Application of Queuing Theory on Toll Plaza to Solve Traffic Problem

Sachin Antil
M.Sc. Student
Department of Applied Mathematics
Amity University Noida, India

Abstract— The biggest disadvantage of building a toll plaza is that it increases the traffic on highway. Therefore the aim of our study is to observe the present situation of traffic congestion at a highway toll plaza and how can we use Queuing Theory and draw the possible solutions to increase the efficiency in order to reduce the waiting time of the customers and their money as well. There are multiple phases in which this study is carried out or identification of the problem, collection of the data, data analysis and then a toll plaza in Delhi was selected to do the analysis. The current operational effectiveness of can be drawn using data analysis and the parameters such as the Arrival Rates, Service Rates and the Number of toll booths. And finally a conclusion is drawn using Queuing Theory which can be recommended or used at other toll plazas throughout the country.

Key words: Traffic Problem, Application of Queuing Theory

I. INTRODUCTION

The rate of arrival of traffic is always high near a metropolitan city. Since vehicles are arriving in very large numbers that directly implies that service time will also increase and hence people have to wait for a long time (waiting time) in the queues.

Fig. 1: Toll Plaza

The most important factor in the operational effectiveness of a toll plaza is its queue length. The length of queue on a toll plaza is directly proportional to the waiting time, service time and the arrival pattern of the vehicles. Longer waiting times in the queues results in losses such as fuel wastage, increases pollution and opportunity cost in terms of time wastage which often results in frustration of the commuters. A person driving to Delhi daily through this toll plaza can save upto Rs.30000 annually including toll and the fuel costs during the wait at the toll. Therefore the goal of the study is to reduce the waiting time for the drivers by analyzing the vehicle arrival pattern at a highway toll plaza in Delhi with applications of queuing theory.

II. PROBLEM DESCRIPTION

A toll plaza has been selected in Delhi (in Narela on Delhi-Sonipat border) consisting of 2 lanes only in which 1 lane is for incoming traffic and 1 lane is for outgoing traffic. The study will be focused on the 1 lane from which the traffic is entering the system. The incoming traffic will be observed from 10am to 6 pm on a Sunday and a Monday in order to show the difference in traffic between a non-working day and a working day. The Analysis is only done for the busiest hour of both the days. A comparison of the traffic situation of both the days is drawn using the queuing theory formulae and hence suggestions are made on how to improve the current scenario.

III. METHODOLOGY

In this problem the solution will be obtained by using the Queuing Theory. We are going to take in the consideration the various parameters of queuing theory such as the average number of customers arriving in the queuing system per minute, the average number of customers getting served per minute, queue length or the average number of customers in the queue.

IV. VARIABLES USED

λ = Average Arrival Rate, μ = Average Service Rate

Lq = Length of Queue (Expected) (it also includes the customers which are currently receiving the service).

Wq = The time for which a customer has to wait in the queue (service time is also included in this as well).

ρ is known as the utilization factor (fraction of time when all individual servers are busy (Expected)).

V. OBSERVATIONS

Time of day	Avg. No. of vehicles on Sunday	Avg. No. of vehicles on Monday
10-11	85	95
11-12	140	187
12-13	110	194
13-14	146	149
14-15	163	176
15-16	169	156
16-17	132	195
17-18	185	205

Table 1: Observation

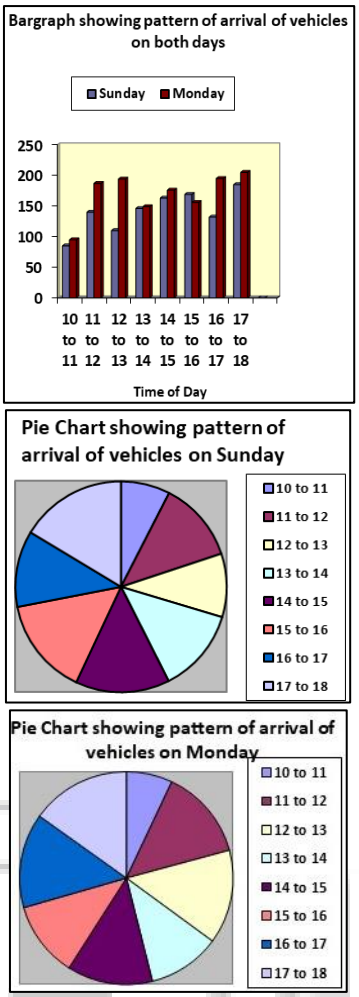

Fig. 2: Observation

VI. Results and Discussions

From the above data we can easily conclude that the busiest time is 5pm-6 pm for both the days and our analysis will be focused on this hour. The given data above shows that the average number of vehicles crossing the toll plaza on Sunday is maximum during 5pm-6 pm (around 185 vehicles). And the average number of vehicles crossing the toll plaza on Monday is 205 during 5 pm-6 pm. The peak hours are 5 pm-6 pm which is clearly evident from the table. Our analysis would be concentrated on 5pm-6pm. The arrival rate on Sunday is found to be 185 vehicles during 5pm-6pm and the observed service rate is 6 vehicle per minute. The arrival rate on Monday is found to be 205 vehicles during 5pm-6pm and the service rate per is 6 vehicles per minute. Since we have 1 server for the incoming traffic and hence the analysis is done using M/M/1 Queuing Model .Also it is assumed that a customer wastes fuel of Rs.4 per minute while waiting in Queue.

VII. Formula Used

Since there is only one lane for incoming vehicles. So only single server is considered. Therefore M/M/1 Queuing Model Formulae are used which are mentioned below

$$\rho = \lambda / \mu$$
$$Lq = \lambda^2 / \mu (\mu - \lambda)$$
$$Wq = \lambda / \mu (\mu - \lambda)$$

VIII. Calculations

A. Performance Analysis (Sunday)

From observations it is found that during the busiest hour (5pm to 6pm) 185 vehicles arrived on Sunday. It was also observed that on an average 3.5 vehicles are served per minute.

So λ = 185 customers during 5pm to 6pm
= 185/60 customers per minute
= 3.08 customers per minute
μ = 3.5 customers per minute

System's Utilization (ρ) = λ/μ = 3.08/3.5 = 0.88 = 88%

Which means that the server remains busy for 88% of the total time.

The average number of customers in the queue (Lq) = $\lambda^2 / \mu (\mu - \lambda) = (3.08)^2 / 3.5(3.5-3.08) = 6.45$

Average waiting time for a vehicle in the queue (Wq) = $\lambda / \mu (\mu - \lambda)$ = 3.08/3.5(3.5-3.08) = 2.09 minutes = 2 minutes 54 seconds

Now a customer is wasting petrol of Rs.4 per minute while waiting in the Queue.

Total loss of the customer = 2.09*4 = Rs.8.36 per journey

B. Performance Analysis (Monday)

From observations it is found that during the busiest hour (5pm to 6pm) 205 vehicles arrived on Sunday. It was also observed that on an average 3.5 vehicles are served per minute.

So λ = 205 customers during 5pm to 6pm
= 205/60 customers per minute
= 3.42 customers per minute
μ = 3.5 customers per minute

System's Utilization (ρ) = λ/μ = 3.42/3.5 = 0.9771 = 97.71%

Which means that the server remains busy for 97.71% of the total time.

The average number of customers in the queue (Lq) = $\lambda^2 / \mu (\mu - \lambda) = (3.42)^2 / 3.5(3.5-3.42) = 41.77$

Average waiting time for a vehicle in the queue (Wq) = $\lambda / \mu (\mu - \lambda)$ = 3.42/3.5(3.5-3.42) = 12.21 minutes = 12 minutes 12 seconds (approx)

Now a customer is wasting petrol of Rs.4 per minute while waiting in the Queue.

Total loss of the customer = 12.21*4 =Rs.48.84 per journey

Now comparing the data of Sunday (non-working day) and Monday (working day)

Parameters	Sunday	Monday
Avg. no. of vehicles in queue (Lq)	6.45	41.77
Average waiting time in Queue (Wq)	2 minutes 54 seconds	12 minutes 12 seconds
System's Utilization	88%	97.71%
Cost of waiting per customer	Rs.8.36 per journey	Rs.48.84 per journey

Table 2: Parameters

IX. Conclusions

On basis of above calculations following conclusions can be drawn.

1) On Sunday (non-working day) during 5pm to 6pm, the number of vehicles in the queue is just 6.25 and a vehicle has to wait for just 2 minutes 54 seconds in the queue and the cost of waiting is Rs.8.36 but these statistics changed completely for Monday (a working day) where the number of vehicles in the queue is 41.77 a vehicle has to wait for 12 minutes 12 seconds in the queue and the cost of waiting is Rs.48.84.
2) On Sunday the waiting time for a vehicle, the queue length, and the cost of waiting is very low so one server (one toll booth) is enough.
3) On Monday the waiting time for a vehicle, the queue length and the cost is high so one server (one toll booth) is not enough to manage the traffic situation.

A. Suggestions

- The government should set up more toll booth on this toll plaza and should deploy more servers for working days especially during peak hours so that the traffic situation can be managed.
- The government can adopt modern technology like smart cards for payments so that the service time can be reduced.

REFERENCES

[1] Introduction part "Operations Research: An introduction" By Humdy a Taha.
[2] The Economic Times "Remove Toll Plazas as initiatives to ease traffic congestion fail" Census Report (2011) "Size, growth rate and distribution of population"
[3] Hindustan Times "Delhi government mulls measures to check vehicle growth"

