Noise Removal in Spatial Domain for MRI Images through Non-Linear Filters with varying Window Size

Smita Agrawal[1] Kamlesh Kumar Samota[2] Umang Varshney[3]
[1,2,3]Department of Electronics Design & Technology
[1,2,3]NIELIT, Gorakhpur, India

Abstract— Noise Removal is a major issue in Biomedical Image Processing. Ultrasound spectroscopy and Magnetic Resonance Image (MRI) require highly efficient output images for analysis. Various techniques exist in literature for filtering noisy images. The type of filtering is highly dependent upon the noise density, window size of the filter and the type of noise. In this paper, various filtering techniques for noise removal from MRI images are discussed. Two types of filtering approaches are employed in noise removal: Linear filtering and Non-Linear filtering. Each type is described in detail along-with their advantages as well as disadvantages in a particular domain. The filters are implemented in MATLAB.

Key words: HMF (Hybrid Median Filter), Gaussian Filter, Adaptive Wiener Filter, Standard Median Filter, Weighted Median Filter, Average Filter, SNR (Signal to Noise Ratio), PSNR (Peak SNR), MSE (Mean Square Error)

I. INTRODUCTION

A major portion of information received by a human from the environment is visual. Hence, processing visual information by computer has been a vital part of research over a last few decades. The process of receiving and analysing visual information by digital computer is called digital image processing. It is concerned primarily with extracting useful information from images. Image processing algorithms may be placed at three levels. At the lowest level are those techniques which deal directly with the raw, possibly noisy pixel values, with denoising and edge detection being good examples. In the middle are algorithms which utilize low level results for further means, such as segmentation and edge linking. At the highest level are those methods which attempt to extract semantic meaning from the information provided by the lower levels, for example, handwriting recognition [2]. Digital image processing may be classified into various sub branches based on methods [2] are (i) input and output are images and (ii) inputs may be images whereas outputs are attributes extracted from those images. Following is the list of different image processing functions based on the above two classes. (i) Image Acquisition (ii) Image Enhancement (iii) Image Restoration (iv) Colour Image Processing (v) Multi-resolution Processing (vi) Compression (vii) Morphological Processing (viii) Image Segmentation (ix) Representation and Description and (x) Object Recognition. For the first seven functions, the inputs and outputs are the images whereas for the rest three the outputs are attributes from the input images. Out of the ten sub branches of digital image processing, cited above, this paper deals with image restoration. The image restoration deals with the noise removal from images. Image restoration is distinct from image enhancement, which is used to manipulate an image in order to produce results more pleasing to an observer, without making use of any particular degradation models [6][11]. On the other hand, restoration emphasizes on recovery of the original image as far as possible from the degraded one. Noise is the result of errors in the image acquisition process that result in pixel values that do not reflect the true intensities of the real scene. There are several ways that noise can be introduced into an image, depending on how the image is created.

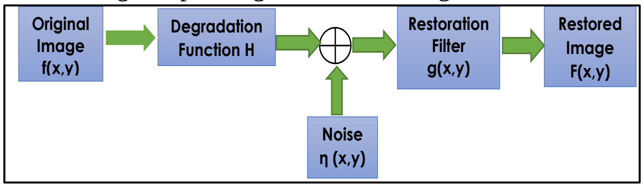

Fig. 1: Image degradation and restoration model

Image restoration employs different filtering techniques. Filtering is a neighbourhood operation, in which the value of any given pixel in the output image is determined by applying some algorithm to the values of the pixels in the neighbourhood of the corresponding input pixel [2][12][13][17]. Certain image processing operations involve processing an image in sections, called blocks or neighbourhoods, rather than processing the entire image at once. Filtering may be done either in spatial domain or in frequency domain. Frequency filtering is more appropriate if no straightforward kernel can be found in the spatial domain, and may also be more efficient [19]. Since the multiplication in the Fourier space is identical to convolution in the spatial domain, all frequency filters can in theory be implemented as a spatial filter [16]. In this paper, different spatial domain restoration techniques are studied and proposed. The spatial domain is the normal image space, in which a change in position in I directly projects to a change in position in S. Distances in I (in pixels) correspond to real distances (e.g. in meters) in S. Filters are broadly classified into two major categories: (i) Linear and (ii) Non-linear.

II. LINEAR FILTERING

In the early development of image processing, linear filters were the primary tools. Their mathematical simplicity with satisfactory performance in many applications made them easy to design and implement. However, in the presence of noise the performance of linear filters is poor. They tend to blur edges, do not remove impulse noise effectively, and do not perform well in the presence of signal dependent noise [16]. A linear filter satisfies both the superposition and homogeneity principles (linearity principle), hence the filter is said to be linear. Linear filtering of an image is accomplished through an operation called convolution. Convolution is a neighbourhood operation in which each output pixel is the weighted sum of neighbouring input pixels. The matrix of weights is called the **convolution kernel**, also known as the filter. A convolution kernel is a correlation kernel that has been rotated 180 degrees. The spatial domain convolution can be represented as:

$$g(x,y) = h(x,y) \odot f(x,y) \qquad (1)$$

Where f(x,y) is the input image and h(x,y) is the filter function. The discrete convolution can be defined as a `shift and multiply' operation, where we shift the kernel over the image and multiply its value with the corresponding pixel values of the image [2]. For a square kernel with size M × M, we can calculate the output image with the following formula:

$$g(x,y) = \sum_{m=-\frac{M}{2}}^{\frac{M}{2}} \sum_{n=-\frac{M}{2}}^{\frac{M}{2}} h(m,n) f(x-m, y-n) \qquad (2)$$

A. *Average (Mean) Filter*

The idea of mean filtering is simply to replace each pixel value in an image with the mean (`average') value of its neighbours, including itself. This has the effect of eliminating pixel values which are unrepresentative of their surroundings [19][26]. Mean filtering is usually thought of as a convolution filter.

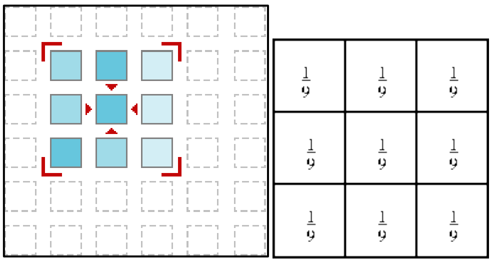

Fig. 2: 3 x 3 kernel used for mean filter

Let S_{xy} represent the set of coordinates in a rectangular subimage window of size m X n, centered at point (x, y). The arithmetic mean filtering process computes the average value of the corrupted image f(x, y) in the area defined by S_{xy}. The value of the restored image at any point (x, y) is simply the arithmetic mean computed using the pixels in the region defined by S. In other words,

$$F(x,y) = \frac{1}{mn} \sum_{(s,t)\in S_{xy}} f(x,y) \qquad (4)$$

B. *Adaptive Wiener Filter*

In signal processing, the Wiener filter is a filter used to produce an estimate of a desired or target random process by linear time-invariant (LTI) filtering of an observed noisy process, assuming known stationary signal and noise spectra, and additive noise [27]. This filter is frequently used in the process of deconvolution. Wiener filters work best for Gaussian noise removal as they tend to minimize the mean square error (MSE) by comparing the received signal with an estimation of a desired noiseless signal. They take a statistical approach by taking into account the spectral properties of the original signal and noise. These are fairly versatile for impulse and speckle noise removal as well, especially for higher levels of noise. When s(x,y) and n(x,y) are stationary Gaussian processes the Wiener filter is the optimal filter. Specifically, when s(x,y) is also a white Gaussian process the Wiener filter has a very simple scalar form:

$$\hat{s}(x,y) = \frac{\sigma_{\hat{s}}^2(x,y)}{\sigma_{\hat{s}}^2(x,y) + \sigma_n^2(x,y)} [f(x,y) - \mu_s(x,y)] + \mu_s(x,y) \quad (4)$$

It is especially useful for eliminating the ringing effect due to blurring of images.

$$\text{MSE}(\hat{s}) = \frac{1}{N} \sum_{i,j=1}^{N} (\hat{s}(x,y) - s(x,y))^2 \qquad (5)$$

The Wiener filter tailors itself to the local image variance. Where the variance is large, it performs little smoothing. Where the variance is small, it performs more smoothing [27]. This approach often produces better results than linear filtering. Wiener filters are non-adaptive by themselves and work well in frequency domain, but these are often unable to restore high frequency components and are time consuming, thus taking inverse FFT of Wiener filter to obtain the impulse response in spatial domain speeds up the filtering process and the filter may be used as an adaptive filter, causing more efficient de-blurring [14].

C. *Gaussian Filter*

A Gaussian filter is a filter whose impulse response is a Gaussian function (or an approximation to it). Gaussian filters have the properties of having no overshoot to a step function input while minimizing the rise and fall time. This behaviour is closely connected to the fact that the Gaussian filter has the minimum possible group delay [11][19]. The Gaussian smoothing operator is a 2-D convolution operator that is used to `blur' images and remove detail and noise. In this sense, it is similar to the mean filter, but it uses a different kernel that represents the shape of a Gaussian (`bell-shaped') hump. In 2-D, an isotropic (*i.e.* circularly symmetric) Gaussian has the form:

$$G(x,y) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}} \quad (6)$$

The idea of Gaussian smoothing is to use this 2-D distribution as a `point-spread' function, and this is achieved by convolution. Since the image is stored as a collection of discrete pixels we need to produce a discrete approximation to the Gaussian function before we can perform the convolution. In theory, the Gaussian distribution is non-zero everywhere, which would require an infinitely large convolution kernel, but in practice it is effectively zero more than about three standard deviations from the mean, and so we can truncate the kernel at this point [26]. The degree of smoothing is determined by the standard deviation of the Gaussian. (Larger standard deviation Gaussians, of course, require larger convolution kernels in order to be accurately represented). The Gaussian outputs a `weighted average' of each pixel's neighbourhood, with the average weighted more towards the value of the central pixels. This is in contrast to the mean filter's uniformly weighted average. Because of this, a Gaussian provides gentler smoothing and preserves edges better than a similarly sized mean filter [6][11]. One of the principle justifications for using the Gaussian as a smoothing filter is due to its frequency response [6][19]. Most convolution-based smoothing filters act as lowpass frequency filters. This means that their effect is to remove high spatial frequency components from an image. The frequency response of a convolution filter, i.e. its effect on different spatial frequencies, can be seen by taking the Fourier transform of the filter. Both filters attenuate high frequencies more than low frequencies, but the mean filter exhibits oscillations in its frequency response. The Gaussian on the other hand shows no oscillations So by choosing an appropriately sized Gaussian filter we can be fairly confident about what range of spatial frequencies are still present in the image after filtering, which is not the case of the mean filter [10][12][13]. Thus, Gaussian filter work best for speckle noise removal [26].

III. NON-LINEAR FILTERING

In contrast to the frequency domain, it is easily possible to implement non-linear filters in the spatial domain [17]. In this case, the summations in the convolution function are replaced with some kind of a non-linear operator $O_{m,n}$.

$$g(x,y) = O_{m,n}[h(m,n)f(x-m, y-n)] \qquad (7)$$

In signal processing, a non-linear filter is a filter whose output is not a linear function of its input. Many nonlinear noise-removal filters operate in the time domain [16][17][18]. They typically examine the input digital signal within a finite window surrounding each sample, and use some statistical inference model (implicitly or explicitly) to estimate the most likely value for the original signal at that point [14][15].

A. *Standard Median Filter*

It is very simple to implement and much efficient as well.

$$F(x,y) = \mathrm{MEDIAN}_{(k,l)\epsilon S}(f(k,l)) \qquad (8)$$

Where f denotes the observed image, F is the restored image and S denotes the neighbouring pixels of f(x,y). The median is calculated by first sorting all the pixel values from the surrounding neighbourhood into numerical order and then replacing the pixel being considered with the middle pixel value. The median is much less sensitive than the mean to extreme values (called outliers). Median filtering is therefore better able to remove these outliers without reducing the sharpness of the image [16]. All smoothing techniques are effective at removing noise in smooth patches or smooth regions of a signal, but adversely affect edges. Often though, at the same time as reducing the noise in a signal, it is important to preserve the edges. Edges are of critical importance to the visual appearance of images, for example. For small to moderate levels of (Gaussian) noise, the median filter is demonstrably better than Gaussian blur at removing noise whilst preserving edges for a given, fixed window size [5][6][7][9]. However, its performance is not that much better than Gaussian blur for high levels of noise, whereas, for speckle noise and salt-and-pepper noise (impulsive noise), it is particularly effective [22].

B. *Weighted Median Filter*

This is another non-linear median filter [20] and [21], which favours the center pixel than others. Let the window size be $(2n+1)^2$ and L=2n(n+1).

$$f^{2k}(x,y) = \mathrm{MEDIAN}\,\{\, f(x-u,y-v)(2k)f(x,y)| -h \le (u,v) \le h \,\} \qquad (9)$$

Where 2k is the weight given to pixel (x,y) and $f^{2k}(x,y)$ represents the repetition operation [3]. Hence, in a 3 x 3 window $f^{2k}(x,y)$ is the median of (9+2k) gray values with the center value of the window repeated (2k+1) times, $f^0(x, y)$ is the standard median filter, whereas $f^{2k}(x, y)$ becomes identity filter when $k \geq L$.

C. *Hybrid Median Filter*

The Hybrid Median Filter performs filtering of the matrix A using an N x N box. It preserves edges better than a square kernel (neighbour pixels) median filter because it is a three-step ranking operation: data from different spatial directions are ranked separately.

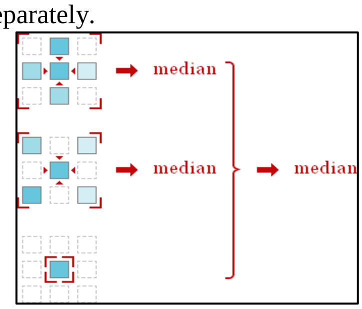

Fig. 3: Hybrid Median filter operation

The basic idea behind this filter is that for any element of the signal (image) apply median technique several times varying window shape and then take the median of the computed median values [1]. In one phrase idea is like "apply median filter with cross-mask, apply median filter with x-mask and take the median of got results and element itself". One advantage of the hybrid median filter is due to its adaptive nature, which allows the filter to perform better than the standard median filter on fast-moving picture information of small spatial extent [25].

IV. RESULTS

Six types of filters are designed for removal of three types of noise; viz., salt and pepper (impulse) noise, speckle noise and gaussian noise. These are Average (Mean) Filter, Adaptive Wiener Filter, Gaussian Filter for linear filtering and Standard Median Filter, Weighted Median Filter and Hybrid Median Filter for non-linear filtering. Four window sizes; viz, 3 x 3, 5 x 5, 7 x 7, 9 x 9 were used subsequently. Some filtered images are shown below. The noise density used is 3% and 10%.

Fig. 4: Original Image

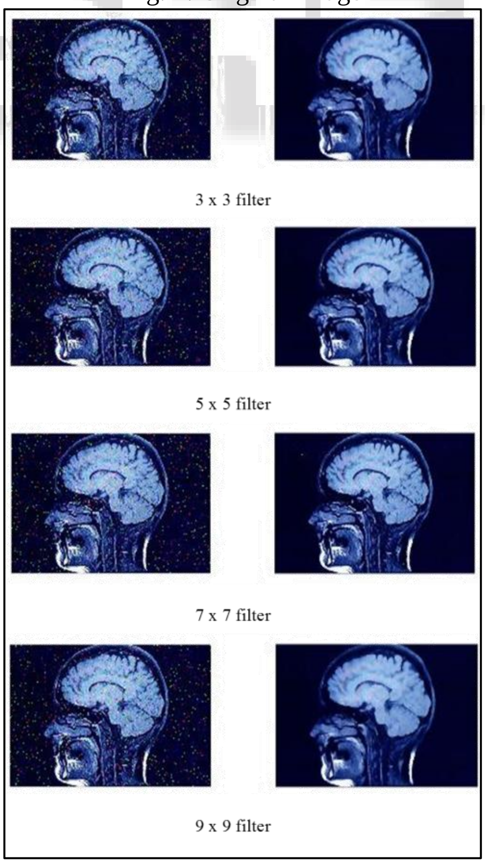

Fig. 5: Hybrid Median Filters for salt and pepper noise removal (3%noise)

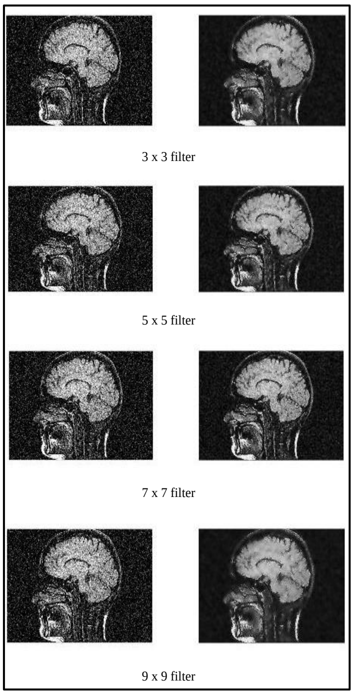

Fig. 6: Adaptive Wiener Filters for Gaussian noise removal (3% noise)

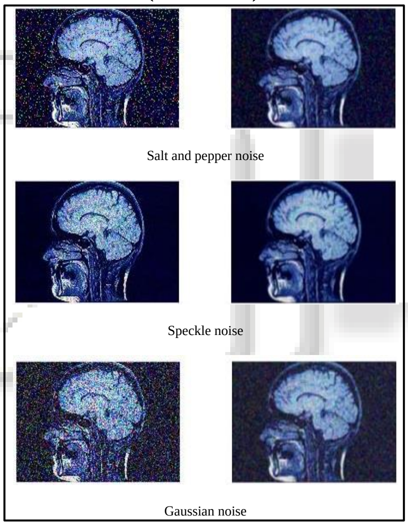

Fig. 7: Average Filtering for 10% noise

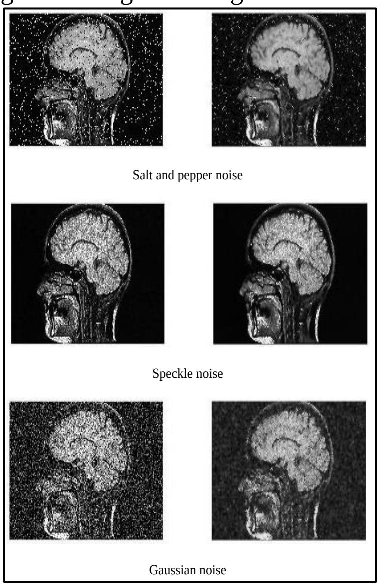

Fig. 8: Adaptive Wiener Filtering for 10% noise

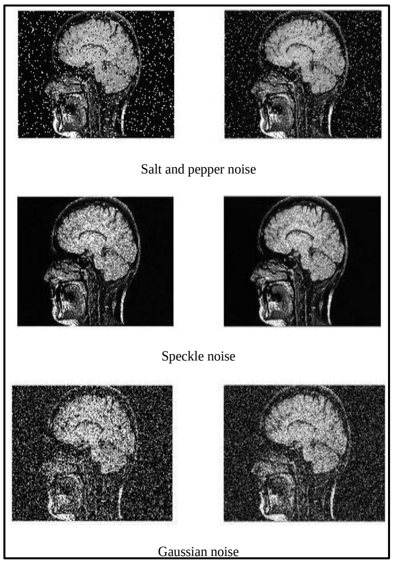

Fig. 9: Gaussian filtering for 10% noise

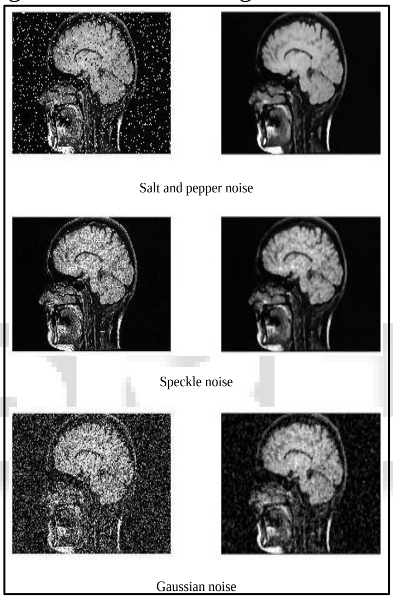

Fig. 10: Standard Median Filtering for 10% noise

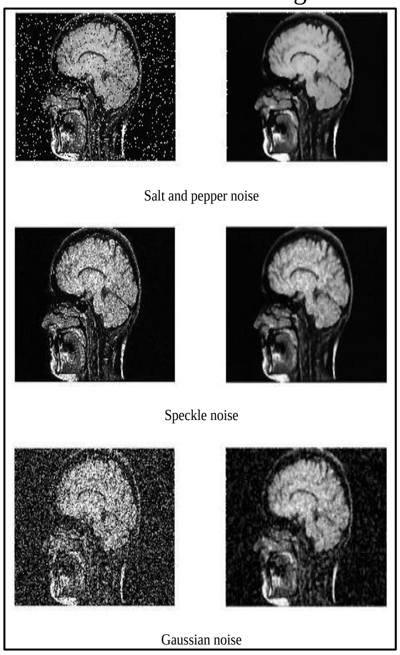

Fig. 11: Weighted Median Filtering for 10% noise

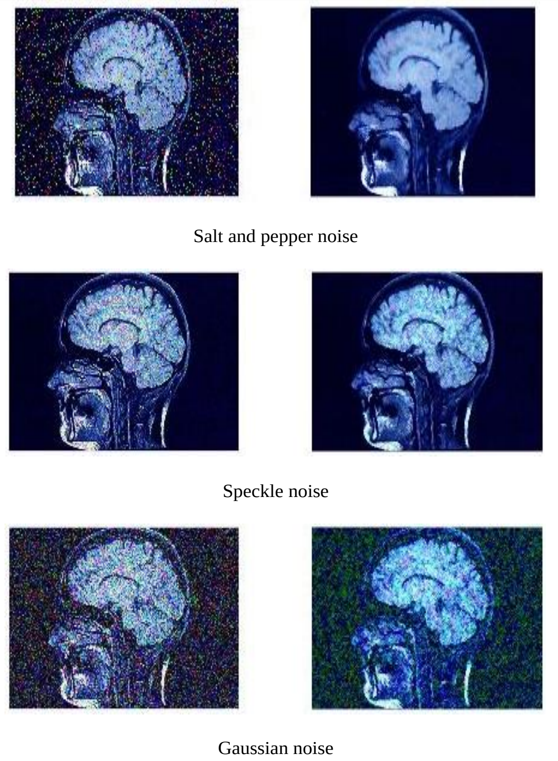

Fig. 12: Hybrid Median Filtering for 10% noise

NOISE	Para-meter	Average (Mean) Filter	Adaptive Wiener Filter	Gaussian Filter	Standard Median Filter	Weighted Median Filter	Hybrid Median Filter
Salt and Pepper (Impulse) Noise	SNR (dB)	14.538	10.319	12.568	16.339	9.208	20.097
	PSNR (dB)	23.373	20.190	22.440	26.212	19.018	28.932
	MSE	0.0046	0.0096	0.006	0.0024	0.0124	0.0013
Speckle Noise	SNR (dB)	15.641	17.598	17.899	15.035	15.473	16.941
	PSNR (dB)	24.476	27.472	27.772	24.909	25.346	25.776
	MSE	0.0036	0.0018	0.0017	0.0032	0.0029	0.0026
Gaussian Noise	SNR (dB)	12.478	11.543	11.11	12.007	7.240	10.88
	PSNR (dB)	21.315	21.416	20.055	21.880	17.114	19.715
	MSE	0.0014	0.0072	0.0102	0.0065	0.0194	0.0095

Table 1: Performance of 3 x 3 filters

NOISE	Para-meter	Average (Mean) Filter	Adaptive Wiener Filter	Gaussian Filter	Standard Median Filter	Weighted Median Filter	Hybrid Median Filter
Salt and Pepper (Impulse) Noise	SNR (dB)	12.323	12.219	12.695	12.2951	9.063	17.340
	PSNR (dB)	21.158	22.092	22.569	22.168	18.936	26.177
	MSE	0.0070	0.0062	0.0055	0.006	0.0128	0.0024
Speckle Noise	SNR (dB)	12.575	16.651	17.924	11.971	15.416	15.539
	PSNR (dB)	21.4100	26.524	27.797	21.845	25.289	24.375
	MSE	0.0072	0.0022	0.0017	0.0065	0.0030	0.0037
Gaussian Noise	SNR (dB)	11.209	11.723	10.15	11.175	7.236	11.439
	PSNR (dB)	20.045	21.597	20.020	21.049	17.109	20.275
	MSE	0.0099	0.0069	0.0100	0.008	0.0195	0.0094

Table 2: Performance of 5 x 5 filters

NOISE	Para-meter	Average (Mean) Filter	Adaptive Wiener Filter	Gaussian Filter	Standard Median Filter	Weighted Median Filter	Hybrid Median Filter
Salt and Pepper (Impulse) Noise	SNR (dB)	11.165	13.083	12.686	10.719	9.291	15.484
	PSNR (dB)	20.003	22.956	22.559	20.593	19.165	24.319
	MSE	0.0100	0.0051	0.0055	0.0087	0.0121	0.0037
Speckle Noise	SNR (dB)	11.266	15.749	17.913	10.556	15.495	14.351
	PSNR (dB)	20.102	25.623	27.787	20.429	25.368	23.187
	MSE	0.0098	0.0027	0.0017	0.0091	0.0029	0.0048
Gaussian Noise	SNR (dB)	10.328	11.602	10.162	10.297	7.245	11.389
	PSNR (dB)	19.165	21.475	20.035	20.17	17.118	20.225
	MSE	0.0121	0.0071	0.099	0.009	0.0194	0.0095

Table 3: Performance of 7 x 7 filters

NOISE	Para-meter	Average (Mean) Filter	Adaptive Wiener Filter	Gaussian Filter	Standard Median Filter	Weighted Median Filter	Hybrid Median Filter
Salt and Pepper (Impulse) Noise	SNR (dB)	10.398	12.905	12.735	9.732	9.076	14.332
	PSNR (dB)	19.234	22.778	22.609	19.605	18.949	23.168
	MSE	0.0119	0.0053	0.0055	0.0110	0.0127	0.0048
Speckle Noise	SNR (dB)	10.454	14.932	17.868	9.635	15.522	13.465
	PSNR (dB)	19.289	24.806	27.741	19.509	25.355	22.301
	MSE	0.0118	0.0033	0.0017	0.0112	0.0029	0.0059
Gaussian Noise	SNR (dB)	9.716	11.223	10.202	9.548	7.270	11.253
	PSNR (dB)	18.552	21.096	20.076	19.421	17.143	20.089
	MSE	0.0140	0.0078	0.0980	0.0114	0.0193	0.0098

Table 4: Performance of 9 x 9 filters

V. CONCLUSION

From the above simulation results and tables, it is clearly evident that for lower window sized filtering, i.e., for 3 x 3 and 5 x 5 dimensions, Hybrid median filters provide the best results for salt and pepper noise removal in terms of SNR and MSE. Their performance, however, is poor for gaussian noise removal as compared to other filters. Gaussian filters provide best filtering for speckle noise and their dominance increases as filter window size is increased. Adaptive Wiener filters are fairly versatile for most types of noise removal as these cause excellent deblurring; marginally being better than other methods for gaussian noise removal. However, these are inefficient for salt and pepper noise removal, especially at lower window sizes. Standard median filters provide fairly average results for salt and pepper noise removal at low window sizes, while conventional weighted median filters provide reasonable results only for speckle noise filtering. Further investigations are required to be carried out for finding techniques to improve the efficiency of median filtering methods at higher filter window dimensions [5][8][14]. Weighted median filters may be improved by improved center weighting [4][9][23].

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