VSGb-Closed Sets in Vague Topological Spaces

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Abstract— In this paper, we introduce vague sgB closed sets and study some of their properties in intuitionistic fuzzy topological spaces.
Key words: Fuzzy Sets, Vague Sets

I. INTRODUCTION

The concept of fuzzy sets was introduced by Zadeh [11] in 1965. The idea of fuzzy set is welcome because it handles uncertainty and vagueness which Cantorian set could not address. In fuzzy set theory, the membership of an element to a fuzzy set is a single value between zero and one. The theory of fuzzy topology was introduced by C.L. Chang [5] in 1967; several researches were conducted on the generalizations of the notions of fuzzy sets and fuzzy topology. In 1986, the concept of intuitionistic fuzzy sets was introduced by Atanassov [2] as a generalization of fuzzy sets.

The theory of vague sets was first proposed by Gau and Buehre [6] as an extension of fuzzy set theory and vague sets are regarded as a special case of context-dependent fuzzy sets. The basic concepts of vague set theory and its extensions defined by [4,6].In this paper we introduce the concept of vague semi generalized b-closed sets and vague semi generalized b-open sets and their properties are obtained. Also as an application we have defined vague T *1/2 spaces and vague T**1/2 spaces.

II. Preliminaries

A. Definition 2.1

[3] A vague set A in the universe of discourse X is characterized by two membership functions given by:

1) A true membership function tA : X → [0, 1] and
2) (b) A false membership function fA : X → [0, 1].

where tA(x) is lower bound on the grade of membership of x derived from the ”evidence for x”, fA(x) is lower on the negation of x derived from the

”evidence for x” and $tA(x) + fA(x) \leq 1$. Thus the grade of membership of x in the vague set A is bounded by a subinterval $[tA(x), 1 - fA(x)]$ of [0,1]. This indicates that if the actual grade of membership μ(x), then $tA(x) \leq \mu(x) \leq 1 - fA(x)$. The vague set A is written as $A = \{ \langle x, [tA(x), 1 - fA(x)] \rangle / x \in X\}$ where the interval $[tA(x), 1 - fA(x)]$ is called the ”vague value” of x in A and is denoted by VA(x).

B. Definition 2.2

[3] Let A and B be VSs of the form $A = \{ \langle x, [tA(x), 1-fA(x)] / x \in X\}$ and $B = \{ \langle x,[tB(x),1 - fB(x)] \rangle / x \in X\}$. Then

1) $A \subseteq B$ if and only if $tA(x) \leq tB(x)$ and $1 - fA(x) \leq 1 - fB(x)$ for all x∈X.
2) A = B if and only if $A \subseteq B$ and $B \subseteq A$.
3) $Ac = \{ \langle x, [fA(x), 1 - tA(x)] \rangle / x \in X\}$.
4) $A \cap B = \{ \langle x, [(tA(x) \wedge tB(x)), ((1 - fA(x)) \wedge (1 - fB(x)))] \rangle / x \in X\}$

(e) $A \cup B = \{ \langle x, [(tA(x) \vee tB(x)), ((1 - fA(x)) \vee (1 - fB(x)))] \rangle / x \in X\}$

For the sake of simplicity, we shall use the notion $A = \{ \langle x, [tA(x), 1 - fA(x)] \rangle \}$ instead of $A = \{ \langle x, [tA(x), 1 - fA(x)] \rangle / x \in X\}$.

C. Definition 2.3

Let (X, τ) be a topological space.A subset A of X is called:

1) b- closed set [10] (bCS) if $cl(int(A)) \cap int(cl(A)) \subseteq A$,
2) α-closed set [9] (αCS) if $cl(int(cl(A))) \subseteq A$,
3) semi-closed [7] (SCS) if $int(cl(A)) \subseteq A$.

D. Definition 2.4

Let (X, τ) be a topological space.A subset A of X is called:

1) generalized αclosed set [10] (GαCS) if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is α open in X,
2) α generalized semi closed set [14] (αGSCS) if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi open in X,
3) w closed set [8] (wCS) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi open in X,
4) π generalized β closed sets[6] (πGβCS) if $\beta cl(A) \subseteq U$ whenever $A \subseteq U$ and U is π open in X.

E. Definition 2.5

[11] A vague topology (VT in short) on X is a family τ of vague sets (VS in short) in X satisfying the following axioms.

1) 0, 1 ∈ τ
2) $G1 \cap G2 \in \tau$, for any G1, G2 ∈ τ
3) $\cup Gi \in \tau$ for any family $\{Gi/i \in J \} \subseteq \tau$.

In this case the pair (X, τ) is called a vague topological space (VTS in short) and any VS in τ is known as a vague open set (VOS in short) in X.

The complement Ac of a VOS in a VTS (X, τ) is called a vague closed set (VCS in short) in X.

F. Definition 2.6:

[11] Let (X, τ) be a VTS and $A = \{ \langle x, [tA, 1 - fA] \rangle \}$ be a VS in X . Then the vague interior and a vague closure are defined by

1) $vint(A) = \cup\{G/G$ is a VOS in X and $G \subseteq A\}$,
2) $vcl(A) = \cap\{K/K$ is a VCS in X and $A \subseteq K\}$.

Note that for any VS A in (X, τ), we have vcl(Ac) = (vint(A))c and vint(Ac) = (vcl(A))c.

Definition 2.7

A VS $A = \{ \langle x, [tA, 1 - fA] \rangle \}$ in a VTS (X, τ) is said to be a

1) vague b- closed set (vbCS)[14] if $vcl(vint(A)) \cap vint(vcl(A)) \subseteq A$,
2) vague α-closed set (vαCS)[11] if $vcl(vint(vcl(A))) \subseteq A$,
3) vague semi-closed (vSCS)[11] if $vint(vcl(A)) \subseteq A$.

G. Definition 2.8

A VS A = { 〈x, [tA, 1 − fA]〉 } in a VTS (X, τ) is said to be a

1) vague b- open set (vbOS)[14] if A ⊆ vint(vcl(A)) ∪ vcl(vint(A)),
2) vague α-open set (vαOS)[11] if A ⊆ vint(vcl(vint(A))),
3) vague semi-open (vSOS)[11] if A ⊆ vcl(vint(A)).

H. Definition 2.9

[11] Let A be a VS of a VTS (X, τ). Then the vague α interior of A(vαint(A) in short) and vague α closure of A(vαcl(A) in short) are defined by

1) vαint(A) = ∪{G/G is a VαOS in X and G ⊆ A},
2) vαcl(A) = ∩{K/K is a VαCS in X and A⊆K}.

I. Definition 2.10

[11] Let A be a VS of a VTS (X, τ), then

1) vαcl(A) = A ∪ vcl(vint(vcl(A)))
2) vαint(A) = A ∩ vint(vcl(vint(A))).

J. Definition 2.11

[11] Let A be a VS of a VTS (X, τ). Then the vague semi interior of A(vsint(A) in short) and vague semi closure of A(vscl(A) in short) are defined by

1) vsint(A) = ∪{G/G is a VSOS in X and G ⊆ A},

K. vscl(A) = ∩{K/K is a VSCS in X and A⊆K}. Definition 2.12

[11] Let A be a VS of a VTS (X, τ),then

1) vscl(A) = A ∪ vint(vcl(A))
2) vsint(A) = A ∩ vint(vcl(A)).

L. Definition 2.13

A VS A of a VTS (X, τ) is said to be a vague generalized α-closed set [10] (V GαCS) if vαcl(A) ⊆ U whenever A ⊆ U and U is vαOS in X.

M. Definition 2.14

A VS A of a VTS (X, τ) is said to be a vague α generalized semi closed set [7] (V αGSCS) if vαcl(A) ⊆ U whenever A ⊆ U and U is V SOS in X.

N. Definition 2.15

A VS A of a VTS (X, τ) is said to be a vague w closed set [8] (V wCS) if vcl(A) ⊆ U whenever A ⊆ U and U is V SOS in X.

O. Definition 2.16

A VS A of a VTS (X, τ) is said to be a vague π generalized β closed sets[6] (V πGβCS) if vβcl(A) ⊆ U whenever A ⊆ U and U is V πOS in X.

P. Definition 2.17

[14] Let A be a VS of a VTS (X, τ). Then the vague b interior of A(vbint(A) in short) and vague semi closure of A(vbcl(A) in short) are defined by

1) vbint(A) = ∪{G/G is a VbOS in X and G ⊆ A},
2) vbcl(A) = ∩{K/K is a VbCS in X and A ⊆ K}.

III. VAGUE SGB CLOSED SETS

A. Definition 3.1

A VS A is said to be vague semi generalized b-closed set (VSG-bCS) in (X,τ) if vbcl(A) ⊆ U whenever A ⊆ U and U is a VSOS in X.

The family of all VSGbCSs of a VTS(X,τ) is denoted by VSGbC(X).

B. Example 3.2

If X = {a,b}. Let τ = {0,G,1} is a VT on X , where G = { 〈 x,[0.3,0.6],[0.2,0.5] 〉 }.Consider the VS A = { 〈 x, [0.2,0.7], [0.4,0.6] 〉 }. Then A is a VSGbCS in X .

C. Theorem 3.3

Every VCS is a VSGbCS.

Proof: Assume that A is a VCS in X. Let A ⊆ U and U is a VSOS in X. vbcl(A) ⊆ vcl(A) = A ⊆ U ,since vbcl(A) ⊆ vcl(A) and A is a VCS in X, Hence A is a VSGbCS in X.

D. Example 3.4

If X = { a,b }. Let τ = {0,G,1} is a VT on X , where G = { 〈 x,[0.2,0.5],[0.3,0.7] 〉 }.Consider the VS A = { 〈 x, [0.3,0.6], [0.4,0.5] 〉 }.Then A is a VSGbCS in X but A is not a VCS in X.

E. Theorem 3.5

Every VαCS is a VSGbCS but not conversely.

Proof: Assume that A be a VαCS in X.Let A ⊆ U and U is a VSOS in X. vbcl(A) ⊆ vαcl(A) = A ⊆ U , since vbcl(A) ⊆ vαcl(A) and A is a VαCS in X. Hence A is a VSGbCS in X.

F. Example 3.6

If X = { a,b }. Let τ = {0,G,1} is a VT on X , where G = { 〈 x,[0.3,0.5],[0.4,0.5] 〉 }.Consider the VS A = { 〈 x, [0.2,0.7],[0.3,0.6] 〉 }.Then A is a VSGbCS in X but A is not a VαCS in X.

G. Theorem 3.7

Every VGαCS is a VSGbCS but not conversely.

Proof: Assume that A be a VGαCS in X. Let A ⊆ U and U is a VSOS in X. vbcl(A) ⊆ vαcl(A) ⊆ U ,since vbcl(A) ⊆ vαcl(A) and A is a VGαCS in X. Hence A is a VSGbCS in X.

H. Example 3.8

If X = { a,b }.Let τ = {0,G,1} is a VT on X , where G = { 〈 x,[0.4,0.6],[0.3,0.6] 〉 }.Consider the VS A = { 〈 x, [0.1,0.6],[0.2,0.7] 〉 }.Then A is a VSGbCS in X but A is not a VGαCS in X.

I. Theorem 3.9

Every VαGSCS is a VSGbCS but not conversely.

Proof: Assume that A be a VαGSCS in X. Let A ⊆ U and U is a VSOS in X. vbcl(A) ⊆ vαcl(A) ⊆ U ,since vbcl(A) ⊆ vαcl(A) and A is a VαGSCS in X. Hence A is a VSGbCS in X.

J. Example 3.10

If X = { a,b }. Let τ = {0,G,1} is a VT on X , where G = { 〈 x,[0.3,0.5],[0.3,0.4] 〉 }.Consider the VS A = { 〈 x, [0.3,0.4],[0.5,0.6] 〉 }.Then A is a VSGbCS in X but A is not a VαGSCS in X.

K. Theorem 3.11

Every VW CS is a VSGbCS but not conversely.
Proof: Assume that A be a VW CS in X. Let A ⊆ U and U is a VSOS in X. vbcl(A) ⊆ vcl(A) ⊆ U ,since vbcl(A) ⊆ vcl(A) and A is a VW CS in X. Hence A is a VSGbCS in X.

L. Example 3.12

If X = { a,b }. Let τ = {0,G,1} is a VT on X , where G = { 〈 x,[0.4,0.5],[0.5,0.6] 〉 }.Consider the VS A = { 〈 x,[0.2,0.5],[0.4,0.5] 〉 }.Then A is a VSGbCS in X but A is not a VW CS in X.

M. Theorem 3.13

Every VSGCS is a VSGbCS but not conversely.
Proof: Assume that A be a VSGCS in X.Let A ⊆ U and U is a VSOS in X. vbcl(A) ⊆ vscl(A) ⊆ U ,since vbcl(A) ⊆ vscl(A) and A is a VSGCS in X. Hence A is a VSGbCS in X.

N. Example 3.14

If X = { a,b }.Let τ = {0,G,1} is a VT on X , where G = { 〈 x,[0.3,0.8],[0.4,0.9] 〉 }.Consider the VS A = { 〈 x,[0.2,0.6],[0.3,0.7] 〉 }.Then A is a VSGbCS in X but A is not a VSGCS in X.

O. Theorem 3.15

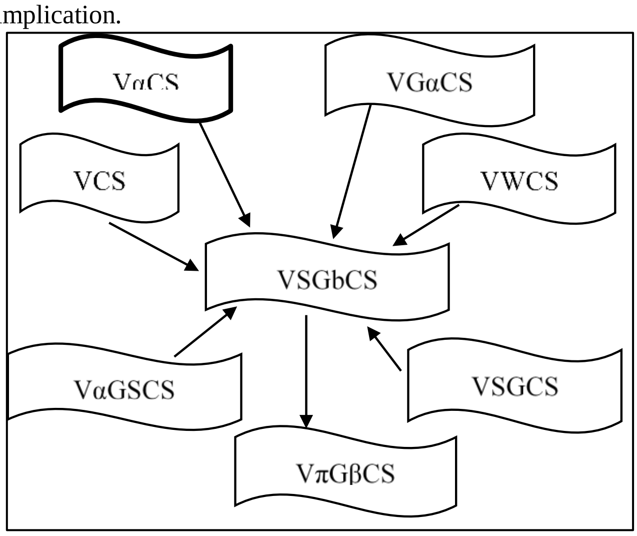
Every VSGbCS is a VπGβCS but not conversely.
Proof: Assume that A be a VSGbCS in X.Let A ⊆ U and U is a VπOS in X. vβcl(A) ⊆ vbcl(A) ⊆ U,
since vβcl(A) ⊆ vbcl(A) and A is a VSGbCS in X. Hence A is a VπGβCS in X.

P. Example 3.16

If X = { a,b }. Let τ = {0,G,1} is a VT on X , where G = { 〈 x,[0.4,0.7],[0.3,0.6] 〉 }.Consider the VS A = { 〈 x,[0.5,0.8],[0.3,0.7] 〉 }.Then A is a VπGβCS in X but A is not a VSGbCS in X.

Q. Remark 3.17

Summing up the above theorems, we have the following implication.



In this diagram by A → B we mean A implies B but not conversely.

R. Remark 3.18

The union of any two VSGbCSs is not a VSGbCS in general as seen in the following example.

S. Example 3.19

If X = { a,b }. Let τ = {0,G,1} is a VT on X , where G = { 〈 x,[0.3,0.8],[0.4,0.9] 〉 }.Consider the VSs A = { 〈 x, [0.2,0.6],[0.3,0.7] 〉 } and B = { 〈 x, [0.2,0.6],[0.3,0.7] 〉 }.Then A and B are VSGbCSs in X but A ∪ B is not VSGbCS in X.

T. Theorem 3.20

Let VSGbCS A in (X,τ) such that A ⊆ B ⊆ vbcl(A) then B is VSGbCSin (X,τ).

Proof: Assume that B is vague set in VTS (X,τ) such that B ⊆ U and U is VSOS in X. Then A ⊆ U . vbcl(A) ⊆ U , since A is VSGbCS. We have cbcl(A) ⊆ vbcl(vbcl(A)) = vbcl(A) ⊆ U . Therefore B is VSGbCS in X.

U. Theorem 3.21

Let VbO and VSGbC in VTS A in (X, τ) then A is VbC in (X, τ).

Proof: Asuume that A is VbO and VSGbC in (X, τ), vbcl(A) ⊆ A. But A ⊆ vbcl(A).Hence vbcl(A) = A.Therefore A is VbC in (X, τ).

IV. Vague SGb Open Sets

A. Definition 4.1

A VS A is said to be a vague semi generalized b-open set (VSG-bOS) in (X, τ) if the complement Ac is a VSGbCS in X . The family of all VSGbOSs of a VTS (X, τ) is denoted by VSGbO(X).

B. Example 4.2

If X = { a,b }. Let τ = {0,G,1} is a VT on X , where G = { 〈 x,[0.3,0.6],[0.2,0.5] 〉 }.Consider the VS A = { 〈 x, [0.3,0.8], [0.4,0.6] 〉 }. Then A is a VSGbOS in X .

C. Theorem 4.3

(1) Every VOS is a VSGbOS, (2) Every VαOS is a VSGbOS, (3) Every VGαOS is a VSGbOS, (4) Every VαGSOS is a VSGbOS, (5) Every VW OS is a VSGbOS, (6) Every VSGOS is a VSGbOS and (7) Every VSGbOS is a VπGβOS.But the converses are not true in general.
Proof: Straight forward.

D. Example 4.4:

If X = { a,b } . Let τ = {0,G,1} is a VT on X , where G = { 〈 x,[0.2,0.5],[0.3,0.7] 〉 }.Consider the VS A = { 〈 x, [0.4,0.7], [0.5,0.6] 〉 }.Then A is a VSGbOS in X but A is not a VOS in X.

E. Example 4.5:

If X = { a,b }. Let τ = {0,G,1} is a VT on X , where G = { 〈 x,[0.3,0.5],[0.4,0.5] 〉 }.Consider the VS A = { 〈 x, [0.3,0.8],[0.4,0.7] 〉 }.Then A is a VSGbOS in X but A is not a VαOS in X.

F. Example 4.6

If X = { a,b }. Let τ = {0,G,1} is a VT on X , where G = { 〈 x,[0.4,0.6],[0.3,0.6] 〉 }.Consider the VS A = { 〈 x, [0.4,0.9],[0.3,0.8] 〉 }.Then A is a VSGbOS in X but A is not a VGαOS in X.

G. Example 4.7

If X = { a,b }. Let τ = {0,G,1} is a VT on X , where G = { 〈 x,[0.3,0.5],[0.3,0.4] 〉 }.Consider the VS A = { 〈 x, [0.6,0.7],[0.4,0.5] 〉 }.Then A is a VSGbOS in X but A is not a VαGSOS in X.

H. Example 4.8

If X = { a,b }. Let τ = {0,G,1} is a VT on X , where G = { 〈 x,[0.4,0.5],[0.5,0.6] 〉 }.Consider the VS A = { 〈 x, [0.5,0.8],[0.5,0.6] 〉 }.Then A is a VSGbOS in X but A is not a VW OS in X.

I. Example 4.9

If X = { a,b }. Let τ = {0,G,1} is a VT on X , where G = { 〈 x,[0.3,0.8],[0.4,0.9] 〉 }.Consider the VS A = { 〈 x, [0.4,0.8],[0.3,0.7] 〉 }.Then A is a VSGbOS in X but A is not a VSGOS in X.

J. Example 4.10

If X = { a,b }. Let τ = {0,G,1} is a VT on X , where G = { 〈 x,[0.4,0.7],[0.3,0.6] 〉 }.Consider the VS A = { 〈 x, [0.2,0.5],[0.3,0.7] 〉 }. Then A is a VπGβOS in X but A is not a VSGbOS in X.

K. Remark 4.11

The intersection of any two VSGbOSs is not a VSGbOS in general as seen in the following example.

L. Example 4.12:

If X = { a,b }. Let τ = {0,G,1} is a VT on X , where G = { 〈 x,[0.6,0.8],[0.2,0.4] 〉 }.Consider the VSs A = { 〈 x, [0.2,0.7],[0.1,0.8] 〉 } , B = { 〈 x, [0.3,0.4],[0.6,0.7] 〉 }. Then A and B are VSGbOSs in X but A ∪ B is not VSGbOS in X.

M. Theorem 4.13

If (X, τ) is a VTS. Let A ∈ V SGbO(X) then V ⊆ vint(vcl(A)) ∪ vcl(vint(A)) whenever V ⊆ A and V is VSCS in X .

Proof: Consider A ∈ V SGbO(X). This implies Ac is a VSGbCS in X . Hence vbcl(Ac) ⊆ U whenever Ac ⊆ U and U is a VSOS in X . That is vcl(vint(Ac)) ∩ vint(vcl(Ac)) ⊆ U .Then Uc ⊆ vint(vcl(A)) ∪ vcl(vint(A)) whenever Uc ⊆ A and Uc is VSCS in X .Replacing Uc by V , we have V ⊆ vint(vcl(A)) ∪ vcl(vint(A)) whenever V ⊆ A and V is VSCS in X .

N. Theorem 4.14

If (X, τ) is a VTS. Then for every A ∈ V SGbO(X) and for every B ∈ V S(X), vbint(A) ⊆ B ⊆ A implies B ∈ V SGbO(X).
Proof: Assume that Ac ⊆ Bc ⊆ (vbint(A))c and let Bc ⊆ U and U be a VSOS. Since Ac ⊆ Bc, Ac ⊆ U .But Ac is a VSGbCS, vbcl(Ac) ⊆ U and Bc ⊆ (vbint(A))c = vbcl(Ac). Hence vbcl(Bc) ⊆ vbcl(Ac) ⊆ U . Therefore Bc is a VSGbCS. Which implies B is a VSGbOS of X.

1) Theorem 4.15

A VS A a VTS (X, τ)and A is a VSGbOS if and only if F ⊆ vbint(A) whenever F is a VSCS and F ⊆ A.

Proof: Necessity: Assume that A is a VSGbOS in X. Consider F be a VSCS and F ⊆ A.This implies Fc is a VSOS in X such that Ac ⊆ Fc . We have vbcl(Ac) ⊆ Fc, since Ac is a VSGbCS. Therefore (vbint(A))c ⊆ Fc . Hence F ⊆ vbint(A).

Sufficiency: Consider A be a VS of X. Let F ⊆ vbint(A) whenever F is a VSCS and F ⊆ A.This implies Ac ⊆ Fc and Fc is a VSOS. By the assumption, (vbint(A))c ⊆ Fc . which implies

vbcl(Ac) ⊆ Fc . Hence Ac is a VSGbCS of X . Therefore A is a VSGbOS of X .

O. Theorem 4.16

A VS A of a VTS (X, τ) and A is a VSGbOS if and only if F ⊆ vint(vcl(A)) ∪ vcl(vint(A)) whenever F is a VSCS and F ⊆ A.

Proof: Necessity: Assume that A is a VSGbOS in X. Consider F be a VSCS and F ⊆ A.This implies Fc is a VSOS in X such that Ac ⊆ Fc. We have vbcl(Ac) ⊆ Fc, since Ac is a VSGbCS

Hence vcl(vint(Ac)) ∩ vint(vcl(Ac)) ⊆ Fc . Therefore vint(vcl(A)) ∪ vcl(vint(A))c ⊆ Fc. Then F⊆vint(vcl(A))∪vcl(vint(A)).

Sufficiency: Consider A be a VS of X.Let F ⊆ vint(vcl(A)) ∪ vcl(vint(A)) whenever F is a VSCS and F ⊆ A.This implies Ac ⊆ Fc and Fc is a VSOS. By the assumption, (vint(vcl(A)) ∪ vcl(vint(A)))c ⊆ Fc. Therefore vcl(vint(Ac)) ∩ vint(vcl(Ac)) ⊆ Fc.Then vbcl(Ac) ⊆ Fc .
Therefore A is a VSGbOS of X .

V. APPLICATIONS OF VAGUE SGB CLOSED SETS

A. Definition 5.1

A VTS (X, τ) is said to be a vague T∗1/2 (VT* 1/2) space if every VSGbCS in X is a VCS in X.

B. Definition 5.2

A VTS (X, τ) is said to be a vague T∗∗1/2 (VT∗∗1/2) space if every VSGbCS in X is a VbCS in X.

C. Theorem 5.3

Every VT*1/2 space is a VT** ½
Proof. Assume that X be a V T∗1/2 space. Consider A be a VSGbCS in X. A is a VCS in X.A is a VbCS in X, since every VCS is a VbCS . Therefore X is a V T∗∗ 1/2 space.
The converse of the above theorem need not be true as seen from the following Example.

D. Example 5.4

If X = { a,b }. Let τ = {0,G,1} is a VT on X , where G = { 〈 x,[0.2,0.5],[0.3,0.7] 〉 }.Consider the VS A = { 〈 x, [0.3,0.6],[0.4,0.5] 〉 }.Since A is VSGbCS in X but A is not a VCS in X. Hence (X, τ) is a V T∗∗1/2 space, but (X,τ) is not a VT*1/2 space.

E. Theorem 5.5

If (X, τ) is a VTS and X is a T*1/2 space then,

1) Any union of VSGbCSs is a VSGbCS.
2) Any intersection of VSGbOSs is a VSGbOS.

Proof: (i) Assume that {Ai}i∈J is a collection of VSGbCSs in a T∗1/2 space (X,τ). Hence every VSGbCS is a VCS. But the union of VCS is a VCS.Therefore the union of VSGbCS is a VSGbCS in X.

(ii) Assume that {Ai}i∈J is a collection of VSGbOSs in a T∗ 1/2 space(X,τ). Hence every VSGbOS is a VOS. But the intersection of VOS is a VOS.Therefore the intersection of VSGbOS is a VSGbOS in X.

F. Theorem 5.6

A VTS X is a V T**1/2 space if and only if VSGbO(X)=VbO(X).

Proof: Necessity: Assume that A be a VSGbOS in X. This implies Ac is a VSGbCS in X . Hence A is a VbOS in X .Therefore V SGbO(X) = V bO(X).

Sufficiency: Consider A is a VSGbCS in X . This implies Ac is a VSGbOS in X. Hence A is a VbCS in X. Therefore X is a V T∗∗ 1/2 space.

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