Comparison of Range of Applicability of Equilibrium and Energy approach to Find Euler's Critical Load for One End Fixed and Other End Pinned Short Colum

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Abstract— In structural engineering, column is a compression member or a structural element which is subjected to compressive stress. Compression is a application of balanced pushing force to different points on a structure due to that force, the resultant stress in member is known as compression stress. Any structure is able to good stand any external force without causing damage and cracking is known as stability of structure. Failure of stability causes due to two different reasons, explained below. 1) Failure with respect to material. 2) Failure with respect to geometry. There are different approach to analysis of stability of column such as equilibrium approach, energy approach, imperfection approach, dynamic approach etc. In this investigation a comparison is made between equilibrium and energy approach to stability analysis the short column of one side fixed and other side pinned by finding the Euler's critical load.

Key words: Stability of Structure, Euler's Critical Load, Equilibrium Approach, Energy Approach

I. INTRODUCTION

Column is a compression member as early mentioned. Under different loading condition column deform or displaced and the failure column under load is known as buckling. Buckling is characterized by sudden sideways deflection of structural member. Critical load is the greatest load that will not causes lateral deflection and if the applying load increased beyond critical load then column will be buckle. Leonhard Euler, Swiss mathematician invented the equations for calculating the critical load under different supporting conditions in 1757. Column can be classified based on slenderness ratio and supporting condition. Based on supporting condition column can be categorized into four type, 1) Both side pinned 2) Both side fixed 3) One side fixed and other side free 4) One side fixed and other side pinned. And based on slenderness ratio, there are three of column,1) Short column 2) Long Column 3) Intermediate column, also shown in figure – 1.

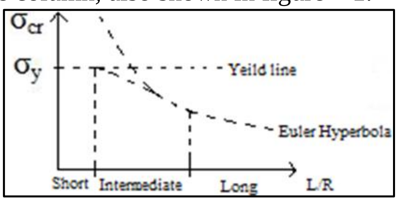

Fig. 1: Classification of Column based on Slenderness ratio.

In this research to do the comparison of range of applicability of equilibrium and energy approach for finding the Euler's critical load for one side fixed and other side pinned column.

II. EQUILIBRIUM APPROACH

A. *Assumption*

- The column is axially loaded.
- Material is perfectly elastic.
- Deformation is small.
- Material obeys Hook's law.
- Flexural rigidity constant.

B. *Solution*

According to equilibrium approach solution of Euler's critical load for one side fixed and other side pinned column (Fig 2) is given below.

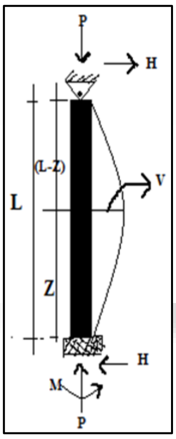

Fig. 2: Free body diagram of one side fixed and other side pinned column

$$EI\frac{d^2v}{dz^2} + M = 0 \qquad (1)$$

From the figure -2,

M= [PV – H(L-Z)], putting the value in equation-1, we got,

$$EI\frac{d^2v}{dz^2} + [PV - H(L-Z)] = 0$$

$$\text{Or, } \frac{d^2v}{dz^2} + \frac{PV}{EI} = \frac{H(L-Z)P}{EIP}$$

Such that $\frac{P}{EI} = K^2$ then,

$$\text{Or, } \frac{d^2v}{dz^2} + K^2V = \frac{H(L-Z)}{P}K^2 \qquad (2)$$

Such as,

$$V = A\sin(KZ) + B\cos(KZ) + \frac{H(L-Z)}{P} \qquad (3)$$

Boundary Condition,

$$@\ Z=0,\ V=0$$

$$@\ Z=0,\ \frac{dv}{dZ} = 0$$

$$V = A\sin(KZ) + B\cos(KZ) + \frac{H(L-Z)}{P}$$

From the B.C, @ Z=0, V =0

$$B = \frac{HL}{P}, \text{ and } @\ Z=0,\ \frac{dv}{dZ} = 0,\ A = \frac{H}{PK}$$

Putting the value of A and B in eqn – 3

$$V= \frac{H}{PK}\sin(KZ) + \frac{HL}{P}\cos(KZ) + \frac{H(L-Z)}{P}$$

Now, @ Z= L, V= 0

$$\tan(KL) = KL$$

From the trigonometry graph of tan(Kl) =4.493

$$KL = 4.493$$

$$Or, (KL)^2 = (4.493)^2$$

$$Or, (KL)^2 = 20.19$$

$$Or, K^2 = \frac{2}{L^2}\Pi^2$$

Where $K = \frac{P}{EI}$

$$Or, P_{cr} = \frac{2EI}{L^2}\Pi^2$$

Euler's Critical load for the one side pinned and other side fixed column is ,

$$P_{cr} = \frac{2\Pi^2 EI}{L^2}$$

III. ENERGY APPROACH

Law of conservation of energy, this principal is taken as a conservative system in equilibrium if strain energy stored will be equal to the work performed by the external load. A conservation system can be defined as a system is on which the work perform by both external and internal force is independent of the path traveled by this forces and depends only on the initial and final position.

Mathematical expression of critical load based on this theory is given below.

Pcr, Critical load,

$$P_{cr} = \frac{\int_0^L EI\left(\frac{d^2V}{dZ^2}\right)^2 dz}{\int_0^L\left(\frac{dv}{dZ}\right)^2 dz}$$

A. Solution

According to energy approach solution of Euler's critical load for one side fixed and other side pinned column (Fig-3) is given below.

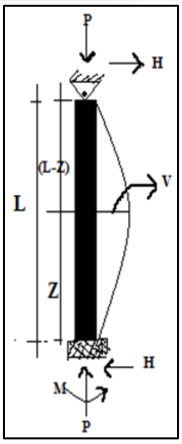

Fig. 3: Free body diagram of one side fixed and other side pinned column

Approximated solution of lateral displacement 'V' can be express as,

$$V= A_0 + A_1Z + A_2Z^2 + A_3Z^3 + A_4Z^4 \quad (4)$$

Applying Boundary condition,

$$@\ Z= 0,\ V= 0$$

$$A_0 = 0.$$

$$@\ z = 0,\ \frac{dv}{dZ} = 0$$

$$A_1 = 0$$

Putting this value of A_0 and A_1 in eqn. no- (4)

$$V= A_2Z^2 + A_3Z^3 + A_4Z^4 \quad (5)$$

Now, applying B.C @Z=L, V= 0 and $\frac{dv}{dZ} = 0$ in eqn. no-(4),

$$A_2 + A_3L + A_4L^2 = 0 \quad (6)$$

$$A_2 + 3A_3L + 6A_4L^2 = 0 \quad (7)$$

From the eqn. no – () and (7)

$$A_2 = 1.5\ A_4L^2$$

$$A_3 = -2.5\ A_4L$$

Putting these values in eqn. no- (5)

$$V= 1.5\ A_4L^2Z^2 - 2.5\ A_4L\ Z^3 + A_4Z^4$$

$$Or, \frac{dv}{dZ} = 3A_4L^2Z - 7.5\ A_4L\ Z^2 + 4A_4Z^3$$

$$Or, \frac{d^2V}{dZ^2} = 3A_4L^2 - 15\ A_4L\ Z + 12A_4Z^2$$

From the energy approach

$P_{cr,}$ Critical load,

$$P_{cr} = \frac{\int_0^L EI\left(\frac{d^2V}{dZ^2}\right)^2 dz}{\int_0^L\left(\frac{dv}{dZ}\right)^2 dz}$$

$$Or\ P_{cr,} = \frac{\int_0^L EI(3A4L2 - 15\ A4L\ Z + 12A4Z2)^2 dz}{\int_0^L (3A4L2Z - 7.5\ A4L\ Z2 + 4A4Z3\ )^2 dz}$$

$$Or, P_{cr} = \frac{1.8EIl^5}{.085\ l^7}$$

$$Or, P_{cr} = \frac{21EI}{l^2}$$

$$Euler's\ Critical\ Load\ P_{cr} = \frac{21EI}{l^2}$$

IV. COMPARISON OF RESULT

According to Indian standard IS: 800:2007 clause no, 7.2.2 and table 11 Euler's critical load for above subjected column will be $\frac{2\Pi^2 EI}{L^2}$. From the analytical investigation, comparative results are clearly shown that equilibrium approach gave the accurate result of Euler's critical load compare to the energy approach. Energy method gives 6.003% error result of the critical load of one side fixed and other side pinned column.

V. CONCLUSION

In this paper the numerical study is presented. From that study on equilibrium and energy approach for finding the critical load of one side fixed and other side pinned column is given below.

- Comparing IS: 800:2007 code clause 7.2.2 and table-11, this equilibrium approach gives the accurate critical load of the subjected column.
- Comparing IS: 800:2007 code clause 7.2.2 and table-11, this energy approach gives the 6.003% error critical load of the subjected column.

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