Game Theory and its Applications use of Repeatation in Zero-Sum Game

Satyaprakash Sahu[1] A J Khan[2]
[1]Research Scholar
[1,2]MATS University, Raipur (C.G.), India

Abstract— Game theory is the formal study of conflict and cooperation. Game theoretic concepts apply whenever the actions of several agents are interdependent. These agents may be individuals, groups, firms, or any combination of these. The concepts of game theory provide a language to formulate, structure, analyze, and understand strategic scenarios. A game is said to be zero-sum if for any outcome, the sum of the payoffs to all players is zero. In a two-player zero-sum game, one player's gain is the other player's loss, so their interests are diametrically opposed. Two players interact repeatedly over an innite horizon and occasionally, one of the players has an opportunity to do a favor to the other player. The ability to do a favor is private information and only one of the players is in a position to do a favor at a time. The cost of doing a favor is less than the benet to the receiver so that, always doing a favor is the socially optimal outcome. Intuitively, a player who develops the ability to do a favor in some period might have an incentive to reveal this information and do a favor if she has reason to expect future favors in return. Two players interact repeatedly over an innite horizon and occasionally, one of the players has an opportunity to do a favor to the other player. The ability to do a favor is private information and only one of the players is in a position to do a favor at a time. The cost of doing a favor is less than the benet to the receiver so that, always doing a favor is the socially optimal Outcome. Intuitively, a player who develops the ability to do a favor in some period might have an incentive to reveal this information and do a favor if she has reason to expect future favors in return. The extreme case of players with fully opposed interests is embodied in the class of two player zero-sum (or constant-sum) games.

Key words: Game Theory, Player, Zero Sum Game Payoff, Strategy
Msc 91A60, 60G40

I. Introduction

A game is a formal description of a strategic situation. Game theory is the formal study of decision-making where several players must make choices that potentially affect the interests of the other players. Game theory is the formal study of conflict and cooperation. Game theoretic concepts apply whenever the actions of several agents are interdependent. These agents may be individuals, groups, firms, or any combination of these. The concepts of game theory provide a language to formulate, structure, analyze, and understand strategic scenarios. When someone overreacts, we sometimes say "it's just a game." Games are often not serious. Mathematical games, which are the subject of this book, are different. It was the purpose of game theory from its beginnings in 1928 to be applied to serious situations in economics, politics, business, and other areas. Even war can be analyzed by mathematical game theory. Let us describe the ingredients of a mathematical game. Rules Mathematical games have strict rules. They specify what is allowed and what isn't. Though many real-world games allow for discovering new moves or ways to act, games that can be analyzed mathematically have a rigid set of possible moves, usually all known in advance. Outcomes and payoffs Children (and grown-ups too) play games for hours for fun. Mathematical games may have many possible outcomes, each producing payoffs for the players. The payoffs may be monetary, or they may express satisfaction. You want to win the game. Uncertainty of the Outcome A mathematical game is "thrilling" in that its outcome cannot be predicted In advance. Since its rules are fixed, this implies that a game must either contain some random elements or have more than one player. Decision making A game with no decisions might be boring, at least for the mind. Running a 100 meter race does not require mathematical skills, only fast legs. However, most sport games also involve decisions, and can therefore at least partly be analyzed by game theory. No cheating In real-life games cheating is possible. Cheating means not playing by the rules. If, when your chess opponent is distracted, you take your queen and put it on a better square, you are cheating, as in poker, when you exchange an 8 in your hand with an ace in your sleeve. Game theory doesn't even acknowledge the existence of cheating. We will learn how to win without cheating.

II. History and Impact of Game Theory

The earliest example of a formal game-theoretic analysis is the study of a duopoly by Antoine Cournot in 1838. The mathematician Emile Borel suggested a formal theory of games in 1921, which was furthered by the mathematician John von Neumann in 1928 in a "theory of parlor games." Game theory was established as a field in its own rightafter the 1944 publication of the monumental volume Theory of Games and Economic Behavior by von Neumann and the economist Oskar Morgenstern. This book provided much of the basic terminology and problem setup that is still in use today. In 1950, John Nash demonstrated that finite games have always have an equilibrium point, at which all players choose actions which are best for them given their opponents' choices. This central concept of noncooperative game theory has been a focal point of analysis since then. In the 1950s and 1960s, game theory was broadened theoretically and applied to problems of war and politics. Since the 1970s, it has driven a revolution in economic theory. Additionally, it has found applications in sociology and psychology, and established links with evolution and biology. Game theory received special attention in 1994 with the awarding of the Nobel prize in economics to Nash, John Harsanyi, and Reinhard Selten. At the end of the 1990s, a high-profile application of game theory has been the design of auctions. Prominent game theorists have been involved in the design of auctions for al locating rights to the use of bands of the electromagnetic spectrum to the mobile telecom munications industry. Most of these auctions were designed with the goal of allocating these resources more efficiently than traditional governmental practices, and additionally raised billions of dollars in the United States and Europe.

III. DEFINITIONS OF GAMES

The object of study in game theory is the game, which is a formal model of an interactive situation. It typically involves several players; a game with only one player is usually called a decision problem. The formal definition lays out the players, their preferences, their information, the strategic actions available to them, and how these influence the outcome. Games can be described formally at various levels of detail. A coalitional (or cooperative) game is a high-level description, specifying only what payoffs each potential group, or coalition, can obtain by the cooperation of its members. What is not made explicit is the process by which the coalition forms. As an example, the players may be several parties in parliament. Each party has a different strength, based upon the number of seats occupied by party members. The game describes which coalitions of parties can form a majority, but does not delineate, for example, the negotiation process through which an agreement to vote en bloc is achieved. Cooperative game theory investigates such coalitional games with respect to the relative amounts of power held by various players, or how a successful coalition should divide its proceeds. This is most naturally applied to situations arising in political science or international relations, where concepts like power are most important. For example, Nash proposed a solution for the division of gains from agreement in a bargaining problem which depends solely on the relative strengths of the two parties' bargaining position. The amount of power a side has is determined by the usually inefficient outcome that results when negotiations break down. Nash's model fits within the cooperative frame work in that it does not delineate a specific timeline of offers and counteroffers, but rather focuses solely on the outcome of the bargaining process.

A. Some definition

1) Game:

A game is a formal description of a strategic situation.

2) Game theory:

Game theory is the formal study of decision-making where several players must make choices that potentially affect the interests of the other players.

3) Mixed strategy:

A mixed strategy is an active randomization, with given probabilities, that determines the player's decision. As a special case, a mixed strategy can be the deterministic choice of one of the given pure strategies.

4) Nash equilibrium:

A Nash equilibrium, also called strategic equilibrium, is a list of strategies, one for each player, which has the property that no player can unilaterally change his strategy and get a better payoff.

5) Payoff:

A payoff is a number, also called utility, that reflects the desirability of an outcome to a player, for whatever reason. When the outcome is random, payoffs are usually weighted with their probabilities. The expected payoff incorporates the player's attitude towards risk.

6) Perfect information:

A game has perfect information when at any point in time only one player makes a move, and knows all the actions that have been made until then.

7) Player:

A player is an agent who makes decisions in a game.

8) Rationality:

A player is said to be rational if he seeks to play in a manner which maximizes his own payoff. It is often assumed that the rationality of all players is common knowledge.

9) Strategic form:

A game in strategic form, also called normal form, is a compact representation of a game in which players simultaneously choose their strategies. The resulting payoffs are presented in a table with a cell for each strategy combination.

10) Strategy:

In a game in strategic form, a strategy is one of the given possible actions of a player. In an extensive game, a strategy is a complete plan of choices, one for each decision point of the player.

11) Zero-sum game:

A game is said to be zero-sum if for any outcome, the sum of the payoffs to all players is zero. In a two-player zero-sum game, one player's gain is the other player's loss, so their interests are diametrically opposed.

Repeatation in zero sum game-The theory of repeated games explores how mutual help and cooperation are sustained through repeated interaction, even when economic agents are completely self-interested beings. This thesis analyzes two models that involve repeated interaction in an environment where some information is private. In the rst chapter, we characterize the equilibrium set of the following game. Two players interact repeatedly over an innite horizon and occasionally, one of the players has an opportunity to do a favor to the other player. The ability to do a favor is private information and only one of the players is in a position to do a favor at a time. The cost of doing a favor is less than the benet to the receiver so that, always doing a favor is the socially optimal outcome. Intuitively, a player who develops the ability to do a favor in some period might have an incentive to reveal this information and do a favor if she has reason to expect future favors in return. We show that the equilibrium set expands monotonically in the likelihood that someone is in a position to do a favor. It also expands with the discount factor. However, there are no fully e¢ cient equilibria for any discount factor less than unity. We nd su¢ cient conditions under which equilibria on the Pareto frontier of the equilibrium set are supported by e¢ cient payo⁄s. We also provide a partial characterization of payo⁄s on the frontier in terms of the action proles that support them.

In two person zero sum game if a player apply this rules, then he will always get profit. Let A and B are two player and they are playing 2 person zero sum game. They used to toss a coin, A choose head and B choose tail. They bet that if head comes then B will give 100 ru. To A and if tail comes then A will give 100ru. to B. If first times toss and tail come then B win and a loss 100ru. And again this bet repeats. If A want to win then A should bet of 300ru. So that he can recover their previous money (100ru.) and take profit of 200 ru. and 2nd time tossed and again tail comes then B win again. Now A is lost of ru.100+300=400. If A want to win then a should follow N= 3n-1*p rules.

Whenever A follow this rule if head will come then A will be always in profit. By using this repeatation method in zero sum game player can increase the probability of profit.
Example: by using this formula N=3n-1*p
Where N= number of opportunities (N=1,2,3,4,5,……..)
n = natural number (n=1,2,3,4,5,…………)
p= principal amount
if principal amount is 100

N =	$N=3^{n-1}*p$	If loss	Finally profit
1.	3^{1-1}*100=100	=100	100
2.	3^{2-1}*100=300	=100+300=400	300-100=200
3.	3^{3-1}*100=900	=900+400=1300	900-400=500
4.	3^{4-1}*100=2700	=2700+1300=4000	2700-1300=1400
5.	3^{5-1}*100=8100	=8100+400=12100	8100-4000=4100
……	……………………….…	……………………….…	……………………….…
……	……………………….…	……………………….…	……………………….…
……	……………………….…	……………………….…	……………………….…

IV. Conclusion

With the help of the mathematical formula a player can increase the probability of winning, but it required a lot of money, passion, continuity, and patience. In a two person model, the meaning of exchanging favors is very clear. With more than two players, when in a position to do a favor, it is not so clear whom a player will provide a favor to. This will require careful modelling with respect to the values of favors from di⁄erent opponents and the cost of doing favors to di⁄erent opponents. If we assume that the benet and cost are identical for all players, we will still have to incorporate in the strategies some rules on how favors are done. For example, a player might do one favor for each opponent before doing any second favors. With an appropriate generalization to the n-player case, it is reasonable to still expect the comparative statics results for the equilibrium set that we see in the current model. It is harder to say what the equilibrium strategies for the Pareto frontier will look like.

References

[1] Theodore L. Turocy Texas A&M University Bernhard von Stengel London School of Economics CDAM Research Report LSE-CDAM-2001-09 October 8, 2001
[2] Erich Prisner Franklin University Switzerland
[3] Christos H. Papadimitriou].Shivani Nayyar, ESSAYS ON REPEATED GAMES, Thomas Palfrey, University of California, Berkeley
[4] Shivani Nayyar, ESSAYS ON REPEATED GAMES, Thomas Palfrey, June 2009
[5] Climate Change and Game Theory Peter John WoodMay 2010
[6] Patrick Bajari,University of Minnesota and NBER, December 3, 2010
[7] Rachel F. Baskerville, Wellington, NewZealand 2007
[8] Patrick Bajari University of Minnesota and NBER, Han Hong,Stanford University December 3, 2010
[9] Joseph Y. Halpern∗ Cornell University,May 18, 2007
[10] Peter John Wood, THE AUSTRALIAN NATIONAL UNIVERSITY, May 2010
[11] Colin F. Camerer,1 Teck-Hua Ho and Juin Kuan Chong
[12] Juin Kuan Chong National University of Singapore, November 14, 2001