On Some Properties of Metric F- Structure Satisfying $F^{2k} – F = 0$

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Abstract— The purpose of this paper is to study various properties of the F structure satisfying F^{2k} - F = 0. Where k is positive integer $k \geq 2$. The metric F- structure, kernel and tangent vectors have also been discussed.
Key words: Differentiable Manifold, Complementary Projection Operators, Metric, Kernel and Tangent Vectors

I. INTRODUCTION

Let V_n be a C^∞ differentiable manifold and $F \neq 0$ be a C^∞ (1, 1) tensor on V_n such that

$$F^{2k} – F = 0 \qquad (1)$$

We define the projection operators l and m on V_n by

$$l = -F^{2k-1}, \qquad m = I - F^{2k-1} \qquad (2)$$

Where I denotes the identify operator

From (1) and (2), we have

$$l + m = I, \quad l^2 = l, \quad m^2 = m, \quad lm = ml = 0$$

$$lF = Fl = F, \quad Fm = mF = 0, \qquad (3)$$

A. *Theorem (1.1):*

If $rank((F)) = n = \dim V_n$ then

$$l = I, \quad m = 0 \qquad (4)$$

1) *Proof: From the fact*

$$rank((F)) + nulity((F)) = \dim V_n = n \qquad (5)$$

We have

$$nulity((F)) = 0 \Rightarrow \quad \ker((F)) = \{0\}$$

Or

$$FX = 0 \Rightarrow X = 0$$

Then

$$FX_1 = FX_2$$
$$F(X_1 - X_2) = 0$$
$$X_1 = X_2 \quad \text{or F is 1-1}$$

Also V_n being finite dimensional F is onto also thus F^{-1} exists. Operating F^{-1} on Fl = F and mF = 0, we get (4)

B. *Theorem (1.2)*

Let l and m satisfying

$$m^2 = m, \; mF = Fm = 0, \quad (m + F^k)(m + F^{k-1}) = I \qquad (6)$$

Then F satisfying

$$(m + F^k)(m - F^{k-1}) = I$$
$$m^2 - mF^{k-1} + F^k m - F^{2k-1} = I$$
$$m - 0 + 0 - F^{2k-1} = I$$
$$mF + F^{2k} = F$$
$$0 + F^{2k} = 0$$
$$F^{2k} - F = 0$$

1) *Definition (1.1)*

$$\ker F = \{X : FX = 0\}$$
$$\text{Tan}\, F = \{X : FX \;\text{II}\; X\}$$

C. *Theorem (1.3)*

For the F-structure satisfying (1.1), we have

$$\ker F = \ker F^2 = ------ = \ker F^{2k} \qquad (7)$$

$$\text{Tan}\, F = \text{Tan}\, F^2 = ------ = \text{Tan}\, F^{2k} \qquad (8)$$

1) *Proof:*

Let $X \in \ker F \Rightarrow FX = 0$

$$F^2X = 0$$
$$X \in \ker F^2$$

Thus

$$X \in \ker F^2 \Rightarrow F^2X = 0 \qquad (9)$$
$$F^3X = 0$$
$$X \in \ker F^3$$
$$\ker F^2 \subseteq \ker F^3 \qquad (10)$$
$$X \in \ker F^{2k-1} \Rightarrow F^{2k-1}X = 0$$
$$F^{2k}X = 0$$
$$X \in \ker F^{2k}$$
$$X \in \ker F^{2k} \Rightarrow F^{2k}X = 0 \qquad (11)$$
$$\Rightarrow F^{2k}X = 0$$
$$\Rightarrow FX = 0$$
$$\Rightarrow X \in \ker F$$
$$\ker F^{2k} \subseteq \ker F \qquad (12)$$

In all $\ker F \subseteq \ker F^2 \subseteq ------ \subseteq \ker F^{2k} \subseteq \ker F$

Thus we get (7). Following similarly we get (8)

II. METRIC F-STRUCTURE

If we define

$$'F(X,Y) = g(FX,Y) \qquad (13)$$

is skew- symmetric. Then,

$$g(FX,Y) = -g(X,FY), \quad \{F,g\} \qquad (14)$$

is called Metric F Structure

A. *Theorem (2.1)*

g satisfying (13) and (1), (2), (3), we have

$$g(F^kX, F^{k-1}Y) = (-1)^k[g(X,Y) - {}'m(X,Y)] \qquad (15)$$

Where

$$'m(X,Y) = g(mX,Y) = g(X,mY). \qquad (16)$$

1) *Proof*

We have

$$g(F^kX, F^{k-1}Y) = (-1)^k g(X, F^{2k-1}Y)$$
$$= (-1)^k g(X, lY)$$
$$= (-1)^k (g,(I - m)Y)$$
$$= (-1)^k [g(X,Y) - g(X,mY)]$$
$$= (-1)^k [g(X,Y) - {}'m(X,Y)]$$

B. *Theorem (2.2): {F, g} is not unique*

1) *Proof*

Let g be nonsingular (1, 1) tensor, such that

$$\mu F' = F\mu, \quad {}'g(X,Y) = g(\mu X, \mu Y) \qquad (17)$$

Then

$$\mu F'^{2k} = F^{2k}\mu = F\mu = \mu F'$$

Thus

$$F'^{2k} = F' \;\text{Or}\; F'^{2k} - F = 0$$

Also

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$$
\begin{aligned}
{}^{/}g(F'^{k}X, F'^{k-1}Y) &= g(\mu F'^{k}X, \mu F'^{k-1}Y)\\
&= g(F^{k}\mu X, F^{k-1}\mu Y)\\
&= (-1)^{k} g(\mu X, F^{2k-1}\mu Y)\\
&= (-1)^{k} g(\mu X, l\mu Y)\\
&= (-1)^{k} g(\mu X, (I-m)\mu Y)\\
&= (-1)^{k}\left[g(\mu X, \mu Y) - g(\mu X, m\mu Y)\right]\\
&= (-1)^{k}\left[{}^{/}g(X,Y) - {}^{/}m(X,Y)\right]
\end{aligned}
$$

C. *Theorem: (2.3)*

With the (2.5), we have

$$\mu l^{/} = l\mu, \quad \mu m^{/} = m\mu \qquad (18)$$

1) Proof

We have

$$
\begin{aligned}
\mu l^{/} &= \mu F^{/\,2k-1}\\
&= F^{\,2k-1}\mu\\
&= l\mu\\
\mu m^{/} &= m\mu\\
\mu m^{/} &= \mu(I - F^{/\,2k-1})\\
&= \mu - F^{/\,2k-1}\\
&= \mu - F^{/\,2k-1}\mu\\
&= m\mu
\end{aligned}
$$

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