Triple Connected Equitable Domination in Fuzzy Graphs

S.Revathi[1] C.V.R.HariNarayanan[2] R.Muthuraj[3]
[1,2,3]Department of Mathematics
[1]Kings College of Engineering, Thanjavur [2]Govt.Arts College, Paramakudi [3]H.H.The Rajah's College, Pudukkottai

Abstract— The concept of triple connected graphs with real life application was introduced in [7] by considering the existence of a path consists of any three vertices of a fuzzy graph G.In[4] G.Mahadevan et.al., was introduced the concepts of triple connected domination number of a graph. In this paper we introduce a new domination parameter called triple connected equitable domination number of a fuzzy graph. A subset S of V of a graph G is said to be fuzzy triple connected equitable dominating set. If S is equitable dominating set and the induced subgraph is triple connected. The minimum cardinality taken over all triple connected equitable dominating sets is called the fuzzy triple connected equitable domination number .We determine this number for some standard fuzzy graphs. Its relationship with other fuzzy graph theoretical parameters is also investigated.

Key words: Fuzzy Equitable dominating set, connected fuzzy equitable dominating set, fuzzy equitable domination number, connected fuzzy equitable domination number, fuzzy triple connected dominating set and fuzzy triple connected equitable domination number

I. Introduction

Fuzzy graphs were introduced by Rosenfeld [8] ten years after Zadeh's Landmark paper "fuzzy sets"[12] .Fuzzy graph theory is now finding numerous applications in modern science and technology especially in the fields of Information theory, neural networks, expert systems, Cluster analysis, control theory, etc. Fuzzy modeling is an essential tool in all branches of science, engineering and medicine.

Rosenfeld has obtained the fuzzy analogues of several basic graph-theoretic concepts like bridge, paths, cycles, trees and connectedness and established some of their properties .Bhattacharya has established some connectivity concepts regarding fuzzy cut nodes and fuzzy bridges. The author has also introduced fuzzy groups and metric notion in fuzzy graphs. The concept of domination and determines the domination number for several fuzzy graphs are discussed in [9].

In this paper, we introduce the concept of fuzzy triple connected equitable dominating set.

II. Preliminaries

A fuzzy graph G= (σ,μ) is a pair of functions $\sigma: V \to [0,1]$ and $\mu: V\times V \to [0,1]$ where for all u, v $\in V$ .We have $\mu(u,v) \le \sigma(u)\wedge\sigma(v)$.

The fuzzy graph H=(τ,σ) is called a fuzzy sub graph of G =(σ,μ) if $\tau(u)\le\sigma(u)$ for all u$\in V$ and $\sigma(u,v)\le\mu(u,v)$ for all u, v $\in V$.

A fuzzy sub graph H= (τ,ρ) is said to be a spanning fuzzy sub graph of G = (σ,μ) if $\tau(u)=\sigma(u)$ for all u .The two graphs have same fuzzy node set they differ only in the arc weights.

Let G=(σ,μ) be a fuzzy graph and τ be any fuzzy subset of σ ,$\tau(u)\le\sigma(u)$ for all u, then the fuzzy sub graph of G=(σ,μ) induced by τ the maximal fuzzy sub graph of G=(σ,μ) that has fuzzy node set τ .The complement of a fuzzy graph G=(σ,μ) is a fuzzy graph $\overline{G}=(\overline{\sigma},\overline{\mu})$ where $\overline{\sigma}=\sigma$ and $\overline{\mu}(u,v)=\sigma(u)\wedge\sigma(v)-\mu(u,v)$ for all u, v in V.

The strength of a path is defined to be the weight of the weakest arc of the path. If a path has length zero, then its strength is defined to be $\sigma(u_0)$.The path ρ is called a cycle if $u_0=u_n$ *and* $n\ge 3$. Two nodes that are joined by a path are said to be connected. A strongest path joining any two nodes u,v is a path corresponding to maximum strength between u and v.

A fuzzy graph is said to be strong fuzzy graph if $\mu(u,v)=\sigma(u)\wedge\sigma(v)$ for all u,v in V.A vertex u is said to be isolated vertex $\mu(u,v)<\sigma(u)\wedge\sigma(v)$ for all v $v\in V-\{u\}$.A fuzzy graph is said to be complete fuzzy graph if all the edges are effective between every pair of vertices and is denoted by K_{cf}. $N(u)=\{v\in V / \mu(u,v)=\sigma(u)\wedge\sigma(v)\}$ is called the open neighborhood of u and N[u] = $N(u)\cup\{u\}$ is the closed neighborhood of u.

A. *Fuzzy Dominating Set*

A subset S of V is called a dominating set in G if every vertex in V-S is adjacent to at least one vertex in S. The minimum cardinality taken over all dominating sets in G is called the domination number of G and is denoted by γ_f .A dominating set S of a fuzzy graph G is said to be connected dominating set of G if the induced subgraph $\langle S\rangle$ is connected. The minimum cardinality taken over all connected dominating sets is the connected domination number and is denoted by $\gamma_{fc}(G)$.

B. *Fuzzy Triple Connected Domionating Set*

A fuzzy graph G is said to be fuzzy triple connected if any three vertices lie on a path in G.A subset S of V of a fuzzy graph G is said to be fuzzy triple connected dominating set ,if S is a dominating set and the induced subgraph $\langle S\rangle$ is triple connected. The minimum cardinality taken over all triple connected dominating sets is called the triple connected domination number of G and is denoted by $\gamma_{ftc}(G)$.Any triple connected dominating set with γ_{ftc} vertices is called a γ_{ftc} set of G.

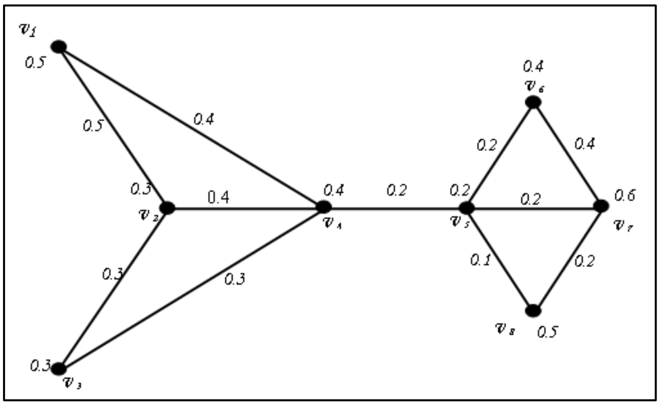
C. *Fuzzy Equitable Domionating Set*

A subset S of V is called an fuzzy equitable dominating set if for every v$\in$ V-S there exists a vertex u$\in S$ such that uv $\in$ E (G) and $\left|\deg(u)-\deg(v)\right|\leq 1$.The minimum cardinality of such a dominating set is denoted by γ_{fe} and is called the fuzzy equitable domination of G.A equitable dominating set S is said to be a minimal equitable dominating set if no proper subset of S is an equitable dominating set. If $u\in V$ *such that* $\left|\deg(u)-\deg(v)\right|\geq 2$ *for every* $v\in N(u)$ then u is in every fuzzy equitable dominating set such points are called an equitable isolates and I_{fe} denotes the set of all fuzzy equitable isolates. A fuzzy equitable dominating set S of connected graph G is called a fuzzy equitable connected dominating set (fecd-set) if the induced subgraph $\langle S\rangle$ is connected.The minimum cardinality of a fecd-set of G is called an fuzzy equitable connected domination number of G is denoted by $\gamma_{fec}(G)$.

III. MAIN RESULTS

A subset S of V of a fuzzy graph G is said to be fuzzy triple connected equitable dominating set. If S is an fuzzy equitable dominating set and the induced subgraph $\langle S\rangle$ is triple connected .The minimum cardinality taken over all fuzzy triple connected equitable dominating set is called the fuzzy triple connected equitable domination number of G and is denoted by $\gamma_{ftce}(G)$ .Any fuzzy triple connected equitable dominating set with γ_{ftce} -set of G.

A. *Example*

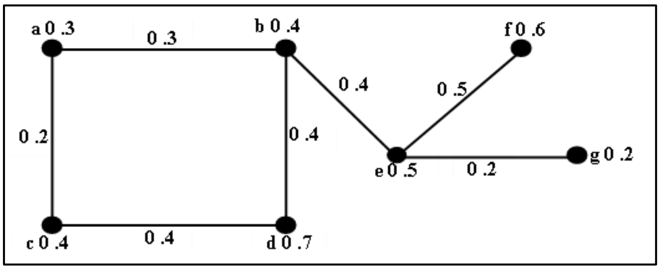


S={v_2,v_4,v_5} is an fuzzy triple connected equitable dominating set.

V-S={ v_1,v_3,v_6, v_7,v_8}

$\langle S\rangle$ is connected.

B. *Example*



S={d,b,e} is an fuzzy triple connected equitable dominating set

V-S={ a,c,f,g}

$\langle S\rangle$ is connected.

From the above examples we obtained the following proposition.

C. *Proposition*

For any fuzzy graph G: $\gamma_{fe}(G)\leq\gamma_{fce}(G)\leq\gamma_{ftce}(G)$

1) *Proof:*

It is clear that for graph G,any fuzzy triple connected equitable dominating set S is connected fuzzy equitable dominating also any connected fuzzy equitable dominating set is an equitable dominating set.

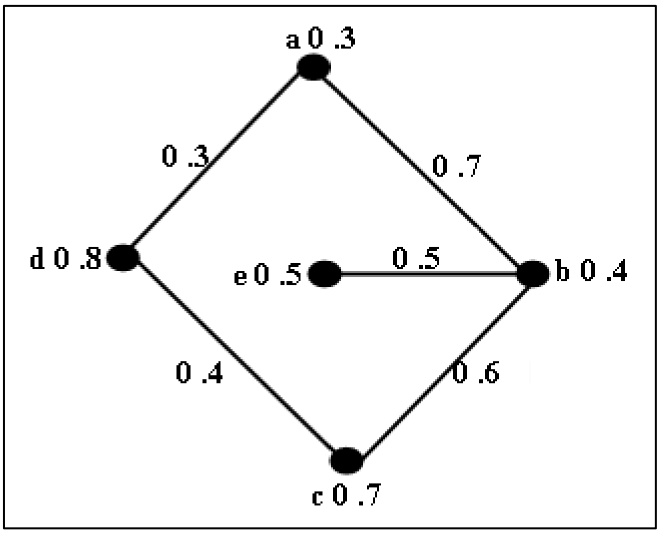
D. *Theorem*

If a spanning fuzzy subgraph H of a graph G has a fuzzy triple connected equitable dominating set, then G also has a triple connected equitable dominating set.

E. *Theorem*

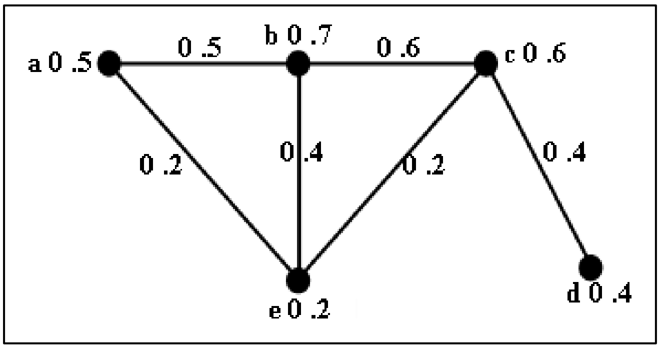
Let G be a connected graph and H be a spanning fuzzy subgraph of G .If H has a fuzzy triple connected equitable dominating set, then $\gamma_{ftce}(G)\leq\gamma_{ftce}(H)$ and the bound is sharp.

F. *Example*

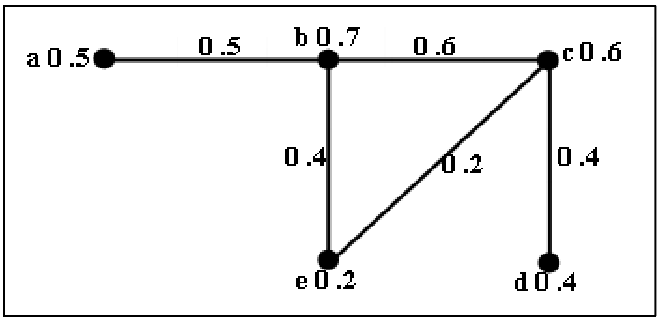


S = {a, b, c}

G. Example



H. Example



For the graph G_5 S= S= {b,c,d} is a fuzzy triple connected equitable dominating set .So $\gamma_{ftce}(G_5) = 1.7$ for the fuzzy spanning subgraph H_5 *of* G_5,S=$\{a,b,c,d\}$is a fuzzy triple connected equitable dominating set so that $\gamma_{ftce}(H_5) = 2.2$

Hence $\gamma_{ftce}(G) \le \gamma_{ftce}(H)$.

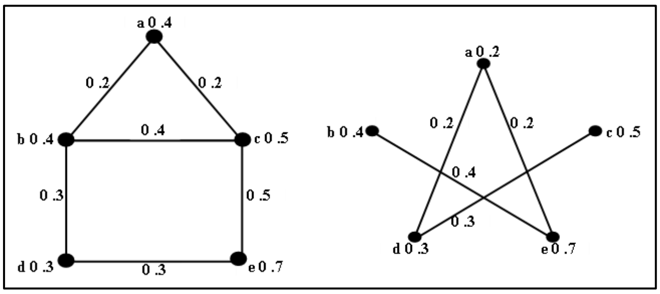
I. Remark

Fuzzy triple connected equitable dominating set doesn't exist for all graphs and if it exists then $\gamma_{ftce}(G) \ge 3\sigma$

J. Remark

The complement of triple connected equitable dominating set need not be a triple connected equitable dominating set.

K. Example



L. Theorem:

For any Fuzzy graph $p - q \le \gamma_{ftce} \le p - \delta_E$ where p,q, δ_E are the order ,size and minimum effective incident degree of G respectively.

1) Proof:

Let S be a dominating set and γ_{ftce} be the minimum fuzzy triple connected equitable domination of G .Then the scalar cardinality of V-S is less than or equal to the scalar cardinality of $V \times V$ .Hence $p - q \le \gamma_{ftce}$ .Now let u be the vertex with minimum effective incident degree δ_E .Clearly v-{u} is a fuzzy triple connected equitable dominating set and hence $\gamma_{ftce} \le p - \delta_E$.*Hence* $p - q \le \gamma_{ftce} \le p - \delta_E$ is true for any fuzzy graph.

M. Remark

If all the vertices having the same membership grade, then

$$p - q \le \gamma_{ftce} \le p - \Delta_E$$

References

[1] E.J.Cockayne, R.M.Dawes And S.T.Hedetniemi, Networks 10 (1980), 211-19.
[2] E.J.Cockayne and S.T.Hedetniemi, Networks 7 (1977), 247-61.
[3] A. Kaufmann(1975), Introduction to the theory of Fuzzy Subsets, Academic Press, Newyork
[4] G.Mahadevn,A.Selvam ,J.Paulraj Joseph,B.Ayisha and Subramaniant.(2012):Complementary triple connected domination number of a graph.
[5] M.Murugan Graph Theory and Algorithms, Muthali Publishing House, Chennai, 2003.
[6] A.NagoorGani and K.RadhaOn Regular Fuzzy Graphs
[7] J.Paulraj Joseph ,M.K.Angel Jebitha ,p.Chithra Devi and G.Sudhana(2012):triple connected graphs ,Indian Journal of Mathematics mathematical Sciences,Vol.8 No1,pp.61-75
[8] A. Rosenfield , Fuzzy graphs In: Zadeh, L.A., Fu, K.S., Shimura, M.(Eds), Fuzzy sets and their applications(Academic Press, New York)
[9] A.Somasundaram, S.Somasundaram, Domination in Fuzzy Graphs - I, February - 1998.
[10] S.Sivakumar ,N.D.Soner Triple connected domination in graphs.
[11] Venkatasubramanian Swaminathan And Kuppusamy Markandan Dharma lingam Degree Equitable Domination on Graph
[12] L.A.Zadeh,(1971), Similarity Relations and Fuzzy Ordering, Information sciences, 3(2),pp.177- 200.