Elasto-Plastic Analysis of Clamped Circular Plates with Central Circular Holes using HOSDT for Different Radius to Thickness Ratios

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Abstract— The present study is implementation of a finite element formulation for elasto-plastic analysis of clamped circular plates by using higher-order shear deformation theory. In this formulation variation of transverse shear strain is taken into account for incremental elasto-plastic plate bending analysis. Parametric studies over clamped circular plates with and without circular holes having different radius to thickness ratios, holes sizes etc. was conducted. The thickness of plate doesn't affect the normalised collapse load which endorses the correctness of normalisation parameter chosen. However a little increase in normalised collapse load was observed in case of thin plates. This may be due to stretching effect which comes into play along-with bending in case of thin plates while only bending effect is seen in case of thick plates.

Key words: Elasto-Plastic Analysis, Higher-Order Shear Deformation Theory, Normalisation Parameter, Normalisation Parameter, Parametric Studies

I. INTRODUCTION

Plate is a structural element, which has applications in construction, fabrication, heavy structural engineering, nuclear engineering, automobiles, aerospace industries etc. The bending behaviour of plate subjected to transverse load is important for design purposes. An accurate prediction of behaviour of plates improves the safety and economy of the whole structure. Various researchers have given their contribution in development and refinements for bending theories of plates. The initial refinements in plate bending theories started with first-order shear deformation theory. First order shear deformation theory accounted for the transverse shear strains by making the normals to the midsurface during deformation remaining straight but not necessarily normal to it. The shear strain energy was corrected by introducing a fictitious shear correction coefficient in its expression. For precise modelling of plate a theory was presented considering the plate as a three-dimensional body. The equations of elasticity were applied for accurate solution, but the resulting governing partial differential equations become too complicated and their solution is very often impossible for many boundary conditions, loads, etc. Then the higher-order deformation theories developed that bridged this gap between the first-order shear deformation theories and three-dimensional elasticity theory. These theories were degenerated from three-dimensional elasticity equations to two-dimensional representations. These theories considered the warping of the cross-sections by higher-order representation of assumed displacement field.

II. HIGHER-ORDER SHEAR DEFORMATION THEORY

Elasto-plastic bending analysis of plate consists of analysis of elastic as well as plastic behaviour. The design based on the elastic bending theories limit the stresses within elastic limit. This uneconomical design of elastic bending theory leads to development of elasto-plastic bending analysis. Initially first-order shear deformation theory was presented. Then with refinement of bending theories higher-order shear deformation theory has been presented. Higher-order shear deformation theory provides more accurate results as compared to first-order shear deformation theory. In this theory, desired accuracy can be obtained by including a sufficient number of terms. In the present study finite element formulation has been developed by using higher-order shear deformation theory. In plate bending, three-dimensional elasticity problem can be reduced to two-dimensional problem by using higher-order shear deformation theory. It is done by expanding the three-dimensional displacements in the power series of the coordinate in the thickness direction. In plate bending, three-dimensional elasticity problem can be reduced to two-dimensional problem by using higher-order shear deformation theory. It is done by expanding the three-dimensional displacements in the power series of the coordinate in the thickness direction.

Taylor series expansion is used to express the three-dimensional displacement components u(x, y, z), v(x, y, z) and w(x, y, z) of any point in the plate space. It is given as

$$\left.\begin{aligned}
u(x,y,z) &= u(x,y,0) + z\left(\frac{\partial u}{\partial z}\right)_0 + \frac{1}{2!}z^2\left(\frac{\partial^2 u}{\partial z^2}\right)_0 + \frac{1}{3!}z^3\left(\frac{\partial^3 u}{\partial z^3}\right)_0 + ... + \infty \\
v(x,y,z) &= v(x,y,0) + z\left(\frac{\partial v}{\partial z}\right)_0 + \frac{1}{2!}z^2\left(\frac{\partial^2 v}{\partial z^2}\right)_0 + \frac{1}{3!}z^3\left(\frac{\partial^3 v}{\partial z^3}\right)_0 + ... + \infty \\
w(x,y,z) &= w(x,y,0) + z\left(\frac{\partial w}{\partial z}\right)_0 + \frac{1}{2!}z^2\left(\frac{\partial^2 w}{\partial z^2}\right)_0 + \frac{1}{3!}z^3\left(\frac{\partial^3 w}{\partial z^3}\right)_0 + ... + \infty
\end{aligned}\right\} \quad ..... 2.1$$

Three-dimensional elasticity problem of plate bending can be reduced to a two-dimensional formulation by using these expressions. Lo *et al* (1977), Kant (1982) and Rode (1995) have shown that the retention of first four terms for the in-plane displacement 'u' and 'v' and first three terms for transverse displacement is sufficient. The warping of transverse cross-sections is incorporated as the expansion of 'u' and 'v' imply a non-linear variation of in-plane displacement through thickness. The expressions given by the equations (2.1) can be written by grouping the terms contributing to membrane and flexural behaviour as given below:

Membrane	Flexure
$u(x,y,z)=u_0(x,y)+z^2u_0^*(x,y)$	$+z\theta_x(x,y)+z^3\theta_x^*(x,y)$
$v(x,y,z)=v_0(x,y)+z^2v_0^*(x,y)$	$+z\theta_y(x,y)+z^3\theta_y^*(x,y)$
$w(x,y,z)=z\theta_z$	$+w_0+z^2w_0^*$

The term u_0, v_0 and w_0 are the displacement component of a point(x, y) on the mid-plane and θ_x, θ_y are the rotations of the normals to the mid-plane about y and x respectively. Terms θ_x^* and θ_y^* are in the Taylor's series expansion represent the higher-order transverse deformation modes. In elasto-plastic analysis, the yielding of material shows the start of plasticity. Before the yield point the material regains its original shape and size, when the applied loads are removed. Above the yield point, some amount of the deformation will be permanent and irreversible. The plastic response of a strain-hardening material is specified by an initial yield condition, a hardening rule and flow rule.

III. RESULTS & DISCUSSION

A higher order theoretical formulation accounting for quadratic variation of transverse shear stresses has been employed along with corresponding matrices and equations for studying elasto-plastic behaviour of clamped circular plates with and without holes. A computer program developed by Rode and Kant (1995) incorporating 5 degrees of freedom and corresponding finite element formulation has been modified and used for the analysis of these plates. The nine-noded Lagrangian element has been used for the finite element discretization. These elements have shown better performance as concluded by Rode (1995) over the serendipity elements when used with selective integration. Hence, the selective integration technique, that is, the normal (3x3) Gauss integration rule for the bending and reduced (2x2) Gauss rule for shear, has been used.

Clamped circular plates with and without holes subjected to uniformly distributed load are considered in present study. A mesh with 14 elements has been adopted for the discretization of a quadrant of a circular plate without holes. A mesh with 12 elements having holes sizes 10%, 20%, 30% of plate area and a mesh with 8 elements having holes sizes 50% of plate area are used for the discretization of quadrant of circular plate. A modified Newton-Raphson method is used for the incremental analysis for the calculation of residual forces using higher order displacements. Tresca and Von-Mises criterion has been incorporated for the analysis of elasto-plastic behaviour of plates. The present analysis is performed by using von-Mises criterion since the earlier experimental work (Taylor and Kinney, 1931) has indicated the superiority of the Von-Mises criterion over the Tresca criterion for the ductile metals.

A. Description of the problem

Clamped circular plates subjected to uniformly distributed load with different thicknesses and holes sizes are considered for elasto-plastic analysis. Circular plates with and without circular holes are considered. The holes sizes considered are 10%, 20%, 30% and 50% of area of full plate.

To study the effect of thickness variation different thicknesses are considered which are given below:

1) Thick plate having R/h = 5
2) Moderately thick plate having R/h = 20
3) Moderately thin plate having R/h = 40
4) Very thin plate having R/h = 100

Where 'R' is radius of plate and 'h' is thickness of plate.

Material and geometric properties considered for these cases have been drawn from Papadopoulos and Taylor (1990) that are: E = 10.92 units, ν = 0.3, σ_0 = 1000 units and radius of plate =10 units.

B. Effect of thickness variation

1) Clamped circular plates without holes:

Thin plates show almost same behaviour as moderately thick plates, but the thick plate show normalised flexible behaviour. The formation of plastic zone starts at the clamped edge. Then another plastic zone starts at the centre. The plastic zone at centre spreads faster than the edge.In case of thick plate the meeting of two zones can be observed with gradual increment of load while in case of thin plate sudden failure occurs. The failure in plates for R/h = 5, 20, 40 and 100 occurs at normalised collapse loads of 14.700, 13.440, 14.103 and14.400 respectively. Thus normalised collapse loads are almost same and are unaffected by thickness of plate.

2) Clamped circular plates having 10% of the plate area as holes area:

Thin plates show almost same behaviour as moderately thick plates, but the thick plate show normalised flexible behaviour. Yielding of plates and formation of plastic zone starts at the clamped edge. Then another plastic zone starts at the edge of the hole. The plastic zone at edge of hole spreads faster as compared to the clamped edge. The failure in plates for R/h = 5, 20, 40 and 100 occurs at normalised collapse loads of 14.175, 14.616, 15.000 and 14.400 respectively. Thus normalised collapse loads are almost same and are unaffected by thickness of plate. In plates having R/h = 5, 20 and 40, meeting of two zones can be observed with gradual increment of load, but in plates with R/h = 100 sudden failure occurs.

3) Clamped circular plate having 20% of the plate area as holes area:

Thin plates show almost same behaviour as moderately thick plates, but the thick plate show normalised flexible behaviour. Formation of plastic zone starts at the clamped edge. Then another plastic zone starts at the edge of hole. The plastic zone at edge of hole spreads faster as compared to that at clamped edge. The failure in plates for R/h = 5, 20, 40 and 100 occurs at normalised collapse load of 15.504, 16.128, 16.670 and 16.640 respectively. Thus normalised collapse loads are almost same and are unaffected by thickness of plate. In plates having R/h = 5 and 20 meeting of two zones can be observed with gradual increment of load, while plates having R/h = 40 and 100 sudden failure occurs.

4) Clamped circular plates having 30% of the plate area as holes area:

Thin plates show almost same behaviour as moderately thick plates, but the thick plate show normalised flexible behaviour. Formation of plastic zone starts at clamped edge. Then it spreads further. Another plastic zone starts at the edge of hole. The plastic zone at the edge of hole spreads faster as compared to the clamped edge. The failure in plate for R/h = 5, 20, 40 and 100 occurs at normalised collapse loads of 17.711, 18.841, 17.950 and18.621 respectively. Thus normalised collapse loads are almost same and are unaffected by thickness of plate. In plates having R/h = 5 and 20 meeting of two zones can be observed with gradual increment of load, while plates having R/h = 40 and 100 sudden failure occurs.

5) Clamped circular plates having 50% of plate area as holes area:

Thin plates show almost same behaviour as moderately thick plates, but the thick plate show normalised flexible behaviour. Formation of plastic zone starts at the clamped edge. Then it spreads further. Another plastic zone starts at edge of hole. The plastic zone at edge of hole spread faster as compared to the clamped edge. The failure in plates for R/h = 5, 20, 40 and 100 occurs at normalised collapse loads of 25.538, 28.001, 26.924 and 26.641 respectively. Thus normalised collapse loads are very close and are unaffected by thickness of plate. In plates having R/h = 20 and 40 meeting of two zones can be observed with gradual increment of load, while plates having R/h = 5 and 100 sudden failure occurs.

Normalised load versus normalised displacement behaviour

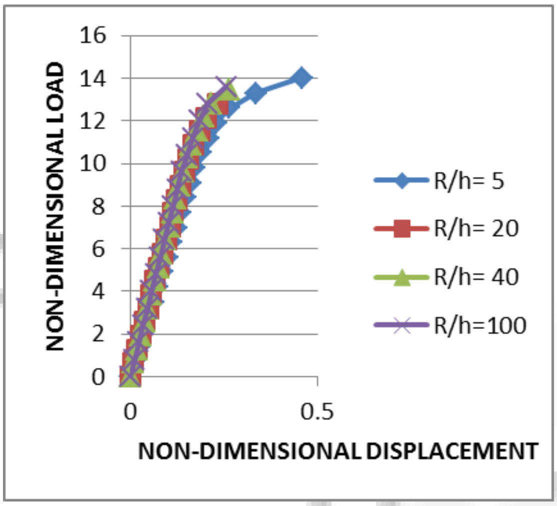

Fig. 3.1: Clamped circular plate without hole and uniform load: normalised load versus normalised displacement graph for R/h=5, 20, 40, and 100.

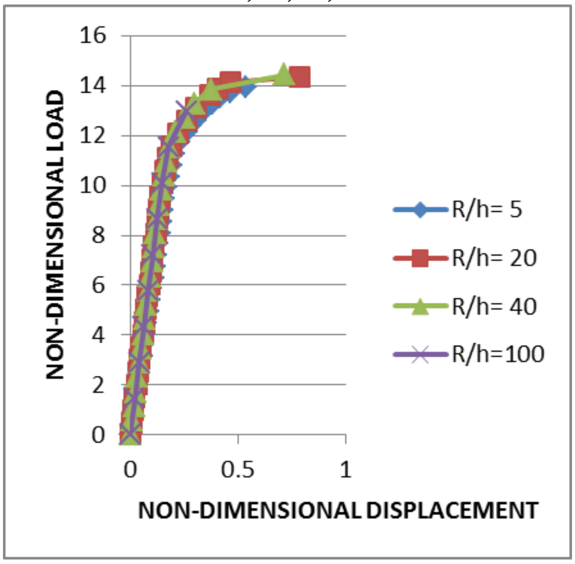

Fig. 3.2: Clamped circular plate having 10% area of plate as hole area and uniform load: normalised load versus normalised displacement graph for R/h = 5, 20, 40 and 100.

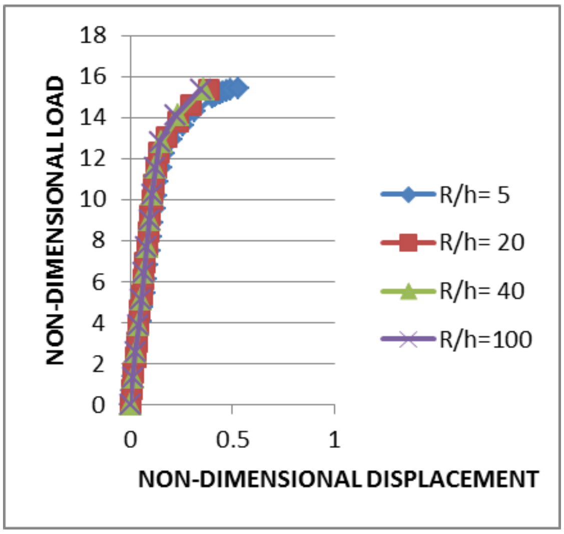

Fig. 3.3: Clamped circular plate having 20% area of plate as hole area and uniform load: normalised load versus normalised displacement graph for R/h = 5, 20, 40 and 100.

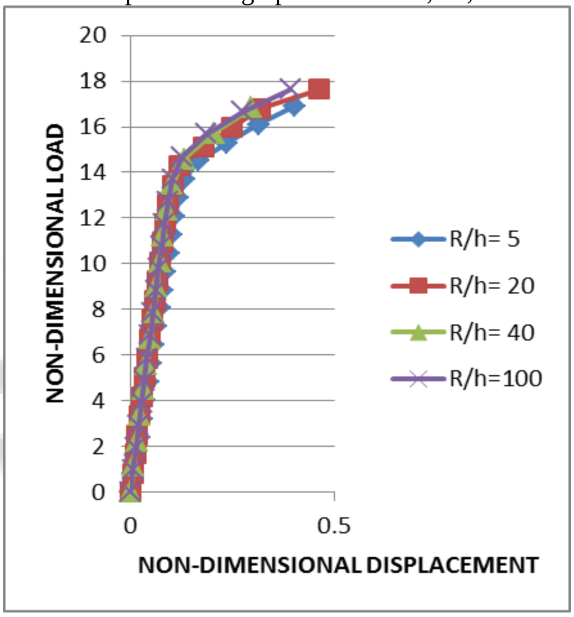

Fig. 3.4: Clamped circular plate having 30% area of plate as hole area and uniform load: normalised load versus normalised displacement graph for R/h = 5, 20, 40 and 100.

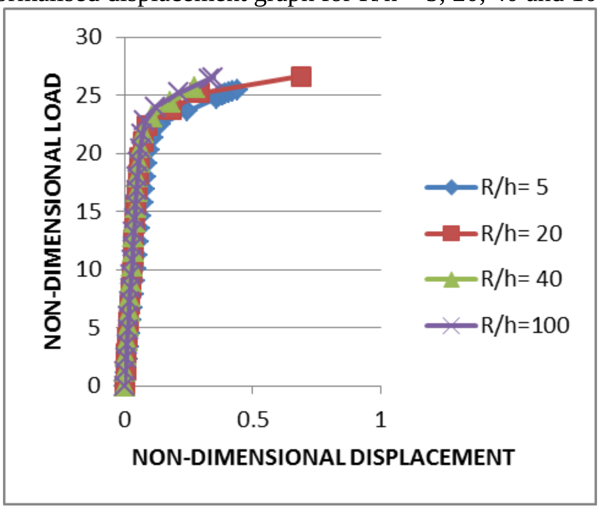

Fig. 3.5: Clamped circular plate having 50% area of plate as hole area and uniform load: normalised load versus normalised displacement graph for R/h = 5, 20, 40 and 100.

IV. CONCLUSIONS

Finite element formulation for elasto-plastic analysis of clamped circular plate by using higher order shear deformation theory is presented. The computer program and theory is validated by comparing it with the other theories presented by researchers wherever available. The normalised collapse load increases with increase in percentage of holes sizes. This may be due to the increased stiffness of the overhanging radial strips of the plates which acts as cantilever beams whose span reduces with increase in size of hole, which in turn increases the bending, that is rotational, stiffness of the beams. In plates without and with holes, progress of the first plastic zone starts at the clamped edge. Then another plastic zone starts at centre (for full plate) or the edge of central circular hole (for plate having holes). The plastic zone at centre or edge of central circular hole spreads faster as compared to that at clamped edge.

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