B. Definition

The order of a fuzzy graph G is O (G) = $\sum_{u\in V} \sigma(u)$.

$\sum$

The size of a fuzzy graph G is S (G) = $\sum_{uv\in E} \mu(uv)$.

C. Definition

A vertex is a fuzzy cut vertex of a fuzzy graph G if removal of it reduces the strength of the connectedness between some other pair of vertices.

D. Definition

A set D of vertices in a fuzzy graph G is dominating set if every vertex $v \in$ V is either an element of D or adjacent to an element of D. A dominating set is called minimal dominating set if no proper subset of D is a dominating set. The minimum fuzzy cardinality of a dominating set is called the domination number of a fuzzy graph G and it is denoted by $\gamma\ (G)$.

E. Definition

A dominating set of a fuzzy graph G is a split (non split) dominating set if the induced sub graph $\langle V - D\rangle$ is disconnected (connected).

The split (non-split) domination number $\gamma_s(G)[\gamma_{ns}(G)]$ is the minimum fuzzy cardinality of a split (non split) dominating set.

F. Definition

Let G be a connected fuzzy graph without isolated vertices. A dominating set D of G is said to be connected dominating set if the induced fuzzy sub graph $\langle D\rangle$ is connected.

G. Definition

The vertex cover in a fuzzy graph G is a set of vertices that covers all the edges of G.

The vertex covering number $\alpha_0(G)$ is a minimum fuzzy cardinality of a vertex cover in G.

H. Definition

An edge cover of G is the set of edges that covers all the vertices. The edge covering number $\alpha_1(G)$ of G is the minimum fuzzy cardinality of an edge cover.

The edge independence number $\beta_1(G)$ of G is the minimum fuzzy cardinality of an independent set of edges.

I. Definition

A dominating set D of M (G) is called connected dominating set of M(G) if the induced $\langle D\rangle$ is connected. The minimum fuzzy cardinality of D is called the connected middle domination number of G and it is denoted by γ_c [M (G)]

J. Definition

A set of edges F in a fuzzy graph G is called an edge dominating set of G if every edge

e$\in E - F$, there exists an edge $e_1 \in F$, such that e and e_1 have an edge in common. The edge domination number of $\gamma'(G)$ of a fuzzy graph G is minimum fuzzy cardinality an edge dominating set.

K. Definition

A connected fuzzy graph $G = (\sigma, \mu)$ is a fuzzy tree if it has a fuzzy spanning subgraph

F: $G = (\sigma, \gamma)$ which is a tree, where for all arcs (u,v) not in F, $\mu(u,v) < \gamma^{\infty}(u,v)$

L. *Definition*

A fuzzy graph G:(σ,μ) with the underlying graph $G^*:(\sigma^*,\mu^*)$ be given. Let G^* be (V, E) be given. Then vertices and edges of G are taken together as the vertex set of the pair

M(G) : (s_M, μ_M) where

$\sigma_M(u) \quad = \sigma(u)$ *if* $u \in \sigma^*$

$\quad = \mu(u)$ *if* $u \in \mu^*$

$\quad = 0$ *otherwise*

$\mu_M(e_i, e_j) = \mu(e_i) \Lambda \mu(e_j)$ *if* $e_i, e_j \in \mu^*$ *and are adjacent in* G^*

$\quad = 0$ *otherwise*

$\mu_M(v_i, v_j) = o$ *if* $v_i, v_j \in \sigma^*$

$\mu_M(v_i, e_j) = \mu(e_j)$ $v_i \in \sigma^*$ *lies on the edge* $e_j \in \mu^*$

$\quad = 0$ *otherwise*

Hence M(G) : (s_M, μ_M) is a fuzzy graph called middle fuzzy graph of G.

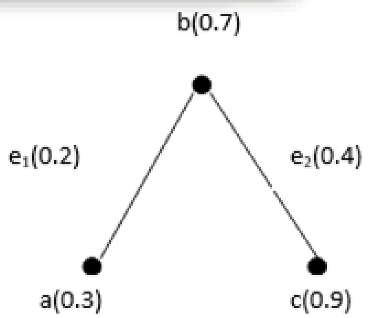
M. *Definition*

A dominating set *D* of M (G) is called split dominating set of M(G) if the induced subgraph $\langle V[M(G)] - D \rangle$ is disconnected. This split dominating set of M (G) is called split middle dominating of G.

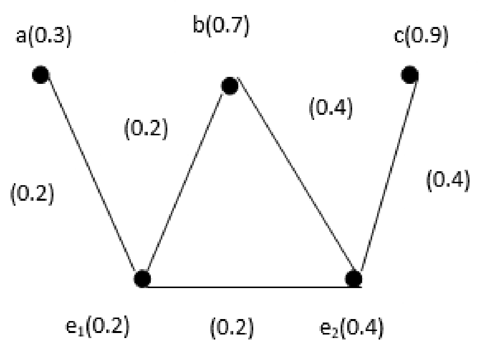
N. *Example*

In the following fuzzy graph, D={e_1,c} is a split dominating set of M(G)
Minimal split dominating set of M(G)={e_2,a}

G



M(G)



III. MAIN RESULTS

A. *Theorem*

A split middle dominating set D$\subseteq V[M(G)]$ is minimal iff for each vertex x$\in D$ one of the following conditions holds:

1) There exists a vertex y$\in V[M(G)] - D$ such that N(y) I $D = \{x\}$
2) x is an isolate in $\langle D \rangle$
3) $\langle V[M(G)] - D \rangle$ Y $\{x\}$ is connected

1) *Proof:*

Suppose D is minimal split middle dominating set of G and there exists a vertex x$\in D$ such that x does not hold any of the above conditions. Then for some vertex v, the set D_1=D-$\{v\}$ forms a split middle dominating set of G by conditions (i) and (ii).

Also by (iii), $\langle V[M(G)] - D \rangle$ is disconnected. This implies that D_1 is a split middle dominating set of G, a contradiction.

Conversely, suppose that for every vertex $x \in D$, one of the above statement hold. Further, if D is not minimal, then there exists a vertex x$\in D$ such that D- $\{x\}$ is a split middle dominating set of G and there exists a vertex y $\in D - \{x\}$ such that y dominates x. That is y$\in N(x)$.

Therefore, x does not satisfy (i) and (ii) hence it must satisfy (iii)

Then there exists a vertex y$\in V[M(G)] - D$ such that N(y) $\cap D = \{x\}$.Since $D - \{x\}$ is a split middle dominating set, then there exists a vertex $z \in D - \{x\}$ such that $z \in N(y)$ Therefore $w \in N(y) \cap D$, where w$\neq x$, a contradiction to the fact that N(y) $\cap D = \{x\}$.

Clearly D is a minimal split middle dominating set of G.

B. *Theorem:*

For a nontrivial connected fuzzy graph G, $\gamma_{SM}(G) \geq \alpha_1(G)$

1) *Proof:*

Let E_1=$\{e_1, e_2, e_3, ... e_n\}$ be the minimal set of edges in G such that $|E_1| = \alpha_1(G.)$. Since V [M(G)]=

(G) ∪(G), let S=$\{s_1, s_2, s_3, ...s_i\}$be the set of vertices subdividing each edge in M(G). Now, let $S_1 = \{S_k \setminus 1 \leq k \leq i\} \subseteq S$ be the set of vertices subdividing each edge

$e_k \in E_1$,$1 \leq k \leq i$ in M (G).

Clearly N (S_1)=V(S-S_1).

Hence $N(S_1) = V(G) \cup V(S - S_1) = V[M(G)]$.

Thus $\langle S_1 \rangle$ forms a minimal dominating set in M (G).

If the sub graph $\langle V[M(G)] - S_1 \rangle$ contains at least two components, then S_1 itself forms the minimal split dominating set in M(G).

Otherwise there exists at least one vertex $\{s_j\} \in V[M(G)] - S_1, 1 \le j \le i$ such that the sub graph $\langle V[M(G)] - (S_1 \cup \{s_j\}) \rangle$ is disconnected. Clearly $S_1 \cup \{s_j\}$ forms a minimal split dominating set inM (G). Thus $|E_1| \le |S_1 \cup \{s_j\}|$ which gives $\gamma_{SM}(G) \ge \alpha_1(G)$.

C. Theorem:

For a connected fuzzy graph $G = (\sigma, \mu), \gamma_{SM}(G) \le p - 1$, where p is the order and q is the size of the fuzzy graph.

1) Proof:

1) Case1: Let G=T be a fuzzy tree. The vertex set and edge set of T are V (T) = $\{v_1, v_2, v_3, \ldots\ldots\ldots v_i\}$

And E (T)= Let S=$\{s_1, s_2, s_3, \ldots\ldots\ldots .s_i\}$ be the vertices subdividing the edge in M(T), which is also the set of cut vertices in M(T). Suppose $S_1 \subseteq S$ be the minimal set of vertices such that $N[S_1]=V[M(T)]$. Hence S_1 forms the minimal dominating set in M(T). Since S_1 is the set of cut vertices, the fuzzy sub graph $\langle V[M(T)] - S_1 \rangle$ is disconnected. Hence S_1 forms the minimal split dominating set in M(T). Therefore $|S_1| \le |E(T)| = |V(T)| - 1$,which gives $\gamma_{SM}(G) \le p - 1$

2) Case2: Let $G \ne T$, then consider a fuzzy spanning tree H of G. Let E_1=$\{e_1, e_2, e_3, \ldots\ldots. e_i\}$ be the edges in H. Let S=$\{s_1, s_2, s_3, \ldots\ldots\ldots .s_i\}$ be the vertices subdividing the edges of E_1 in M(H). Again $S_2 \subseteq S_1$ be the minimal set of vertices which covers all the vertices in M(H) and the fuzzy sub graph $\langle V[M(H)] - S_2 \rangle$ is disconnected. Thus $N[S_2]=V[M(H)]$. By adding the edges $E_2=E(G)-E_1$ of G to H. Again consider $S_2^{'} = \{s_1{'}, s_2{'}, \ldots\ldots\ldots .s'_i\}$ be the vertices subdividing the edges $E_2^{'} = \{e_1{'}, e_2{'}, \ldots\ldots\ldots .e_i^{'}\}$ in M(G). Now since $(s_i) \cap N(s_i^{'}) \ne \Phi, \forall s_i \in S_2 \text{ and } s_i^{'} \in S_2^{'}$ in M (G).

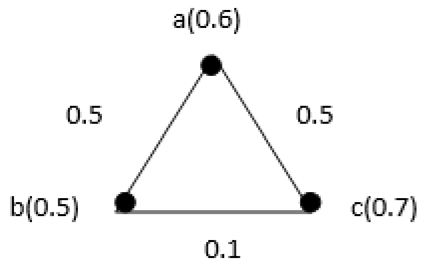
Clearly $N[S_2] = S_2^{'} \cup V[M(H)] = V[M(G)]$.

Thus S_2 forms a minimal split dominating set of M (G) with $|S_2| \le |\gamma_{SM}(G)|$

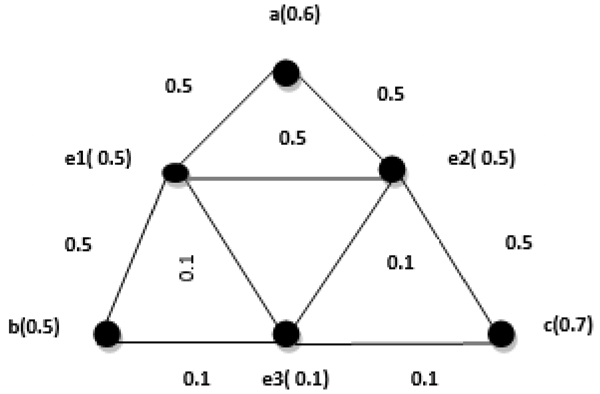
Since $S_2 \subseteq E_1$, then $|S_2| \le |E_1| \le p - 1$.

Therefore $\gamma_{SM}(G) \le p - 1$

D. Example: Fuzzy tree T



1) Middle fuzzy tree M(T)



Split middle dominating set D={a,e3}

Split middle domination number $\gamma_{SM}(G) = 0.7$

Fuzzy subgraph induced by D is disconnected.

p=0.6+0.5+0.7=1.8 and p-1=1.8-1=
0.8

Clearly 0.7<0.8

Hence $\gamma_{SM}(G) \le p - 1$

E. Theorem:

For a connected fuzzy graph G, $\gamma_{SM}(G) \le p - \gamma'(G)$

1) Proof:

Let E_1=$\{e_1, e_2, e_3, \ldots\ldots. e_n\} \subseteq E(G)$ be the minimal set of edges, such that for each edge $e_i \in E_1$, i=1,2…………….n, $N(e_i) \cap E_1 = \Phi$. Then $|E_1| = \gamma'(G)$

In M (G), V [M(G)]= $V(G) \cup E(G)$. Let D= $\{v_1, v_2, v_3, \ldots\ldots\ldots v_i\}$ be the set of vertices subdividing the edges of G in M (G). Let $D_1 \subseteq D$ be the set of vertices subdividing the edges in $E(G)-E_1$ in M(G). Suppose $N[D_1]=V[M(G)]$. Then $\langle D_1 \rangle$ forms a minimal dominating set in M (G).Now assume $\langle V[M(G)] - D_1 \rangle$ is disconnected. Then D_1 itself forms the minimal split dominating in M(G). Otherwise let $D_1^{'} \subseteq D_1, D_2^{'} \subseteq D_2$ such that $N[D_1^{'} \cup D_2^{'}] = V[M(G)]$ and the sub graph $\langle V[M(G)] - D_1^{'} \cup D_2^{'} \rangle$ is disconnected. Hence $D_1^{'} \cup D_2^{'}$ forms a minimal split middle dominating set of G. Clearly, it follows that $\left|D_1^{'} \cup D_2^{'}\right| \leq \left|V(G)\right| - \left|E_1\right|$ implies that $\gamma_{SM}(G) \leq p - \gamma'(G)$

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