Assessment of Blast Pressure using Cantilevers

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Abstract— History has shown that the effects of explosion targeted at civilian and military facilities cause widespread damage to infrastructure, inflict human casualties and generate confusion in peacetime. Although such events are tragic, a detailed study of the post-detonation effects on structures offers valuable lessons for engineers. One simple device generally overlooked by investigators is the response of cantilever structures such as lamp-posts or traffic sign-posts after an explosion has occurred. These are vital structures for the engineer to determine weapon yield. In this way, the integrity of the damaged structure and other undamaged buildings in the vicinity can be assessed. This paper outlines a preliminary study to identify a passive economical mode of assessing blast wave characteristics using a simple solid circular aluminium cantilever. A review of two conventional approaches to assess the blast overpressure from an explosion using a cantilever is presented. The development of mathematical formulations and design of simple cantilevers to measure air-blast pressure is discussed. A comparative analysis is provided to enable designers to better appreciate the significance of this analytical tool in blast pressure assessment.

Key words: Dynamic Loading, Pressure-Impulse, Quasi-Static Loading, Strain Energy, Kinetic Energy

I. INTRODUCTION

Historical data are important sources of information. In one of the earliest records on weapons output, Lord Penny[1] determined the effective energy yields of the two nuclear weapon explosions at Hiroshima and Nagasaki in 1945. He observed the damage that occurred to various simple structures such as bent poles, toppled grave stones, crushed paint cans and broken glass windows in the vicinity of the explosion. It occurred to him that a simple cantilever structure can effectively gauge the blast overpressure. This research was subsequently extended by various investigators in the development of various types of passive gauges. These simple gauges were used to assess the properties of blast wave by the various failure modes. Ewing, et. al.[2] and Baker, et. al.[3] developed cantilever gauges with rectangular cross-section to assess the effects of shockwaves, and Dewey[4] considered the effect of TNT and ammonium nitrate explosions on the deformations of solder cantilevers.

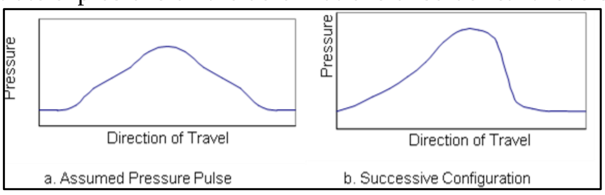

Fig. 1: Variation of Pressure with Time

The net load on a structure subjected to air-blast depends on the physical properties of the blast wave as well as the shape and size of the resisting form. An important feature of any large explosion in air is the blast wave it generates by forcibly pushing the surrounding atmosphere outwards due to the sudden release of energy. The formation of the blast wave may be illustrated in Fig.1, which represents a pressure pulse imparting an arbitrarily initial configuration.

The response of structures to such dynamic loads has been studied extensively and a large volume of literature is available which deals with this problem in a comprehensive manner. In the literature, Thompson[5], Irvine[6], Berg[7] and Paz[8] have been in the forefront of such research. Further, to study the response of a cantilever, it is necessary to quantify the dynamic pressure-time history, including the effect of drag on a structural configuration. Hence, drag coefficient, which is a function of the Mach and/or Reynolds number of the flow, has to be established. A number of effects that could influence the response of a cantilever have been identified but these are not considered in the following analysis.

II. RESPONSE OF A CANTILEVER

A. *Impulse technique*

The impulse technique is based on the Rayleigh-Ritz method of analysis. In this procedure, the deformed shape is expressed as a mathematical equation, which satisfies all the necessary boundary conditions relating to the displacement. By operating on the deformed shape, the curvature and hence the strain of deformation is obtained, and from which strain energy per unit volume of material can be evaluated. By integrating over the entire element, the total strain energy can be calculated.

Consider next the nature of the blast load acting on the structure. If the loading is determined as being impulsive, then a calculation of total kinetic energy delivered to the structure is made. If, however, the load is quasi-static in nature, then the work done by the load is found by considering the work done on a small element of the structure and then integrating over the loaded area. In the impulsive regime, the response of the cantilever is evaluated by equating the imparted kinetic energy to the stored strain energy in the structure. In the quasi-static loading case, the response is assessed by equating the work done by the load to the stored strain energy. Thus, the response of a cantilever (maximum displacement, strains and stresses) can be evaluated.

Consider the cantilever shown in Fig. 2. To calculate the maximum deflection and maximum bending strain experienced by the structure, a deformed shape is assumed which satisfies the boundary conditions.

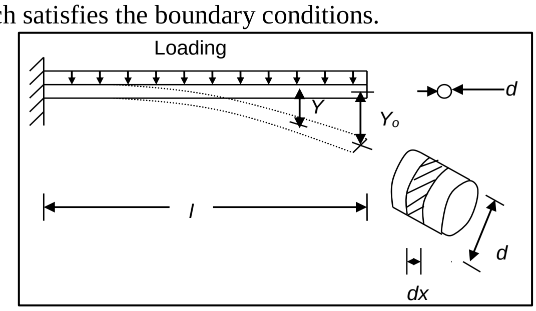

Fig. 2: Impulsively Loaded Elastic Cantilever

Consider the deformation of a cantilever in the form of a parabola, given by:

$$Y = Y_0\left[\frac{62.65}{12}\left(\frac{x}{l}\right)^2 - 11\left(\frac{x}{l}\right)^3 + \frac{20.35}{3}\left(\frac{x}{l}\right)^4\right] \quad (1)$$

Where Y is the displacement at distance x from the built-in end of the cantilever of length l, and Y_o is the maximum displacement at the free end of the cantilever.

If the cantilever is slender, it may be assumed that the deformed structure acquires strain energy due to bending alone. Thus, the elastic beam bending equation is:

$$\frac{M}{I} = \frac{E}{R} = \frac{\sigma}{Y} \quad (2)$$

Where M is bending moment, I is second moment of area of the section about the neutral axis and E is Young's modulus for the beam material. R is the radius of curvature given by (approximately):

$$R \approx \frac{1}{\left[\frac{d^2Y}{dx^2}\right]} \quad (3)$$

The stress, σ, is the direct stress taken at a distance y from the neutral axis of the cross-section. In this case:

$$\frac{1}{R} = \frac{62.65Y_0^2}{l^2}\left[\left(\frac{1}{6}\right) - 1.053\left(\frac{x}{l}\right) + 1.3\left(\frac{x}{l}\right)^2\right] \quad (4)$$

Thus, the bending moment can be expressed as

$$M = \frac{EI}{R} = EI\frac{d^2Y}{dx^2} = \frac{62.65\,EIY_0^2}{l^2}\left[\left(\frac{1}{6}\right) - 1.053\left(\frac{x}{l}\right) + 1.3\left(\frac{x}{l}\right)^2\right] \quad (5)$$

The strain energy in bending 'U' can be evaluated by expressing the strain energy per unit volume for an element of the beam of length dx as:

$$\frac{dU}{dV} = \int \sigma d\varepsilon = \int \frac{\sigma}{E} d\sigma = \frac{\sigma^2}{2E} = \frac{M^2Y^2}{2I^2E} \quad (6)$$

If the element has cross-sectional area dA and is of length dx then the elemental volume $dV = dA \times dx$. Summing up the entire cross-section, the strain energy U can be expressed as:

$$U = \int \frac{M^2Y^2}{2I^2E} dA.dx = \int \frac{M^2I}{2I^2E} dx = \int \frac{M^2}{2EI} dx \quad (7)$$

Since by definition

$$\int Y^2 dA = I \quad (8)$$

Then, integrating over the entire length of the beam gives

$$U = \int_0^l \frac{M^2}{2EI} dx = \int_0^l \frac{E^2I^2}{2EI}\left[\frac{d^2Y}{dx^2}\right]^2 dx$$

$$= \frac{EI}{2}\int_0^l \left[\frac{d^2Y}{dx^2}\right]^2 dx = \frac{1962\,EIY_0^2}{l^4}\int_0^l \left[\left(\frac{1}{6}\right) - 1.053\left(\frac{x}{l}\right) + 1.3\left(\frac{x}{l}\right)^2\right]^2 dx$$

$$= \frac{307.5EIY_0^2}{l^3} \quad (9)$$

As an example, consider a solid circular cross-section as the cantilever. The kinetic energy delivered to the system may be evaluated by considering a small element of the circular section shown in Fig. 2. By summing up the kinetic energy acquired by each element, the total kinetic energy, KE, can be obtained from:

$$KE = \sum_{beam} \frac{1}{2} \times \text{mass of element x (initial velocity)}^2 \quad (10)$$

Thus

$$KE = \int_0^l \frac{1}{2}\left[\rho \times \frac{\pi}{4} \times d^2 \times dx\right]\left[\frac{i_r \times ddx}{\rho \times \frac{\pi}{4} \times d^2 dx}\right]^2 = \int_0^l \frac{2i_r^2 dx}{\pi\rho}$$

$$= \frac{2i_r^2 l}{\pi\rho} \quad (11)$$

The initial velocity is the ratio of the impulse delivered to each element to the mass of the element. Equating kinetic energy and strain energy of bending, gives

$$\frac{307.5EIY_0^2}{l^3} = \frac{2i_r^2 l}{\pi\rho} \quad (12)$$

For a solid circular cross-section, $I = \frac{\pi}{64}d^4$. Thus,

$$\frac{2i_r^2 l}{\pi\rho} = \frac{307.5E\frac{\pi}{64}d^4Y_0^2}{l^3} \quad (13)$$

Equation (13) can be rearranged to give

$$\frac{Y_0}{l} = \sqrt{\frac{128}{307.5\pi^2}}\left[\frac{l}{d^2}\right]\left[\frac{i_r}{\sqrt{E\rho}}\right] \quad (14)$$

This is a generalised equation and can be used for the analysis of solid circular cantilevers of any ductile material. For example, if the cantilever was forced to deflect by 45°, then $Y_o = l$. Thus, equation (14) can be further simplified to:

$$\left[\frac{l}{d^2}\right] = \frac{\pi}{i_r}\left[\sqrt{\frac{E\rho}{0.416}}\right] \quad (15)$$

Consider the material made of aluminium, where Young's modulus E = 70GPa and the density, ρ = 2700kg/m^3. Substituting these values into equation (15) yields

$$\left[\frac{l}{d^2}\right] = \left[\frac{66939772}{i_r}\right] \quad (16)$$

Equation (16) provides the required diameter to length combination of a solid circular aluminium cantilever. A few examples have been calculated and are tabulated in Table 1. The values are taken from an imaginary detonation of two relatively large charges at prescribed distances.

Charge	Impulse (kPa-ms)	Cantilever	
		Diameter (mm)	Length (mm)
1	6266	5	267
		3	96
	3862	5	433
		3	156
	2602	5	643
		3	232
2	11000	5	152
		3	55
	6729	5	249
		3	90
	4577	5	366
		3	132

Table 1: Solid Circular Aluminium Cantilever with given Impulse and Diameter

B. *Quasi-Static Load Approach*

In this approach the work-done by the external pressure acting over the cantilever (or any structure) is first determined. Then this work is equated to the strain energy to establish a simple relationship between the geometric properties of the cantilever and pressure. This approach gives realistic results for low values of pressure. Evaluation of the strain energy is identical to the impulsive technique but now the maximum work done by the blast load is calculated by considering the work done by the blast load on a beam element and integrating over the length of the beam:

$$Workdone = \int_0^l p_r \, dY dx \qquad (17)$$

Where p_r is the pressure per unit length

Substituting the expression for displacement, the work done may be written as:

$$Workdone = \int_0^l p_r dY_0 \left[\frac{62.65}{12} \left(\frac{x}{l} \right)^2 - 11 \left(\frac{x}{l} \right)^3 + \frac{20.35}{3} \left(\frac{x}{l} \right)^4 \right] dx$$

After integration and applying the limits:

$$Workdone = 0.347 \, p_r l dY_0 \qquad (18)$$

Equating the work done and strain energy gives

$$\frac{307.5 EIY_0^2}{l^3} = 0.347 \, p_r l dY_0 \qquad (19)$$

Thus, by substituting the value of *I* for the solid circular section, the simplified equation may be written as:

$$\frac{307.5 E \left(\frac{\pi d^4}{64} \right) Y_0^2}{l^3} = 0.347 \, p_r l dY_0$$

or

$$\frac{Y_0}{l} = \frac{0.0722}{\pi E} \left(\frac{l}{d} \right)^3 p_r \qquad (20)$$

For an assumed deflection of 45°, i.e. $Y_o = l$, equation (20) simplifies to:

$$l = d \times \sqrt[3]{\frac{\pi E}{0.0722 \, p_r}} \qquad (21)$$

If the material is aluminium, where *E* = 70GPa, then

$$l = d \times \sqrt[3]{\frac{\pi \times 70 \times 10^9}{0.0722 \times p_r}} = 14495.62 \, d \times \sqrt[3]{\frac{1}{p_r}} \qquad (22)$$

A number of examples on the response of cantilevers to low overpressures are tabulated in Table 2. In this case, the overpressure has been used instead of the impulse.

Charge	Overpressure (kPa)	Cantilever	
		Diameter (mm)	Length (mm)
1	1075	5	708
		3	425
	336	5	1043
		3	626
	145	5	1380
		3	828
2	1056	5	712
		3	427
	330	5	1049
		3	629
	145	5	1380
		3	828

Table 2: Solid Circular Aluminium Cantilever with given Overpressure and Diameter

The response of solid circular aluminium cantilevers for the two cases considered above has been plotted in Fig. 3. The plot shows the variation of specimen length with air-blast overpressure for the two different diameters. The results show an almost linear relationship in both cases.

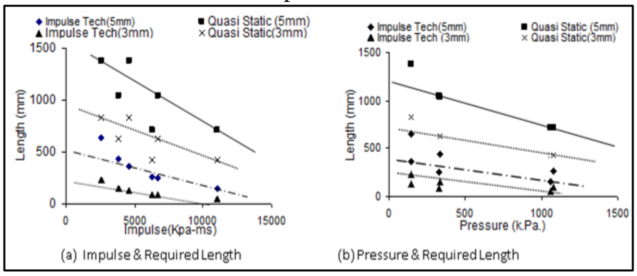

Fig. 3: Variation of Required Length

III. CONCLUSIONS

Two methods for assessing the response of cantilevers to air-blast loading have been presented. The methods were used to predict the required length of solid circular aluminium cantilevers with two different diameters and forced to deflect 45° from its original position. The predicted response is approximately linear. Other effects, such as the influence of air-flow, strain rate and plasticity, were not considered in the analysis. The extent to which each of these parameters influences the response of simple cantilevers is currently being investigated. Explosion tests are planned to verify some aspects of behaviour of such simple structures. From this preliminary study on solid circular aluminium cantilever, both the *impulse technique* and *quasi-static* analyses give an almost linear relationship for the same diameter and explosive charge.

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