A Modify approach for Fuzzy Assignment Problem

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Abstract— The Fuzzy assignment problem is one of the main problems while assigning task to the Worker. It is one of the fundamental combinatorial optimization. It is a particular case of Transportation problem where the objective is to assign the resources to the activities so as to minimize total profit of allocation. In this paper we proposed modified Fuzzy assignment problem. The algorithm of this approach is presented and explained briefly. Also its Algorithm shown comparison with Hungarian.

Key words: Fuzzy Assignment Problem, Hungarian Method, Optimization, Ranking of Triangular Fuzzy Number, Fuzzy Sets

I. Introduction

The assignment problem is a special type of linear programming problem in which our objective is to assign number of salesman's to number of areas at a minimum cost (time). The mathematical formulation of the problem suggests that this is a 0-1 programming problem and it's highly degenerate all the algorithms developed to find optimal solution of transportation problem are applicable to the assignment problem. However, due to its highly degeneracy nature a specially designed algorithm. Widely known as Hungarian method proposed by Kuhn (1) is used for its solution.

In this paper, we investigate more realistic problem and namely the assignment problem, with fuzzy costs (C_{ij}). Since the objectives are to minimize the total cost (or) to maximize the total profit. Subject to some crisp constraints, the objective function is considered also as a fuzzy number. The methods are to rank the fuzzy objective values of the objective function by some ranking method for fuzzy number to find the best alternative.

On the basic of idea the Robust's ranking methods (2) has been adopted to transform the fuzzy Assignment problem to a Crisp one so that the conventional solution methods may be applied to solve assignment problem by R.PanneerSelvam (7).Dominance of fuzzy numbers can be explained by many ranking methods (3,4,5,6) of these, Robust's ranking method (2) which satisfies the properties of compensation, linearity and additive. In this paper we have applied Robust's ranking technique (2).

II. Preliminaries

In this section, some basic definitions and arithmetic are reviewed. Zadeh(16) in 1965 first introduced fuzzy set as a mathematical way of representing impreciseness or vagueness in everyday life.

A. *Definition: 2.1*

A fuzzy set is characterized by a membership function mapping elements of a domain, space or universe discourse X to the unit interval [0,1], is A={X, μ_A(X): $x \in X$ } Here: μ_A: $x \to [0,1]$ is a mapping called the degree of membership function of the fuzzy set A and μ_A(X) is called the membership value of $x \in X$ in the fuzzy set A, there membership grates are after represented by real numbers ranging from[0,1].

B. *Definition: 2.2*

A fuzzy set A of the universe, of discourse X is called a normal fuzzy set implying that there exists at least one $x \in X$ such that μ_A(X) =1.

C. *Definition: 2.3*

A fuzzy set A is convex if and only if for any $x_1, x_2 \in X$ the membership function of A satisfies the inequality.

$$\mu_A\{\lambda x_1 + (1-\lambda)x_2\} \geq \min\{\mu_A(x_1), \mu_A(x_2)\}, 0 \leq \lambda \leq 1$$

D. *Definition: 2.4: (Triangular fuzzy number)*

For a triangular fuzzy number A(x), it can be represented by A (a, b, c; 1) with membership function $\mu(x)$ given by

$$\mu(x) = \begin{cases} \frac{x-a}{b-a}; a \leq x \leq b \\ 1; x = b \\ \frac{c-x}{c-b}; b \leq x \leq c \\ 0; otherwise \end{cases}$$

E. *Definition: 2.5: (K- cut of a trapezoidal fuzzy number)*

The K – cut of a fuzzy number A(x) is defined as $A(k) = \{x; \mu(x) \geq k; k \in [0,1]\}$

III. Assignment problem

Assignment problem is a special case of the transportation problem in which the number of square and destination are the same and the objective is to assign the given job to most appropriate person so as to optimize the objective function like minimize cost.

A. *Balanced Assignment Problem*

When the number of rows equals to the number of columns.

Number of rows = Number of columns.

B. *Unbalanced Assignment Problem*

When the number of rows not equals to the number of columns and vice versa.

Number of rows ≠ Number of columns.

C. *Dummy Matrix*

We introduce dummy rows/columns in the matrix. These rows or columns have a zero cost elements. Here we introduce the fuzzy one cost elements instead of fuzzy zero cost.

D. Unbalance Assignment Problem to Change into Balanced Assignment Problem

The number of rows (areas) is not equal to the number of columns (salesman's) then the problem is termed as unbalanced assignment problem then this problem into change balanced assignment problem as follows necessary number of dummy row(s) / column(s) are added such that the cost matrix is a square matrix the values for the entries in the dummy row(s) / column(s) are assumed to be zero.

E. Robust's Ranking Techniques Algorithms

The assignment problem can be stated in the form of $n \times n$ cost matrix [C_{ij}] of real numbers as given in the following.

	Sales man 1	Sales man 2	…… …	Sales man i	… ….	Salesman N
Area 1	C_{11}	C_{21}	…… …	C_{1j}	… ….	C_{1n}
Area 2	C_{12}	C_{22}	…… …	C_{2j}	… …	$z=\sum_{i=1}^{n}\sum_{j=1}^{n} C_{ij}x_{ij}$
…….	……..	……..	……..	…….	… …	…….
Area N	C_{n1}	……..	……..	C_{nj}	… ….	C_{nn}

Table 1: Robust Ranking Techniques Algorithms

Mathematically assignment problem can be stated as

$$\text{Minimize } z = \sum_{i=1}^{n}\sum_{j=1}^{n} C_{ij}x_{ij}$$

Subject to $\sum_{i=1}^{n} x_{ij} = 1$ & $\sum_{j=1}^{n} x_{ij} = 1$ & $x_{ij} \in [0,1] \rightarrow (1)$

Where, $x_{ij} = \begin{cases} 1; \textit{if the ith area is assigned the jth salesman} \\ 0; \textit{Otherwise} \end{cases}$

i.e., the decision variable denoting the assignment of the area I to job j $(\tilde{C}_{ij})$ is the cost of assigning the j[th] salesman to the i[th] area. The objective is to minimize the total cost of assigning all the salesmen's to the available persons (one salesman to one area).

When the costs or time $(\tilde{C}_{ij})$ are fuzzy numbers then the total costs becomes a fuzzy number.

$$Z^{r^*} = \sum_{i=1}^{n}\sum_{j=1}^{n} C_{ij}X_{ij}$$

Hence it cannot be minimized directly for solving the problem. We defined by the fuzzy cost Co-efficient into crisp ones by a fuzzy number ranking method. Robust's ranking technique (3) which satisfies compensation linearity, and addictive properties and provides results which are consistence with human intuition. Given a convex fuzzy number $(\tilde{c})$, the Robust's ranking index is defined by

$$R(\tilde{C}) = \int_0^1 0.5(C_k^L, C_{dk}^U)dk$$

Where (C_k^L, C_k^U) is the k-Level cut of the fuzzy number $(\tilde{c})$

Here we use this method for ranking the objective values. The Robust's ranking index $R(\tilde{c})$ gives the representative value of the fuzzy number $(\tilde{c})$ it satisfies the linearity and additive property.

If $\tilde{P} = l\tilde{E} + m\tilde{y}$ and $\tilde{Q} = S\tilde{c} - t\tilde{N}$ where l, m, t, s are constant then we have $R\hat{p} = lR(\tilde{E}) + nR(\tilde{y})$ and $R\hat{\phi} = SR(\hat{c}) - tR(\tilde{N})$ on the basis of this property the fuzzy assignment problem can be transformed into a crisp assignment problem linear programming problem from.

The ranking technique of the Robust's is

if $R(\tilde{G}) \leq R(\tilde{H})$ then $\tilde{G} \leq H^v$

i.e., $min\{\hat{G}, H^v\} = \tilde{G}$ from the assignment problem (1) with fuzzy objective function.

$$\min z^{r^A} = \sum_{i=1}^{n}\sum_{j=1}^{n} R(\tilde{C}_{ij})X_{ij}$$

We apply Robust's ranking method (3) (using the linearity and associative property) to get the minimum objective value z^{r^A} from the formulation.

$$R(z^{r^A}) = \min z = \sum_{i=1}^{n}\sum_{j=1}^{n} R(C_{ij})X_{ij}$$

Subject to

$$\sum_{i=1}^{n} x_{ij} = 1$$

$$\sum_{j=1}^{n} x_{ij} = 1$$

$$x_{ij} \in [0,1] \rightarrow (2)$$

Where x_{ij} =

$\begin{cases} 1; \textit{if the } i^{th} \textit{ area is assigned the } j^{th} \textit{ salesman} \\ 0; \textit{Otherwise} \end{cases}$

Is the decision variable denoting the assignment of the area i[th] to j[th] salesman is the cost of designing the j[th] job to the i[th] area. The objective is to minimize the total cost of assigning all the salesman to the available areas.

Since R(C_{ij}) are crisp values, this problem (2) is obviously the crisp assignment problem of the form (1) which can be solved by the conventional method, namely the Hungarian method or simple method to solve the linear programming problem form of the problem. Once the optimal solution x* of model (2) is found the optimal fuzzy objective value z^{v^*} of the original problem can be calculated as $(z^{v^*}) = \sum_{i=1}^{n}\sum_{j=1}^{n} (\tilde{C}_{ij})x_{ij}^*$.

IV. PROPOSED METHOD ALGORITHM

1) Step 1: Convert the fuzzy numbers into crisp numbers. Using Robust Ranking Technique.
2) Step 2: Determine fuzzy cost matrix from the given problem. Then test whether it is balanced or not.
   – If it is balanced one. (i.e.) The no.of persons are equal to the no.of jobs). Then go to step 4.
   – If it is an unbalanced one.(i.e, The no.of persons are not equal to the no.of jobs). Then go to step 3.
3) Step 3: Add dummy row or dummy column. The fuzzy cost entries of rows/columns are always fuzzy zero.
4) Step 4: Now form two columns, where column 1 represents resource and column 2 represents an activity.
5) Step 5: Find minimum unit cost for each row, whichever minimum value is available in the representing column. Select it and write it in terms of activities under column 2. Continue this process for all the rows and write in terms of activities.

6) Step 6: For each resource; if there is unique activity then assigned that activity for the corresponding resource, hence we get our optimal solution.
7) Step 7: If there is no unique activity for corresponding resource then the assignment can be made using following steps:
   – Find at which of any one resource unique activity has and then assign that activity for the corresponding resource. Next delete that row and it is corresponding column for which resource has already been assigned.
   – Again find the minimum unit cost for the remaining rows. Check if it satisfy step 4 then perform it. Otherwise, check which rows have only one same activity.
   – Next find difference between minimum and next minimum unit cost for all those rows which have same activity. Assign that activity which has maximum difference. Delete those rows and corresponding columns for which those resource have been assigned.
   – However if there is tie in difference for two and more than to activity than further take the difference between minimum and next to next minimum unit cost. Next check which activity has maximum difference, assign that activity.
8) Step 8: Repeat steps 4 to 5 till all resources are assigned uniquely to the corresponding activity.
9) Step 9 : Once all the jobs are assigned then calculate the total cost by

$$\text{Total Cost} = \sum_{i=1}^{n}\sum_{j=1}^{n} C_{ij}x_{ij}\,.$$

V. NUMERICAL EXAMPLE

Let us consider a fuzzy unbalanced assignment problem with rows representing 4 area A_1, A_2, A_3, A_4 and columns representing the salesman's B_1, B_2, B_3. The cost matrix $(\tilde{C}_{ij})$ is given whose elements are triangular fuzzy numbers. The problem is to find the optimal assignment so that the total cost of area assignment becomes minimum.

$$(\tilde{C}_{ij}) = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} \begin{array}{c} \begin{matrix} B_1 & B_2 & B_3 \end{matrix} \\ \begin{pmatrix} (-2,1,4) & (1,4,7) & (6,7,8) \\ (6,10,14) & (2,5,8) & (7,8,9) \\ (6,7,8) & (10,12,14) & (5,6,7) \\ (-2,3,8) & (7,8,9) & (8,9,10) \end{pmatrix} \end{array}$$

A. Solution

The given problem is a fuzzy unbalanced assignment problem. We have to change into the fuzzy balanced assignment problem as follows. $(\tilde{C}_{ij}) =$

$$\begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} \begin{array}{c} \begin{matrix} B_1 & B_2 & B_3 & B_4 \end{matrix} \\ \begin{pmatrix} (-2,1,4) & (1,4,7) & (6,7,8) & (0,0,0) \\ (6,10,14) & (2,5,8) & (7,8,9) & (0,0,0) \\ (6,7,8) & (10,12,14) & (5,6,7) & (0,0,0) \\ (-2,3,8) & (7,8,9) & (8,9,10) & (0,0,0) \end{pmatrix} \end{array}$$

The fuzzy balanced assignment problem can be formulated in the following mathematical programming form.

$$\begin{aligned} \text{Min}\{ & R(-2,1,4)x_{11} + R(1,4,7)x_{12} + R(6,7,8)x_{13} + R(0,0,0)x_{14} \\ & + R(6,10,14)x_{21} + R(2,5,8)x_{22} + R(7,8,9)x_{23} + R(0,0,0)x_{24} + \\ & R(6,7,8)x_{31} + R(10,12,14)x_{32} + R(5,6,7)x_{33} + R(0,0,0)x_{34} + \\ & R(-2,3,8)x_{41} + R(7,8,9)x_{42} + R(8,9,10)x_{43} + R(0,0,0)x_{44}\} \end{aligned}$$

Subject to the constraints:

$$x_{11}+x_{12}+x_{13}+x_{14}=1$$
$$x_{11}+x_{21}+x_{31}+x_{41}=1$$
$$x_{21}+x_{22}+x_{23}+x_{24}=1$$
$$x_{12}+x_{22}+x_{32}+x_{42}=1$$
$$x_{31}+x_{32}+x_{33}+x_{34}=1$$
$$x_{13}+x_{23}+x_{33}+x_{43}=1$$
$$x_{41}+x_{42}+x_{43}+x_{44}=1$$
$$x_{14}+x_{24}+x_{34}+x_{44}=1$$
$$x_{ij} \in [0,1]$$

Now we conclude R(-2,1,4) by applying Robust's Ranking method. The membership function of the triangular fuzzy number R(-2,1,4) is

$$\mu(x) = \begin{cases} \dfrac{x+2}{1+2}; & -2 \le x \le 1 \\ \dfrac{4-x}{4-1}; & 1 \le x \le 4 \\ 0 \; ; & otherwise \end{cases}$$

Ranking of Triangular fuzzy number

$$\Re(A) = \frac{a_1 + 4a_2 + a_3}{6}$$

$$\Re(C_{11}) = \frac{-2+4(1)+4}{6} = \frac{-2+4+4}{6} = \frac{6}{6} = 1$$

Proceeding similarly, the ranking of triangular fuzzy number for the fuzzy costs $(\tilde{C}_{ij})$ are calculated.

We replace these values for the corresponding $(\tilde{C}_{ij})$ which results in assignment problem in the linear programming problem.

We solve it by Hungarian method to get the following optional solution.

$$(\tilde{C}_{ij}) = \begin{pmatrix} 1 & 4 & 7 & 0 \\ 10 & 5 & 8 & 0 \\ 7 & 12 & 6 & 0 \\ 3 & 8 & 9 & 0 \end{pmatrix}$$

1) *Step 1: Row reduction*

$$(\tilde{C}_{ij}) = \begin{pmatrix} 1 & 4 & 7 & 0 \\ 10 & 5 & 8 & 0 \\ 7 & 12 & 6 & 0 \\ 3 & 8 & 9 & 0 \end{pmatrix}$$

2) *Step 2: Column reduction*

$$\begin{pmatrix} 0 & 0 & 1 & 0 \\ 9 & 1 & 2 & 0 \\ 6 & 8 & 0 & 0 \\ 2 & 4 & 3 & 0 \end{pmatrix}$$

3) *Step 3: Assignments*

$$\begin{pmatrix} \cancel{0} & \boxed{0} & 1 & \cancel{0} \\ 9 & 1 & 2 & \cancel{0} \\ 6 & 8 & \boxed{0} & \cancel{0} \\ 2 & 4 & 3 & \boxed{0} \end{pmatrix}$$

Have not assignment for each row and each column. Go to next step.

4) Step 4

$$\begin{pmatrix} 0 & 0 & 1 & 0 \\ 9 & 1 & 2 & 0 \\ 6 & 8 & 0 & 0 \\ 2 & 4 & 3 & 0 \end{pmatrix}$$

5) Step 5

$$\begin{pmatrix} 0 & 0 & 1 & 1 \\ 8 & 0 & 1 & 0 \\ 6 & 8 & 0 & 1 \\ 1 & 3 & 2 & 0 \end{pmatrix}$$

6) Step 6

$$\left(\tilde{C}_{ij}\right) = \begin{pmatrix} \boxed{0} & 0 & 1 & 1 \\ 8 & \boxed{0} & 1 & 0 \\ 6 & 8 & \boxed{0} & 1 \\ 1 & 3 & 2 & \boxed{0} \end{pmatrix}$$

$$A \rightarrow 1 ; B \rightarrow 2 ; C \rightarrow 3 ; D \rightarrow 4$$

The optimal Assignment

$$A_1 \rightarrow B_1, A_2 \rightarrow B_2, A_3 \rightarrow B_3, A_4 \rightarrow B_4$$

The optimal total minimum cost = Rs.1+5+6+0

= Rs.12

The fuzzy optimal Assignment

$$A_1 \rightarrow B_1, A_2 \rightarrow B_2, A_3 \rightarrow B_3, A_4 \rightarrow B_4$$

The fuzzy optimal total minimum cost

$$= \tilde{C}_{11} + \tilde{C}_{22} + \tilde{C}_{33} + \tilde{C}_{44}$$

= R(-2,1,4) + R(2,5,8) + R(5,6,7) + R(0,0,0)

= R(5,12,19)

= Rs.12

VI. PROPOSED METHOD

$$\left(\tilde{C}_{ij}\right) = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} \begin{array}{c} \begin{matrix} B_1 & B_2 & B_3 & B_4 \end{matrix} \\ \begin{pmatrix} 1 & 4 & 7 & 0 \\ 10 & 5 & 8 & 0 \\ 7 & 12 & 6 & 0 \\ 3 & 8 & 9 & 0 \end{pmatrix} \end{array}$$

A. Step 1

For the given cost matrix, Find minimum unit cost for each row, whichever minimum value is available in the respecting column, select it and write it in term of activities under column 2. Continue this process for all the rows and write in term of Activities.

	B_1	B_2	B_3	B_4
A_1	1	4	7	0
A_2	10	5	8	0
A_3	7	12	6	0
A_4	3	8	9	0

A_1	B_1
A_2	B_2
A_3	B_3
A_4	B_1

Since two unique salesman's is there so find the difference between minimum to next minimum for that salesman's and then select the maximum difference. Here brand A_2 having maximum difference so assign this salesman B_2 to brand A_2 and delete the corresponding row and column. Again repeat the process.

B. Step 2

	B_1	B_3	B_3
A_1	1	7	0
A_2	7	6	0
A_4	3	9	0

A_1	B_1
A_3	B_3
A_4	B_1

Now min. to next min. difference is 1, 3 and 6 and we assign on max. Difference salesman B_3. And delete that row A_3 and column B_3. And hence is assigned A_1 to B_1 and then brand is A_4 assigned to B_4.

C. Step 3

	B_1	B_4
A_1	1	0
A_2	3	0

A_1	B_1
A_2	B_4

Hence the optimal assignment is:

Assign Brands	To Salesman	Cost (Rs)
A_1	B_1	1
A_2	B_2	5
A_3	B_3	6
A_4	B_4	0

The Minimum total cost = Rs. 12

VII. CONCLUSION

In this paper main goal is to find out the minimum optimal assignment by assigning all tasks to the worked. The proposal method uses a new liner space solution to solve the fuzzy assignment problem. Based on an experiment our proposed method provide same optimal cost than of the Fuzzy Hungarian method. It has been found that although the resultant obtained via this method is same but the No of iteration have been reduced which consecutively saves time and earlier to perform.

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