Effect of Soil Structure Interaction on Seismic Response of Building

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Abstract— Masonry infill wall panels may not contribute towards resisting gravity loads, but contribute significantly, in terms of enhanced stiffness and strength under earthquake (or wind) induced lateral loading. However, in practice, the infill stiffness is commonly ignored in building frame analysis, resulting in an under-estimation of stiffness and natural frequency. Also, the infill has energy dissipation characteristics that contribute to improved seismic resistance. Thus a reasonably accurate model for the soil-foundation-structure interaction system with computational validity, efficiency and accuracy is needed in the improved seismic design of structures. Most of the times, the effect of soil flexibility is not considered by structural designer with the feeling that such omission is conservative and simple, this could be true for static analysis but not for dynamic analysis. This omission is because of experimental and computational difficulties of the soil data and complicated modelling behaviour of substructure and superstructure. It is instructive to study the implications of the common practice of ignoring the infill stiffness with regard to performance under seismic loading. The present study makes an attempt to show the effect of flexibility and rigidity of foundation in earthquake analysis of structure by the considering the combined effect of soil structure interaction and infill wall stiffness of building. For superstructure G+3 simple rectangular building is considered for seismic analysis. The infill wall is replaced by equivalent diagonal member. The total work is divided into two parts. In first part, the manual analysis is done in which response spectra applied in both X (longitudinal) and Y (transverse)direction and in second part the analysis is done with ANSYS software considering soil structure interaction effect. The more emphasis is given on manual earthquake analysis by using model superposition response method as per IS 1893-2016 (part-I). The stiffness of soil is calculated by the formulae as per the FEMA 356. After studying this behaviour, it is found that base shear decreases in seismic analysis of superstructure by considering the effect of soil structure interaction in X (Longitudinal) and Y (Transverse) direction.

Key words: Soil Structure Interaction, Soil Stiffness, Infill Wall, Earthquake, Base Shear

I. INTRODUCTION

The soil response analyses is one of the most important aspects of earthquake engineering, as it will determine the ground motion that will be experienced at the top of soil without the presence of a structure or the so-called free field response. The analysis involves estimation of the seismologic characteristics of the region, and determination and modeling of the soil profile and its dynamic characteristics. Further, it accounts for the multiple reflections and refractions that will occur at the soil layer interfaces as the seismic waves propagate though the soil deposits. Although special purpose computer programs exist for this purpose, the validity of the results depend greatly on how accurate dynamic soil properties are estimated, which in spite the improvements in the in situ testing, is still a challenging task. In the present study, no soil amplification analysis was performed; rather, they considered accelerograms were used directly to excite the structure.

The actual seismic input motion to the structural foundation is the result of kinematic interaction analysis considering only the geometry and stiffness properties of the structural foundation and soil. The second aspect of the soil structure interaction analysis involves the deformations and stresses in supporting soil, induced due to the base shears and moments generated in the vibrating structure.

II. OBJECTIVE

Present study will be carried out with following objectives:

1) To compare the total base shear with and without considering effect of SSI.
2) To compare the effect of SSI by considering the different type of soil.
3) To compare the total base shear using response spectrum method by manually and software (ANSYS 14.0).

The building considered for analysis is shown in fig.

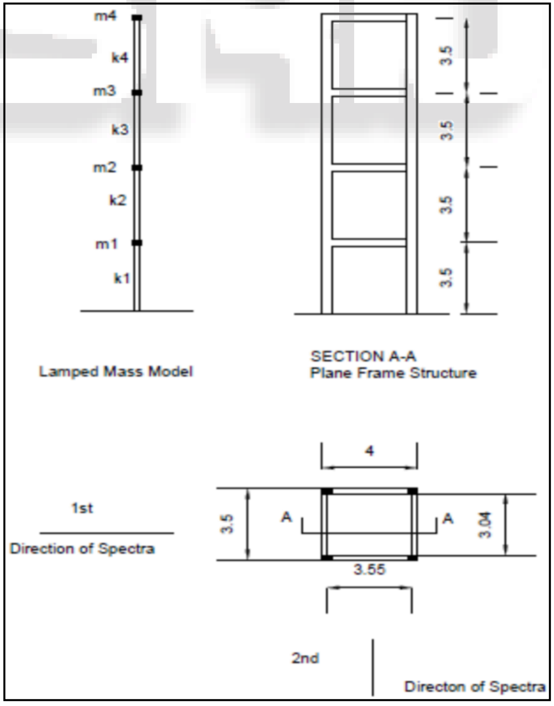


Fig. 1: Simple Geometrical G + 3 Structure

III. CALCULATION OF SOIL PARAMETER

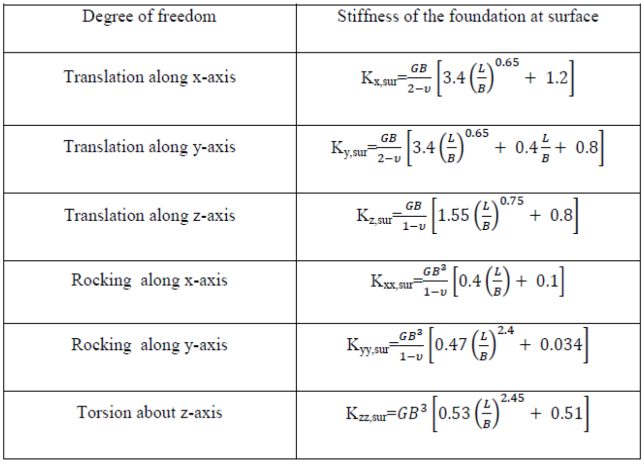
The stiffness is calculated by as per the guidelines of FEMA -356 by considering the elastic parameters of soil dimensions of foundations. The relation between modulus of elasticity, poison’s ratio and shear modulus is given by the following formula,

$$G = \frac{E}{2(1 + \mu)}$$

Where,
E= modulus of elasticity
G = shear modulus
μ = poison ratio

Sr No.	Constant	Value	Remark
1	Z	0.16	Zone- III
2	I	1.0	Importance factor
3	R	5	Response reduction factor
4	M-25		Grade of concrete
5	Fe-500		Grade of steel

Table 1 Constant used in Calculation (IS1893: 2016 PART-I)



Degree of freedom	Stiffness of the foundation at surface
Translation along x-axis	$K_{x,sur} = \frac{GB}{2-\upsilon}\left[3.4\left(\frac{L}{B}\right)^{0.65} + 1.2\right]$
Translation along y-axis	$K_{y,sur} = \frac{GB}{2-\upsilon}\left[3.4\left(\frac{L}{B}\right)^{0.65} + 0.4\frac{L}{B} + 0.8\right]$
Translation along z-axis	$K_{z,sur} = \frac{GB}{1-\upsilon}\left[1.55\left(\frac{L}{B}\right)^{0.75} + 0.8\right]$
Rocking along x-axis	$K_{xx,sur} = \frac{GB^3}{1-\upsilon}\left[0.4\left(\frac{L}{B}\right) + 0.1\right]$
Rocking along y-axis	$K_{yy,sur} = \frac{GB^3}{1-\upsilon}\left[0.47\left(\frac{L}{B}\right)^{2.4} + 0.034\right]$
Torsion about z-axis	$K_{zz,sur} = GB^3\left[0.53\left(\frac{L}{B}\right)^{2.45} + 0.51\right]$

IV. MANUAL CALCULATION

1) Step I: Calculation of lumped masses to various floor levels

Roof /floor = mass of infill + mass of column +mass of beam in longitudinal and transverse direction of that floor + mass of slab + imposed load on that floor if permissible

Roof = {((0.23 x 3.1 x (3.5/2)) + (0.23 x 3.04 x 2 x(3.5/2))) x22 } + {(0.23 x 0.4 x (4x2 + 3.5x2)x 25} + {0.12 x 4 x 3.5 x25 } + {(0.23 x 0.45 x 3.5/2) x 4)x 25} + 0
=203.35 kN
=20.73 Ton

Floors = Third, Second and First floor
= {((0.23 x 3.1 x 2 x 3.5) + (0.23 x 3.04 x 2 x 3.5)) x 22 } + {(0.23 x 0.4 x (4x2 + 3.5x2)x 25} + {0.12 x 4 x 3.5 x25 } + {(0.23 x 0.45 x 3.5 x 4)x 25} + {(3.5/2)** x 3.5 x 4 }
= 354.69kN i.e. 36.15 Ton

* Imposed load on roof not consider.

** 50% of Imposed load, if imposed load is greater than3 kN/m^2

2) Step - II: Frame considering the stiffness of infill

The frame considered in previous section is again analysed by considering the stiffness of infill walls. This is modelled as equivalent diagonal strut.

The mass matrix [M] for lumped plane frame model is

$$= \begin{bmatrix} 36.15 & 0 & 0 & 0 \\ 0 & 36.15 & 0 & 0 \\ 0 & 0 & 36.15 & 0 \\ 0 & 0 & 0 & 20.73 \end{bmatrix}$$

Column stiffness of storey

$$k = \frac{12EI}{L^3} = \frac{12 \text{ X } 22360 \text{ X } 10^3 \text{ X } 0.23 \text{ X } 0.45^3}{12 \text{ X } 3.5^3}$$

= 10930.32 kN /m

Stiffness of infill is determined by modelling the infill as an equivalent diagonal strut in which, width of strut

$E_f = 22360$ N/m2 $\qquad E_m = 13800$ N/m2

t = thickness of infill wall = 230 mm
h = height of infill wall = 3.5 m
l = length of infill wall = 4 m

$I_c =$ moment of inertia of column $= \frac{0.23\,x 0.45^3}{12} = 0.001746$ m4

$I_b =$ moment of inertia of beam $= \frac{0.23 x 0.4^3}{12} =$ 0.001226 m4

$$\theta = \tan^{-1}\frac{3.5}{4} = 41.18^0$$

$$\alpha_h = \frac{\pi}{2}\left[\frac{22360\, x\, .001746\, x\, 3.5}{2\, x\, 13800\, x\, 0.23\, x \sin(2\, x\, 41.18)}\right]^{\frac{1}{4}}$$

= 0.602

$$\alpha_l = \pi\left[\frac{22360\, x\, 0.001226\, x\, 4}{13800\, x\, 0.23\, x \sin(2\, x\, 41.18)}\right]^{\frac{1}{4}}$$

= 1.357

$$W = \frac{1}{2}\sqrt{0.602^2 + 1.357^2}$$

= 0.7422 m

A= area of diagonal stiffness
= 0.7422 x 0.23
= 0.1707 m^2

Size of diagonal strut = 0.7422 x 0.23

Diagonal length $= \sqrt{3.5^2 + 4^2} = 5.315$ m

Stiffness of infill is $= \frac{\text{AE}_\text{m}}{\text{I}_\text{d}}(\cos\theta)^2$

$$= \frac{0.1707 \text{ x } 13800 \text{ x } 10^6 \text{ x } (\cos 41.18)^2}{5.315}$$

= 251.066898 x 10^6 N/m

For the frame with two bays there are two struts participating in each direction total lateral stiffness of each storey

$k_1 = k_2 = k_3 = k_4$ = 4 x 10930.32 + 2 x 251.066898 x 10^6
= 500.163 x 10^6 N/m
= 500163 kN/m.

Stiffness matix [k]=

$$\begin{bmatrix} 1.000326 & -0.500163 & 0 & 0 \\ -0.500163 & 1.000326 & -0.500163 & 0 \\ 0 & -0.500163 & 1.000326 & -0.500163 \\ 0 & 0 & -0.500163 & 0.500163 \end{bmatrix} 10^6 \text{kN/m}$$

For the above stiffness and mass matrix, eigen values and Eigen vectors are work out as follows therefore quadratic equation in ω is,

$$[\omega] = \begin{bmatrix} 45.02 & 0 & 0 & 0 \\ 0 & 128.43 & 0 & 0 \\ 0 & 0 & 191.84 & 0 \\ 0 & 0 & 0 & 227.47 \end{bmatrix}$$

$\therefore \omega_1 = 45.02$ rad/sec
ω_2= 128.43 rad/sec
ω_3= 191.84 rad/sec
ω_4= 227.47 rad/sec

The mode shapes corresponding to each natural frequency is determined from the equation

$$\begin{vmatrix} 2K-\omega^2 m & -K_2 & 0 & 0 \\ -K_2 & 2K-\omega^2 m & -K_3 & 0 \\ 0 & -K_3 & 2K-\omega^2 m & -K_4 \\ 0 & 0 & -K_4 & K-\omega^2\,0.575m \end{vmatrix} \begin{Bmatrix} \emptyset_1 \\ \emptyset_2 \\ \emptyset_3 \\ \emptyset_4 \end{Bmatrix} = 0$$

3) Step –III Mode Shapes

Mode - 1:

$\boldsymbol{\omega}_1$= 45.02 rad/sec

(100032636.15 $\boldsymbol{\omega}^2$)$\emptyset_{11}$ – 500163$\emptyset_{21}$= 0

Put $\emptyset_{11}$=1therefore, $\emptyset_{21}$= 1.85

Similarly $\emptyset_{31}$= 2.43, $\emptyset_{41}$ = 2.65

Mode - 2

$\boldsymbol{\omega}_2$= 128.43 rad/sec

Put $\emptyset_{12}$=1therefore, $\emptyset_{22}$= 0.807

Similarly $\emptyset_{32}$= -0.348, $\emptyset_{42}$ = -1.088

Mode – 3

$\boldsymbol{\omega}_3$= 191.84 rad/sec

Put $\emptyset_{13}$=1therefore, $\emptyset_{23}$= -1.51

Similarly $\emptyset_{33}$= 0.00348, $\emptyset_{43}$ = 0.00659

Mode - 4

$\boldsymbol{\omega}_4$= 227.47 rad/sec

Put $\emptyset_{14}$=1therefore, $\emptyset_{24}$= -0.574

Similarly $\emptyset_{34}$= -0.0138, $\emptyset_{44}$ = 0.576

Eigen vectors (mode shapes):

$\{\emptyset\}=\{\emptyset_1\emptyset_2\emptyset_3\,\emptyset_4\}=$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1.85 & 0.807 & -1.51 & -0.574 \\ 2.43 & -0.348 & -0.00138 & -0.00138 \\ 2.65 & -1.088 & 0.00659 & 0.576 \end{bmatrix}$$

4) Step –IV Determination of modal participation factor

The modal participation factor

$$P_k = \frac{\sum_{i=1}^{n} W_i\,\emptyset_{ik}}{\sum_{i=1}^{n} W_i\,\emptyset_{ik}^2}$$

$$P_1 = \frac{W_1\emptyset_{11} + W_2\emptyset_{21} + W_3\emptyset_{31} + W_4\emptyset_{41}}{W_1\emptyset_{11}^2 + W_2\emptyset_{21}^2 + W_3\emptyset_{31}^2 + W_4\emptyset_{41}^2}$$

$$= \frac{36.15\times 1 + 36.15\times 1.85 + 36.15\times 2.43 + 20.73\times 2.65}{36.15\times 1 + 36.15\times 1.85^2 + 36.15\times 2.43^2 + 20.73\times 2.65^2}$$

P_1= 0.4739

Similarly

P_2= 0.342

P_3= -0.155

P_4=0.496

Step –V Determination of modal mass

$$m_k = \frac{\left(\sum_{i=1}^{n} W_i\,\emptyset_{ik}\right)^2}{g\left(\sum_{i=1}^{n} W_i\,\emptyset_{ik}^2\right)}$$

Where,

g= acceleration due to gravity

$\emptyset_{ik}$= mode shape coeff. at i^th floor in mode k

$$m_1 = \frac{\left(\sum_{i=1}^{4} W_1\,\emptyset_{ik}\right)^2}{g\left(\sum_{i=1}^{4} W_1\,\emptyset_{i1}^2\right)}$$

m_1= 116.43 tonne

m_2= 32.85 tonne

m_3=30.34 tonne

m_4= 3.20 tonne

Total modal mass = 116.43 + 30.34 + 32.85 + 3.20

= 182.82 tonne

Modal contribution for each mode

For mode 1 = 116.43/182.82 = 63.68%

For mode 2 = 32.85/182.82 = 17.96 %

For mode 3 = 30.34/182.82 = 16.59 %

For mode 4 = 3.20/182.82 = 1.75 %

5) Step –VI Determination of lateral forces at each floor at each mode

The design lateral force is given by(Q_{ik}) at floor i in mode k is,

$Q_{ik}=A_k\emptyset_{ik}P_k\,W_i$

The design horizontal seismic coefficient A_h, for various mode are

$$A_h = \frac{Z}{2}\,\frac{I}{R}\,\frac{Sa}{g}$$

$$\frac{Sa}{g} = \begin{cases} 1+15T & T < 0.10\,s \\ 2.5 & 0.10\,s < T < 0.40s \\ \dfrac{1}{T} & 0.40\,s < T < 4.0\,s \\ 0.25 & T > 4.00\,s \end{cases}$$

For $T_{1=}$ 0.139 $\frac{S_{a1}}{g}$ =2.5

For T_2 = 0.0489 $\frac{S_{a1}}{g}$ =1.734

For T_3 = 0.0327$\frac{S_{a1}}{g}$ =1.491

For T_4 = 0.0276$\frac{S_{a1}}{g}$ =1.414

$A_{h1}=\frac{Z}{2}\,\frac{I}{R}\,\frac{Sa}{g} = \frac{0.16}{2}\,\frac{1}{5}\;2.5$

∴ A_{h1}= 0.04

Similarly,

A_{h2} = 0.0277

A_{h3} = 0.0238

A_{h4} = 0.0226

$$Q_{i1} = \begin{bmatrix} (A_{h1}P_1\,\emptyset_{11}\,W_1) \\ (A_{h2}P_1\,\emptyset_{21}\,W_1) \\ (A_{h3}P_1\,\emptyset_{31}\,W_1) \\ (A_{h4}P_1\,\emptyset_{41}\,W_1) \end{bmatrix}$$

$$= \begin{bmatrix} (0.0400)(0.473)(1.00)(36.15\times 9.81) \\ (0.0277)(0.473)(1.85)(36.15\times 9.81) \\ (0.0238)(0.473)(2.43)(36.15\times 9.81) \\ (0.0226)(0.473)(2.65)(36.15\times 9.81) \end{bmatrix}$$

$$\therefore Q_{i1} = \begin{bmatrix} 6.71 \\ 8.59 \\ 9.70 \\ 5.76 \end{bmatrix}$$

Similarly,

$$Q_{i2} = \begin{bmatrix} 4.85 \\ 2.71 \\ -1.00 \\ -1.76 \end{bmatrix}$$

$$Q_{i3} = \begin{bmatrix} -2.20 \\ 2.30 \\ 0.0018 \\ -0.0047 \end{bmatrix}$$

$$Q_{i4} = \begin{bmatrix} 7.04 \\ -2.80 \\ -0.0058 \\ 2.29 \end{bmatrix}$$

6) Step VII Determination of storey shear forces in each mode

$V_{ik}=\sum_{i=i+1}^{n} Q_{ik}$

$$V_{i1} = \begin{bmatrix} V_{11} \\ V_{21} \\ V_{31} \\ V_{41} \end{bmatrix} = \begin{bmatrix} (Q_{11}+Q_{21}+Q_{31}+Q_{41}) \\ (Q_{21}+Q_{31}+Q_{41}) \\ (Q_{31}+Q_{41}) \\ (Q_{41}) \end{bmatrix} = \begin{bmatrix} 30.05 \\ 24.05 \\ 15.46 \\ 5.76 \end{bmatrix}$$

Similarly,

$$V_{i2}=\begin{bmatrix}V_{12}\\V_{22}\\V_{32}\\V_{42}\end{bmatrix}=\begin{bmatrix}4.80\\-0.05\\-2.76\\-1.76\end{bmatrix}$$

$$V_{i3}=\begin{bmatrix}V_{13}\\V_{23}\\V_{33}\\V_{43}\end{bmatrix}=\begin{bmatrix}0.097\\2.29\\-0.0029\\-0.0047\end{bmatrix}$$

$$V_{i4}=\begin{bmatrix}V_{14}\\V_{24}\\V_{34}\\V_{44}\end{bmatrix}=\begin{bmatrix}6.52\\-0.52\\2.28\\2.29\end{bmatrix}$$

7) Step VIII Determination of storey shear forces due to all modes

$V_1=[(V_{11})^2+(V_{12})^2+(V_{13})^2+(V_{14})^2]^{0.5}$

$V_1=[(30.05)^2+(4.80)^2+(0.097)^2+(6.52)^2]^{0.5}$

$= 31.12$ kN

Similarly,

$V_2 = 24.16$ kN, $V_3 = 15.87$ kN, $V_4= 6.44$ kN

Step IX Determination of lateral forces at each storey

Final base shear at each floor

F_4 at roof floor= 6.44kN

F_3 at third floor = V_3- V_4=15.87-.44=9.43 kN

F_2 at second floor = V_2- V_3=24.16-15.87=8.29 kN

F_1 at first floor = V_1- V_2=31.12-24.16=6.96 kN

Total Base shear = 6.44 + 9.43 + 8.29 + 6.96 = 31.12 kN

V. CALCULATION WERE DONE WITH ANSYS WITHOUT CONSIDERING SSI & WITH SSI FOR DIFFERENT SOIL TYPE

A. *ANSYS Result in X direction*

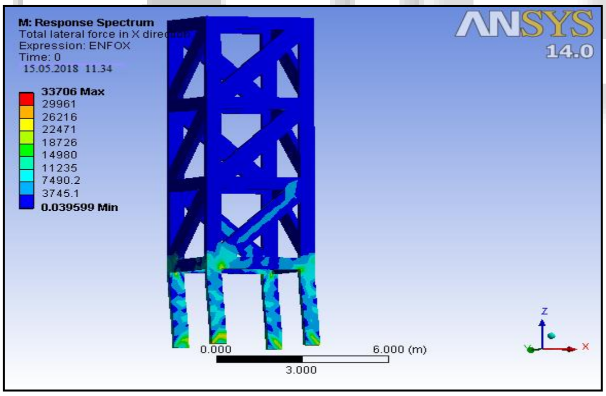


Fig 2: Building model without considering SSI after analysis in X direction

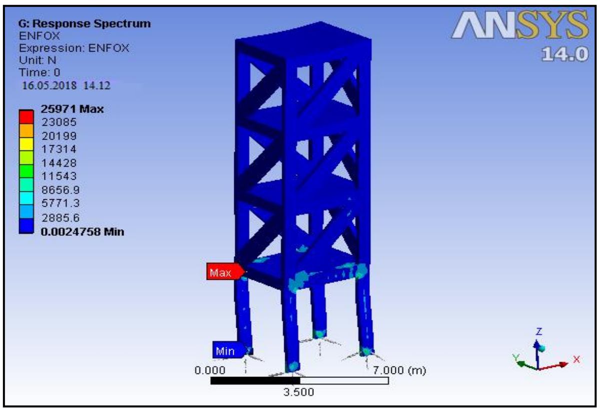


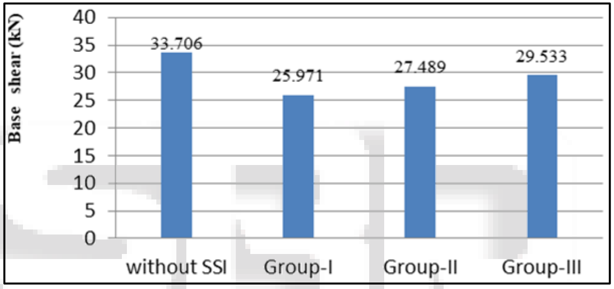
Fig. 3: Building model with considering SSI after analysis for soil group I in X direction

VI. RESULTS

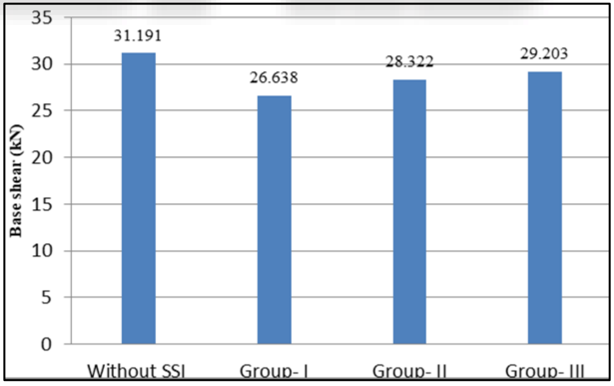
In order to accomplish a satisfactory and safe result structural performance during seismic events of reinforced concrete multi-storied building on soil media, proper seismic design should be taken into account the effect of soil structure interaction.

Direction of spectra applied	Without SSI (kN) Group- I (Rock or hard soil)		With SSI (kN)		
	Manually	ANSYS	Group- I (Rock or hard soil)	Group- II (Medium soil)	Group- III (Soft soil)
X direction	31.120	33.706	25.971	27.489	29.533
Y direction	29.330	31.193	26.638	28.322	29.203

Table 2: Analysis Result of base Shear is tabulated as Below



Graph 1: Base shear when spectra in X direction



Graph 2: Base shear when spectra in Y direction

VII. CONCLUSIONS

In this study, the importance of subsoil dynamic properties on behaviour of mid-rise concrete moment resisting building frames under influence of soil-structure interaction is investigated. The study is conducted for the 4 storey concrete moment resisting building frame resting on soil Group I (Rock or hard soil), II (Medium soil) and III (Soft soil).

1) In analysis performed without considering SSI for the same soil type, the value of base shear obtained by manual calculation is less by 8.30 % in longitudinal (X) direction and by 6.35 % in transverse (Y) direction as compared to value obtained by analysis using ANSYS.

This is because in manual calculation lumped mass modeling is used whereas ANSYS software uses discrete modeling.

2) For Group-I (Rock or hard soil), the base shear values in longitudinal (X) direction and transverse (Y) direction are decreased by 29.78% and 17.09% respectively when SSI effect is considered as compared to values without considering SSI.
3) For spectra is in longitudinal direction (X) the base shear value considering SSI is maximum in Group-III (Soft soil) and minimum in Group-I (Rock or hard soil). The value is more in Group-III (Soft soil) by 13.71 % than Group-I (Rock or hard soil) and by 7.43 % than Group-II (Medium soil).
4) For spectra is in transverse direction (Y) the base shear value considering SSI is also maximum in Group-III (Soft soil) and minimum in Group-I (Rock or hard soil). The value is more in Group-III (Soft soil) by 9.62 % than Group-I (Rock or hard soil) and by 3.11 % than Group-II (Medium soil).
5) For the soil with SSI, the spectra along transverse (Y) direction of building is more critical than spectra along longitudinal (X) direction because base shear value in transverse (Y) direction is more than base shear value along longitudinal (X) direction for Group-I (Rock or hard soil) and Group-II (Medium soil). But in Group-III spectra along both directions are almost equal.

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