Finding Optimum Solution for Fuzzy Transportation Problem using Heptagonal Fuzzy Numbers

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Abstract— In this paper, we introduce a new concept for the solution of fuzzy transportation problem under fuzzy environment in which the transportation costs, supplies at sources and demands at destinations are represented by heptagonal fuzzy numbers. This fuzzy transportation problem is solved by converting to its crisp equivalent problem from using a roluts ranking techniques and new fuzzy heptagonal fuzzy numbers. Further, we provide the solution of the proposed approach with numerical example.

Key words: Fuzzy Sets, Normal Fuzzy Set, Fuzzy Number, Heptagonal Fuzzy Numbers

I. Introduction

The fuzzy transportation problem (FTP) is one of the special kinds of fuzzy linear programming problems. A fuzzy transportation problem is a transportation problem in which the transportation costs, supply and demand quantities are fuzzy quantities. Transportation problem was originally introduced and developed by Hitchcock in 1941[7], in which the parameters like transportation cost, demand and supply are crisp values. But in the present world the transportation parameters may be uncertain due to many uncontrolled factors. So to deal the problems with imprecise information Zadeh [18] introduced the concept of fuzziness. Chen [2] introduced the concept of generalized fuzzy numbers to deal problems with unusual membership function. Pandian and Natarajan [15] proposed a new algorithm namely fuzzy zero point method to find optimal solution of a FTP with trapezoidal fuzzy numbers. Many researchers applied generalized fuzzy numbers to solve the real life problems. Kaur and Kumar solved FTP with generalized trapezoidal fuzzy numbers. Chandrasekaran .S, Kokila .G, Junu Saju [4] proposed a Ranking of Heptagon Number using Zero Suffix Method. In the present paper we discuss the fuzzy transportation problem using heptagonal fuzzy numbers.

II. Preliminaries

A. Fuzzy Set

[17] A fuzzy set is characterized by a membership function mapping element of a domain space or the universe of discourse X to the unit *interval* [0,1]. (i. e.) $\mu_A : X \to [0,1]$.

B. Normal Fuzzy Set

[16] A fuzzy set A of the universe of discourse X is called a normal fuzzy set implying that there exist at least one x ∈ X such that $\mu_A(x) = 1$.

C. Fuzzy Number

[4] A fuzzy set A defined on the set of real numbers R is said to be a fuzzy number if its membership function μ_A: R→ [0,1] has the following properties

1) A must be a normal fuzzy set.
2) A must be a closed interval for every [0, 1].
3) The support of A, O + A must be bounded.

D. Heptagonal Fuzzy Number:[8]

A heptagonal fuzzy number of a fuzzy set A is defined as $\overline{A_h}$ ={a1,a2,a3,a4,a5,a6,a7}, where a1,a2,a3,a4,a5,a6,a7 are real numbers and its membership function is given by

$$\mu = \begin{cases} \frac{1}{4}\left(\frac{x-a_1}{a_2-a_1}\right), & \text{for } a_1 \le x \le a_2; \\ \frac{1}{4}, & \text{for } a_2 \le x \le a_3; \\ \frac{1}{4}+\frac{3}{4}\left(\frac{x-a_3}{a_3-a_2}\right), & \text{for } a_3 \le x \le a_4; \\ \frac{1}{4}+\frac{3}{4}\left(\frac{a_4-x}{a_5-a_4}\right), & \text{for } a_4 \le x \le a_5; \\ \frac{1}{4}, & \text{for } a_5 \le x \le a_6; \\ \frac{1}{4}\left(\frac{a_7-x}{a_7-a_6}\right), & \text{for } a_6 \le x \le a_7; \\ 0, & \text{otherwise;} \end{cases}$$

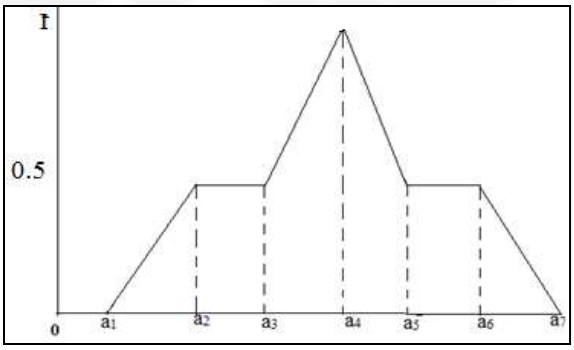

Fig. 1: Graphical Representation of Heptagonal Fuzzy Numbers

III. Solution of Fuzzy Transportation Problem

A. Arithmetic Operation

Let $\tilde{A} = (p_1, q_1, r_1, s_1, t_1)$ and $\tilde{B} = (p_2, q_2, r_2, s_2, t_2)$ are two fuzzy numbers where $p_1 \le q_1 \le r_1 \le s_1 \le t_1$ similarly $p_2 \le q_2 \le r_2 \le s_2 \le t_2$ then the arithmetic operations are defined as:

1) Addition

$$\tilde{A} + \tilde{B} = (p_1 + p_2, q_1 + q_2, r_1 + r_2, s_1 + s_2, t_1 + t_2)$$

2) *Subtraction*

$$\tilde{A}-\tilde{B}=(p_1-t_2,q_1-s_2,r_1-r_2,s_1-q_2,t_1-p_2)$$

B. *Fuzzy Transportation Problems*

The Fuzzy Transportation problems deal with the Transportation of a single product from several sources to several sinks. In general, let there be m sources $S_1,S_2,.......S_m$ with $\tilde{a}_i(i=1,2,.....m)$ available supplies or capacity at each source i , to be allocated among n destinations $D_1,D_2,.......D_N$ with $\tilde{b}_j=(j=1,2,......n)$ specified requirements at each destination j . Let $\tilde{c}_{ij}$ be the cost of shipping one from i to destination j for each route. Then, if $\tilde{x}_{ij}$ be the units shipped per route from source i to destination j , the problem is to determine the transportation schedule so as to minimize the total transportation cost satisfying the supply and demand condition.

The problem may as stated as follows:

$$\text{Minimize } \tilde{z}=\sum_{i=1}^{m}\sum_{j=1}^{n}\tilde{c}_{ij}\tilde{x}_{ij}$$

subject to the constraints:

$$\tilde{x}_{i1}+\tilde{x}_{i2}+..........\ .....+\tilde{x}_{in}=\tilde{a}_i;i=1,2,.........,m$$

$$\tilde{x}_{1j}+\tilde{x}_{2j}+..........\ ...+\tilde{x}_{mj}=\tilde{b}_j;\ j=1,2,........,\ n$$

And $\tilde{x}_{ij}\ \geq,$ for all i and j

For a feasible solution to exist, it is necessary that supply equals total requirement,

i.e.. $\sum_{i=1}^{m}\tilde{a}_i=\sum_{j=1}^{n}\tilde{b}_j$ the fuzzy transportation problem is explicitly represented by the following fuzzy transportation table:

....	D_1	D_2	......	D_n	Supply
S_1	$\tilde{c}_{11}$	$\tilde{c}_{12}$		$\tilde{c}_{1n}$	$\tilde{a}_1$
S_2	$\tilde{c}_{21}$	$\tilde{c}_{22}$		$\tilde{c}_{2n}$	$\tilde{a}_2$
....	......	....	....	....	....
S_m	$\tilde{c}_{m1}$	$\tilde{c}_{m2}$		$\tilde{c}_{mn}$	a_m
Demand	$\tilde{b}_1$	$\tilde{b}_2$		b_n	

Table 1:

C. *Solution Procedure*

Fuzzy transportation problems require two methods for its optimal basic feasible solution. Fuzzy version of VAM method:

1) Step 1: Calculate the reduced values by using the ranking method.
2) Step 2: Calculate penalties by taking differences between the minimum and next to transportation cost in each row and each column.
3) Step 3: Circle the largest Row Difference and Column Difference. In the event of a tie, choose either.
4) Step 4: Allocate as much as possible in the lowest cost cell of the row (or column) having a circled Row (or Column) Difference.
5) Step 5: In the case the allocation is made fully to a row (or column), ignore that row (or column) for further consideration, by crossing it.
6) Step 6: Revise the differences again and cross out the earlier figures. Go to step2.
7) Step 7: Continue the procedure until all rows and columns have been crossed out. i.e., distribution is complete.

D. *Numerical Example*

Consider a fully fuzzy Transportation problem in which supply at sources, demand at destinations and fuzzy unit transportation costs, etc…. are assumed to be heptagonal fuzzy numbers.

DESTINATIONS						
		D_1	D_2	D_3	D_4	Fuzzy supply
S OURC E S	O_1	(1,2,3,4,5,6,7)	(2,4,6,8,10,12,14)	(3,4,5,6,7,8,9)	(4,5,6,7,8,9,10)	(2,3,4,5,6,7,8)
	O_2	(3,6,9,12,15,18,21)	(5,6,7,8,9,10,11)	(1,3,5,7,9,11,13)	(1,4,7,10,13,16, 19)	(6,8,10,12,14,16, 18)
	O_3	(8,9,10,11,12,13,14)	(4,6,8,10,12,14, 16)	(3,5,7,9,11,13, 15)	(5,7,9,11,13,15, 17)	(7,8,9,10,11,12,13)
	O_4	(9,10,11,12,13,14, 15)	(10,15,20,25,30, 35,40)	(2,5,8,11,14,17, 20)	(4,7,10,13,16,19, 22)	(6,7,8,9,10,11,12)
	Fuzzy Demand	(11,13,15,17,19,21, 23)	(9,11,13,15,17,19, 21)	(11,12,13,14,15, 16,17)	(7,9,11,13,15,17, 19)	

Table 2:

Supply ≠ demand. So the given fuzzy transportation problem is unbalanced. Now we convert the unbalanced fuzzy transportation problem into balanced fuzzy transportation.

DESTINATIONS						
		D_1	D_2	D_3	D_4	Fuzzy supply
S OUR CE S	O_1	(1,2,3,4,5,6,7)	(2,4,6,8,10,12,14)	(3,4,5,6,7,8,9)	(4,5,6,7,8,9,10)	(2,3,4,5,6,7,8)
	O_2	(3,6,9,12,15,18,21)	(5,6,7,8,9,10,11)	(1,3,5,7,9,11,13)	(1,4,7,10,13,16, 19)	(6,8,10,12,14,16, 18)
	O_3	(8,9,10,11,12,13,14)	(4,6,8,10,12,14, 16)	(3,5,7,9,11,13, 15)	(5,7,9,11,13,15, 17)	(7,8,9,10,11,12, 13)

	O_4	(9,10,11,12,13,14, 15)	(10,15,20,25,30, 35,40)	(2,5,8,11,14,17, 20)	(4,7,10,13,16,19, 22)	(6,7,8,9,10,11, 12)
	O_5	(0,0,0,0,0,0,0)	(0,0,0,0,0,0,0)	(0,0,0,0,0,0,0)	(0,0,0,0,0,0,0)	(1,2,2,3,4,5,6)
	Fuzzy Demand	(11,13,15,17,19,21, 23)	(9,11,13,15,17, 19,21)	(11,12,13,14,15, 16,17)	(7,9,11,13,15,17, 19)	

Table 3:

Using ranking function

$R(\bar{A}) = \frac{P_1 + P_2 + P_3 + P_4 + P_5 + P_6 + P_7}{7}$

$R(1,2,3,4,5,6,7) = \frac{1+2+3+4+5+6+7}{7} = \frac{28}{7} = 4$

$R(2,4,6,8,10,12,14) = \frac{2+4+6+8+10+12+14}{7} = \frac{56}{7} = 8$

$R(3,4,5,6,7,8,9) = \frac{3+4+5+6+7+8+9}{7} = \frac{42}{7} = 6$

$R(4,5,6,7,8,9,10) = \frac{4+5+6+7+8+9+10}{7} = \frac{49}{7} = 7$

$R(2,3,4,5,6,7,8) = \frac{2+3+4+5+6+7+8}{7} = \frac{35}{7} = 5$

$R(3,6,9,12,15,18,21) = \frac{3+6+9+12+15+18+21}{7} = \frac{84}{7} = 12$

$R(5,6,7,8,9,10,11) = \frac{5+6+7+8+9+10+11}{7} = \frac{56}{7} = 8$

$R(1,3,5,7,9,11,13) = \frac{1+3+5+7+9+11+13}{7} = \frac{49}{7} = 7$

$R(1,4,7,10,13,16,19) = \frac{1+4+7+10+13+16+19}{7} = \frac{70}{7} = 10$

$R(6,8,10,12,14,16,18) = \frac{6+8+10+12+14+16+18}{7} = \frac{84}{7} = 12$

$R(8,9,10,11,12,13,14) = \frac{8+9+10+11+12+13+14}{7} = \frac{77}{7} = 11$

$R(4,6,8,10,12,14,16) = \frac{4+6+8+10+12+14+16}{7} = \frac{70}{7} = 10$

$R(3,5,7,9,11,13,15) = \frac{3+5+7+9+11+13+15}{7} = \frac{63}{7} = 9$

$R(5,7,9,11,13,15,17) = \frac{5+7+9+11+13+15+17}{7} = \frac{77}{7} = 11$

$R(7,8,9,10,11,12,13) = \frac{7+8+9+10+11+12+13}{7} = \frac{70}{7} = 10$

$R(9,10,11,12,13,14,15) = \frac{9+10+11+12+13+14+15}{7} = \frac{84}{7} =$ 12

$R(10,15,20,25,30,35,40) = \frac{10+15+20+25+30+35+40}{7}$

$= \frac{175}{7} = 25$

$R(2,5,8,11,14,17,20) = \frac{2+5+8+11+14+17+20}{7} = \frac{77}{7} =$ 11

$R(4,7,10,13,16,19,22) = \frac{4+7+10+13+16+19+22}{7} = \frac{91}{7} = 13$

$R(6,7,8,9,10,11,12) = \frac{6+7+8+9+10+11+12}{7} = \frac{63}{7} = 9$

$R(11,13,15,17,19,21,23) = \frac{11+13+15+17+19+21+23}{7} = \frac{119}{7}$ =17

$R(9,11,13,15,17,19,21) = \frac{9+11+13+15+17+19+21}{7} = \frac{105}{7}$ =15

$R(11,12,13,14,15,16,17) = \frac{11+12+13+14+15+16+17}{7} = \frac{98}{7}$ =14

$R(7,9,11,13,15,17,19) = \frac{7+9+11+13+15+17+19}{7} = \frac{91}{7}$ =13

E. Reduced Table

	D_1	D_2	D_3	D_4	Supply
S_1	4	8	6	7	5
S_2	12	8	7	10	12
S_3	11	10	9	11	10
S_4	12	25	11	13	9
Demand	17	15	14	13	

Table 4: Reduced Table

F. Solution:

1) Step 1

	D_1	D_2	D_3	D_4		
S_1	4	8	6	7	5	(2)
S_2	12	8	7	10	12	(1)
S_3	11	10	9	11	10	(1)
S_4	12	25	11	13	9	(1)
S_5	0	13 0	0	0	23	(0)
	17	15	14	13		
	(4)	(8)	(6)	(7)		

And finally,

	D_1	D_2	D_3	D_4	
S_1	[5] 4	8	6	7	5
S_2	12	8	[12] 7	10	12
S_3	11	10	[2] 9	[5] 11	10
S_4	[9] 12	25	[3] 11	13	9
S_5	0	[15] 0	0	[8] 0	23
	17	15	14	13	

The fuzzy initial basic feasible solution
= (5*4) + (12*7) + (2*9) + (5*11) + (9*12) + (3*11) + (15*0) + (8*0)
= 318

IV. Conclusion

In this paper, we have proposed a simple method for solving fully fuzzy transportation problems with heptagonal fuzzy numbers. A numerical example is given by using the proposed method with converting the problem to crisp equivalent problem.

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