An Overview of Applications of Fourier Transforms and Z-Transforms in Engineering and Technology

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Abstract— Applied mathematics is used in all the fields of science and technology. In order to manipulate physical and engineering problems, one can apply mathematical tools like Fourier-Transforms [1], Laplace Transforms and Z-Transforms [2]. In this paper we will study an overview application of Fourier Transforms and Z Transforms in Engineering and Technology.

Key words: Fourier Series, Fourier Transforms Z-Transforms, Periodic Function, Signals

I. Fourier Transforms

A. Introduction

In most of the physical and engineering applications complicated functions are approximated by simpler functions which can be easily handled for further computations. In that sense, to find the solution of engineering problems, periodic functions need to be converted into series expansion in terms of sine and cosine functions. That is to express them in terms of a sum of sine and cosine functions of various amplitudes and frequencies. This analysis is basically concerned with representing and analyzing periodic phenomena, through series, and then by extending these concepts to non-periodic phenomena, with the use of Fourier Tr*ansform*. Discrete set of frequencies in the periodic functions transforms into a continuum of frequencies in the non-periodic case.

The Fourier series expansion of a periodic function f(x) in (a, a+2l) is

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n \cos\frac{n\pi x}{L} + \sin\frac{n\pi x}{L}\right)$$

For example, Fourier series of x^2 in $(0,2\pi)$ is

$$x^2 = \frac{4\pi^2}{3} + \sum_1^{\infty}\left(\frac{4cosnx}{n^2} - \frac{4\pi sinnx}{n}\right)$$

The Fourier –Transform of g(t) is

$$F(s) = F(g(t)) = \int_{-\infty}^{\infty} e^{-ist}\, g(t)dt.$$

For example, Fourier transform of the function

$$f(t) = \begin{cases} x^2 & if \quad |x| < a \\ 0 & if \quad |x| > a \end{cases} \text{ is}$$

$$\frac{2((a^2s^2 - 2)sinas + 2ascosas)}{s^3}$$

Techniques based on the Fourier transforms are used in all branches of science and technology. This plays an important role in solving problems pertaining to heat conduction, digital signal processing, pattern recognition, image reconstruction and so on.

B. Applications of Fourier series and Fourier transforms

1) Electric Circuits

Electricity consumption [3] in several types of civil constructions can be viewed in terms of periodic and irregular activity. These different observations vary in terms of time frame, from hourly to seasonal, making it difficult for construction models to accurately predict electricity consumption. Accurate predictions are possible through Fourier models.

2) Pattern recognition

A discrete cosine transform (DCT) is defined and an algorithm to compute it using the Fast Fourier transform. It is possible that the discrete cosine transform can be used in the area of digital processing for the purposes of pattern recognition[3] and Wiener filtering.

3) Magnetic resonance imaging

(MRI) is usually implemented as a Fourier transform-based technique. During data acquisition, spatially resolved information relating to spin density, relaxation rates, chemical shifts, and other parameters is phase and frequency encoded in the measured data. Image reconstruction is accomplished through the use of the Fourier series model, which can be evaluated deficiently using a Fast Fourier transform (FFT) algorithm.

II. Z-Transforms

A. Introduction

The Z-transform is a powerful method for solving difference equations and, in general, to present discrete systems. Although applications of Z-transforms are relatively new, the main features of this mathematical technique date back to the early 17^{th} century when De-Moivre's introduced the concept of a generating functions which is similar to that for the Z-transform. Now a days, the development and extensive uses of the Z-transform are much enhanced as a result of the use of advanced digital computers.The z-transform defines the relationship between the time domain signal, *x* [*n*], and the z-domain signal, *X*(*z*).

For the sequence $\{a_1, a_2, a_3 \text{..........} a_n \text{.......}\}$, the Z-transform is given by

$$U(z) = \sum_{n=0}^{\infty} (a_n\ z^{-n}).$$

As analog filters are developed using the technique of Laplace transform, recursive digital filters are designed with the aid of the z-transform.

B. Applications of Z-Transform

1) Digital signal processing

With the vast development in the field of digital communication networks [4] and multimedia, the need for techniques to study, understand, analyze and process digital data is becoming more and more important these days. JPEG images, MP3/4 music, MPEG-2 audios and videos, and ZIP files etc. are all processed using digital processing techniques. For such systems Z-transform is a powerful available mathematical tool for the design, analysis and monitoring the stability. Z-transform converts a sequence, {x[n]}, into a function, X(z), of an arbitrary complex-valued variable z. This is very important as complex functions are easy to manipulate than sequence. Hence Z-transform is highly utilized in the digital signal processing and digital filters analysis.

2) Solutions of Difference Equations with Constant Coefficients.

Z-Transforms [5] are applied to those mathematical models consisting of difference equations, these are very usual and different from those described by differential equations. Z-Transform is employed in computer controlled systems that take measurements with digital I/O boards, calculate an output voltage. Also we can observe that it can used in the development of Digital Filters - which are found in digital signal processing where digital filtering is done. These include digital signal transmission systems like the telephone and mobiles network that process audio signals. For example, a CD contains digital signal information that can be processed with a digital filter. There are an incredible number of systems where we use every day that have digital components which satisfy difference equations.

Some other applications are in

- Finite Impulse Responses (FIR) Filters.
- Linear, Time-Invariant, Discrete-Time, Dynamical Systems.
- Linear Discrete-Time Filters.
- Optimum Linear Filtering.

III. Conclusion

Applications of transform methods are widely utilized in all streams of science and engineering. Students and research scholars of software engineering and electronics & communication engineering can employ the techniques of transform methods for further research and development.

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