Image Compression using Alpha-Molecules

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Abstract— This paper describes a simple and efficient method of image compression using Alpha-molecules and Huffman coding technique. Image compression using Wavelet transform, Curvelet transform and Contourlet transform followed by Huffman coding exists in the literature. The work in this paper is proposed using Shearlet transform which has not been done yet. Various biomedical images of MRI of different parts of body such as lungs, shoulder, cardiac and brain are compressed using transform based coding. Wavelet and its variants such as Curvelet, Contourlet and Shearlet are used for transformation. The approximation and detail coefficients being extracted are processed for coding followed by decoding and inverse transformation. A comparative analysis is drawn between various transforms on the basis of various quantitative measures such as PSNR (Peak Signal to Noise ratio), MSE (Mean Square error), AD (Average Difference), NAE (Normalized Absolute error).

Key words: Curvelet Transform, Contourlet Transform, MSE, NAE, PSNR, Shearlet Transform, Wavelet Transforms

I. INTRODUCTION

Image compression means to conserve the storage space required for images. The aim of compression is to remove the redundancy and irrelevant components of the image. The need for compression arises because of the fact that the size of uncompressed image is large which enhances the memory requirements of the system. Moreover the transmission bandwidth and time occupied by uncompressed data is more. Various compression techniques have been developed, lossy [1] and lossless [2] image compression.

The advantages of wavelets over Fourier methods such as faster implementations, rich mathematical structure, ability to analyze images in time and frequency domain, ability to reconstruct images with fewer coefficients has led to its success. Besides it, the wide range of applications of wavelets in compression, enhancement, feature extraction, classification etc. has revolutionized the image processing area [3].

But when multivariate data comes into consideration, representation of wavelets is not sparse. Moreover for pointwise singularities, wavelets are optimal but for curve like structures, wavelets are not effective. In order to overcome this limitation, directional wavelets were introduced such as and 2 D directional wavelets by Antoine and steerable pyramids by Simoncelli etc.

In 2004, to overcome the drawbacks of wavelet, another variant of wavelet viz curvelet was introduced by Candes and Donoho [4]. Curvelets are analyzed with respect to scale, orientation and location parameters.The inability of wavelets to deal with multivariate data was overcomed by curvelets as very few shearlet coefficients could approximate edges in the images leading to an perfect adaptive representation system.

A tree structured filterbank version was introduced in 2005 by Do and Vitterli known as contourlet [5]. It has good spatial localization which was not possible with curvelets and it can remove redundancy to a much better extent. But when it comes to the number of directions available for shearing, contourlet is not effective. This limitation was overcomed by another variant of wavelet viz shearlet as it has affine representative structure which make it more efficient when dealing with multivariate data.

Traditional multiscale systems are optimal in dealing point-discontinuities only. They have a limited capability to deal singularities in higher dimensions such as edges and moving fronts. In 2005, shearlets emerged from composite wavelets whose developers were a group of people-Guo, Kutyniok, Labate, Lim and Weiss. The advantage of shearlet was that there could be as many number of directions for performing shear operation as compared to contourlet. As a result of this excellent directional selectivity could be obtained from shearlets [6].

Image compression using Wavelet transform and Huffman coding approach is illustrated in [17].

In this paper, 2-D wavelet transform of images is taken at various levels of decomposition. The approximation coefficients are taken for processing and detail coefficients are discarded. They are subjected to uniform quantization followed by Huffman coding. In the later stage both approximation and detail coefficients are processed and results are compared for test images of Lena, Barbara in terms of PSNR and compression ratio.

In the paper [18], a comparative analysis is drawn for medical image compression using Wavelet and Contourlet transform. The transformed coefficients are quantized and encoded by Huffman coding technique. CT images are taken for comparison. PSNR values are computed for different images. At low compression ratios, wavelet also shows good results but for high compression ratios Contourlet is effective for compression.

In this paper, we propose an efficient image compression technique with the aim of improving the peak signal to noise ratio. Initially, the input medical image of MRI of lungs, shoulder, cardiac, brain etc. is taken and is subject to transformation. The shearlet coefficients are extracted by applying discrete shearlet transform to the images, with the aim of doing lossy compression in shearlet domain. Then, decompression stage, the original medical image is extracted using Huffman decoding and inverse transformation. A Comparative analysis is drawn by using other transforms such as Wavelets, Curvelets and Contourlets in place of Shearlet and the results are quantified using various measures available for compression.

The remaining paper is outlined as follows: A brief description of Alpha-Molecules is given in Section II. Section III highlights the proposed approach. The results and discussion are summed in Section IV. Section V compares the proposed and existing results. Section VI concludes the work in the paper.

II. ALPHA-MOLECULES

Alpha-Molecules include all multiscale systems such as wavelet, contourlet, ridgelet, curvelet and shearlet. It is a generalization of these systems with the parameter alpha representing the degree of anisotropy. A system has to be associated with a particular (α) scaling

$$\begin{bmatrix} s & 0 \\ 0 & s^{\alpha} \end{bmatrix} \quad (1)$$

Parameter α ranging from $\alpha = 1$ (wavelet), $\alpha = 1/2$ (curvelet and shearlet), $\alpha = 0$ (ridgelet).

The mathematical definitions of alpha molecules viz wavelets, curvelets and shearlets are explained in [7].

A. Shearlet

The shearlet representation belongs to a class of representations which have been introduced to provide sparse representations for multidimensional functions with distributed discontinuities.

Shearlet system is derived from Composite wavelet systems which provide sparse representation when dealing with functions involving two variables. Shearlet provides a directional multiscale decomposition of functions within the framework of affine systems.

A Composite wavelet system in dimension n=2 are:

$$\psi_{i,j,k}(x) = |\det A|^{i/2}\psi(B^{j}A^{i}x - k): \ i, j \ \varepsilon \ Z, k \ \varepsilon \ Z$$

Where, A= $\begin{pmatrix} 2 & 0 \\ 0 & \sqrt{2} \end{pmatrix}$ and B = $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ (2)

Here, ψ is the well localized function, A represents the anisotropic dilation matrix and B represents the shear matrix.

The dilation matrix is based on the principle of parabolic scaling represented by $diag\,(a, a^{\alpha})$ where the parameter α controls the measure of an-isotropy. Here $a = 2$ and $\alpha = 1/2$.

The continuous shearlet transform is dependent on scale and translation parameters as wavelets, in addition it has a shear parameter which gives the measure of directionality and is given by equation (3)

$$SH_f(a,s,t) = < f, \Psi_{a,s,t} > \quad (3)$$

Where a represents scale parameter, s represents shear parameter and t represents translation parameter.

Here, the continuous shearlets is described by

$$\Psi_{a,s,t}(x) = a^{-3/4}\psi((D_{a,s}^{-1}(x - t))) \quad (4)$$

Where $D_{a,s} = [a, -a^{1/2}s; 0, a^{1/2}]$ is the dilation matrix.

The shear variable s is associated with slope rather than angle.

B. Properties of Shearlets

1) Shearlets have anisotropy property i.e. shearlet coefficients have different values when measured in different directions.
2) Shearlets are appropriate for multivariate data.
3) The shearlet elements are well localized in space and frequency domain.
4) Very few shearlet coefficients are needed for reconstruction. Therefore, we can say that shearlets have sparse approximation.
5) The reconstruction of image is perfectly stable.
6) Shearlet leads to an efficient implementation.

C. Curvelet

Curvelet preserves the same time frequency localization as for wavelets and it has additional directional properties due to elongated support in the frequency domain. Curvelet transform is a multiscale directional transform which has scale, translation and orientation parameters.

In spatial domain curvelet is Gabor along the width and Gaussian along the length.

Curvelet basis function is:

$$\gamma(x1, x2) = \psi(x1).\phi(x2) \quad (5)$$

Where $\psi(x1)$ is Gabor(x1) and $\phi(x2)$ is Gaussian(x2).

The first generation Curvelet transform is explained in [9]. Fast Curvelet transform is described in [10].

D. Contourlet

The Contourlet transform [11] is computed by making use of Laplacian pyramid along with directional banks of filters. It decomposes the image into several directional subbands and variable scales. This is illustrated with the help of figure below:

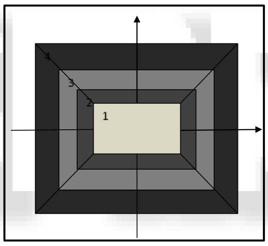


Fig. 1: Decomposition of image into subbands

The original image is reconstructed by using inverse Contourlet transform.

E. Wavelet

Wavelets are an extension to Fourier analysis. The drawbacks of Fourier transform such as lack of time information, non-suitability for non-stationary signals prompted the mathematicians and engineers to go for short term Fourier transform invented by Gabor in 1946.

To overcome the problem of resolution in STFT, wavelets were developed. Wavelets adapt to different components of the signal, the low frequency signals give non-detailed view of the signal and high frequency signals give the detailed view [12].

Wavelets convert the information in the form of coefficients. The coefficients are stored, transmitted and analyzed to reconstruct the original signal.

A function ψ ε L2(R) is known as wavelet if it has zero average over the entire interval specified.

$$\int_{-\infty}^{\infty} \Psi(t)dt = 0 \quad (6)$$

$$C_{\Psi} = 2\pi \int_{R} \frac{|\hat{\Psi}(w)|^2}{|w|} dw < \infty$$

If ψ ε L2(R) $\cap L1(R)$ satisfying

$$C_{\psi} = 2\pi \int \left(\frac{|\hat{\Psi}(w)|}{|w|} \right) dw \qquad (7)$$

Where, $\hat{\psi}(w)$ gives the Fourier transform of ψ, and then equation (3) is satisfied. The condition is called the wavelet admissibility condition.

III. PROPOSED WORK

To achieve higher signal to noise ratio without distorting the important data in a medical image, we propose an image compression algorithm using shearlet transform and Huffman coding. The proposed approach is illustrated with the help of figure 1.

A. Image Compression Algorithm

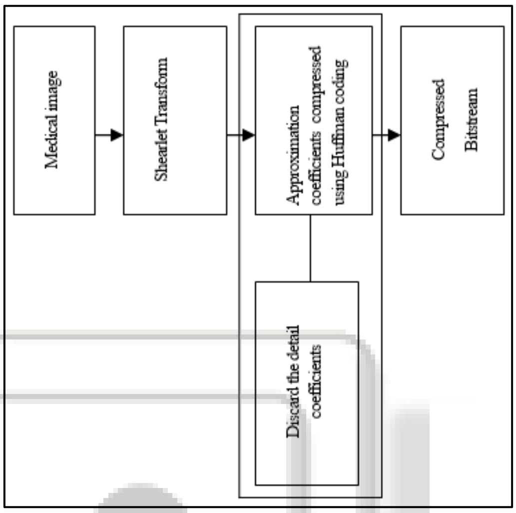


Fig. 2: Proposed image compression algorithm

The medical images of MRI of different parts of body are taken [13]. To identify the location of the edges we use shearlet transform and convert the pixels into bit stream using encoding techniques. The approximation coefficients are compressed using Huffman coding, the detail coefficients are discarded. The resultant is the compressed bit stream.

1) Shearlet Transform

The medical image is subjected to discrete shearlet transform which converts it into shearlet coefficients.

- Step 1: The first step is to get the shearing filters for every scale j and shear parameter k. Meyer based window function is used for carrying out windowing given by:

$$y = 35x^4 - 84x^5 + 70x^6 - 20x^7 \qquad (8)$$

- Step 2: The norm of shearlets is computed for every scale and direction, taking laplacian pyramid filter, input image size and directional shearing filters array. The translation invariant shearlet transform is computed [14] with the known inputs. The input image is decomposed into sub-bands of varying scales by using a-trous decomposition. Then directional shearing filters are used for decomposition for each scale j.

2) Huffman Coding

The approximation coefficients are subjected to Huffman coding, whereas the detail coefficients are discarded.

The Huffman coding algorithm [15] is described as below:

- Step 1: Input the image from the path of the location of the image into the matlab.
- Step 2: Manipulate the image by changing it to gray level color, adjusting its size according to the requirement.
- Step 3: Obtain the symbols such that the pixel values are not repeated.
- Step 4: Obtain the probability of all the symbols.
- Step 5: Arrange the probabilities of all the symbols in the descending order. The two lower probabilities are added and this process is repeated until only two symbol probabilites are left. Then the assignment of codes is done in such a way that the symbols having maximum probability of occurrence have shorter length code.
- Step 6: Then, Huffman encoding is done such that the. codewords are mapped to the corresponding symbols which results in a compressed data.
- Step 7: Finally, decompression is done by using huffman decoding to reconstruct the original image.
- Step 8: A tree equivalent to the encoding tree is generated.
- Step 9: Scan the input character until the last element is reached.
- Step 10: Obtain the output from the character encoded in the leaf node and return to the starting node, and repeat the above step till all the codes corresponding to symbols probabilities are known.

B. Image Decompression Algorithm

The proposed image decompression process is given in the figure below.

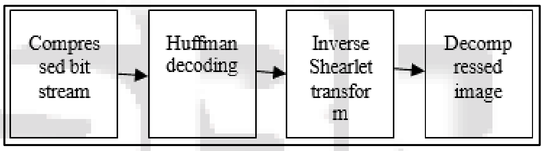


Fig. 3: Proposed image decompression algorithm

The compressed bit stream is subjected to Huffman decoding followed by inverse Shearlet transform to obtain the reconstructed image/decompressed image. Various quantitative measures [16] are in use for finding out the reconstructed image quality.

1) Compression ratio

Image compression ratio of the image is found by finding the ratio between the size or rate of uncompressed image and compressed image size as shown in equation (9).

$$Compression\ Ratio = \frac{Size\ of\ Original\ image}{Size\ of\ compressed\ bit\ stream} \qquad (9)$$

2) Peak Signal to noise ratio

The description of PSNR is given below:

$$PSNR = 10 \log_{10} \frac{E_{max}^2 \times (DC^M[x,y])_w \times (DC^M[x,y])_h}{\sum (DC^M[x,y])_{ij} \times (DC^M[x,y])^*_{ij}} \qquad (10)$$

Where,

$(DC^M[x,y])_w$ → Width of the decompressed image

$(DC^M[x,y])_w$ → Height of the decompressed image

$(DC^M[x,y])_{ij}$ → Pixel value of the original image whose co-ordinates are x and y

$(DC^M[x,y])^*_{ij}$ → Pixel value of the decompressed image whose co-ordinates are x and y

E_{max}^2 → Highest energy of the image pixels (i.e., E_{max} =255 if there are 256 gray-level images)

3) Average Difference

The average difference (AD) of the input and compressed image is given by

$$AD = \frac{1}{MN}\sum_{i=0}^{M}\sum_{j=0}^{N}\left\{ I_m[x.y] - DC^{M}[x.y] \right\} \quad (11)$$

4) Normalized Absolute error

The normalized absolute error (NAE) is given by

$$NAE = \frac{\sum_{i=1}^{M}\sum_{j=1}^{N}\left|\left[I_m(x,y) \bullet DC^{M}(x,y)\right]\right|}{\sum_{i=1}^{M}\sum_{j=1}^{N}\left|I_m(x,y)\right|} \quad (12)$$

Where;

$I_m[x,y]$ → Original input image

$DC^{M}[x,y]$ → Decompressed image

IV. Experimental Results

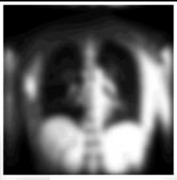
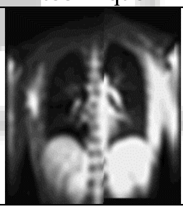
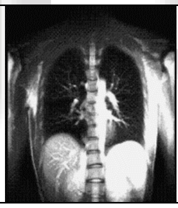
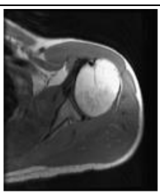
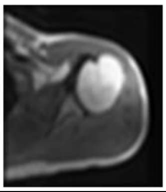
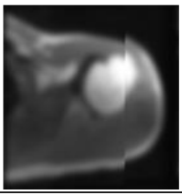
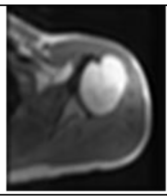
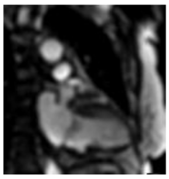
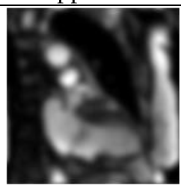
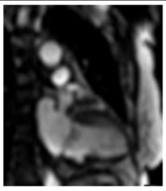
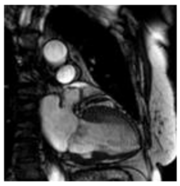
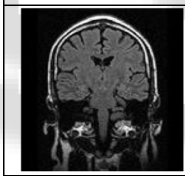
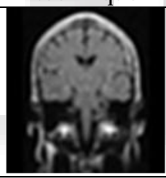
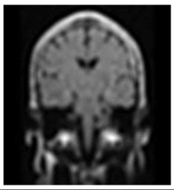
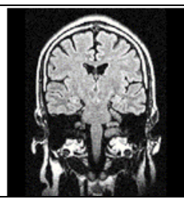
Original image MRI Lungs	Image compressed by proposed Shearlet technique	Image compressed by Curvelet based approach
		
	Image compressed by Contourlet technique	Image compressed by Wavelet
		
MRI Shoulder	Image compressed by proposed Shearlet technique	Image compressed by curvelet based approach
		
	Image compressed by Contourlet technique	Image compressed by Wavelet
		
MRI Cardiac	Image compressed by proposed Shearlet technique	Image compressed by curvelet based approach
		
	Image compressed by Contourlet technique	Image compressed by Wavelet
		
MRI Brain	Image compressed by proposed Shearlet technique	Image compressed by curvelet based approach
		
	Image compressed by Contourlet technique	Image compressed by Wavelet
		

Table 1: Experimental Results

V. Comparative Analysis

We have done a Comparative analysis by comparing the shearlet based proposed approach with curvelet, contourlet and wavelet based approach. The performance of the comparative analysis is tabulated in table 1.

	PSNR (dB)				Mean Square error			
	Proposed work	Contourlet transform	Curvelet transform	Existing [17] transform	Propose d work	Contourlet transform	Curvelet transform	Existing [17]
1	40.72	31.23	39.27	28.31	5.51	49.04	7.70	95.89
2	35.02	24.57	23.39	24.92	20.49	226.79	298.16	209.30
3	40.29	33.00	32.78	27.50	6.09	32.56	34.28	115.61
4	31.25	18.36	18.44	26.58	48.76	947.63	932.02	142.91
	Average Difference				Normalized Absolute Error			

	Proposed work	Contourlet transform	Curvelet transform	Existing [17] transform	Proposed work	Contourlet transform	Curvelet transform	Existing [17]
1	0.01	0.03	0.01	95.89	0.20	0.43	0.19	1.02
2	-0.01	-0.02	0.02	-44.5	0.10	0.18	0.19	0.90
3	-0.03	-0.06	0.01	-14.3	0.16	0.27	0.28	1.00
4	-0.04	-0.10	0.00	-35.4	0.34	0.43	0.42	0.87

Table 2: Shearlet transform vs. Contourlet transform vs. Curvelet transform vs. Wavelet transform comparison

The comparison in these tables implies that the performance of the shearlet based proposed approach have outperformed by having better PSNR value, low value of Mean Square error and better Average difference compared with contourlet, curvelet and wavelet based algorithms.

VI. CONCLUSION

This paper describes an efficient image compression approach based on shearlet transform and Huffman coding. At first, the input medical images of different parts of body are resized and converted to grayscale image. Then, shearlet transform is proposed to be applied on these .Subsequently, the approximation coefficients extracted are compressed by Huffman coding. Decompression stage is performed by Inverse shearlet transform and Huffman decoding. Experimental results show that our image compression approach is competitive in comparison with other techniques available in the literature.

REFERENCES

[1] K. Sayood, “Introduction to data compression,” New Delhi: Harcourt India private limited, 2000.
[2] M. J. Weinberger, G. Seroussi and G. Sapiro, “The LOCO-I lossless image compression algorithm: Principles and standardization into JPEG-LS,” IEEE Trans. on Image Processing, vol. 2, 2000, pp. 1309-1324.
[3] Siddiqi A.H., “Emerging Applications of Wavelet methods, American institute of Physics,” AIP Conference Proceedings, Meluille, New York.
[4] E.Candes, D.Donoho, “New tight frames of curvelets and optimal representations of objects with piecewise C2 singularities,” Commun. Pure Appl. Math, vol. 57, no. 2, 2004, pp. 219-266.
[5] M. Do, M. Vetterli, “The contourlet transform: An efficient directional multiresolution image representation,” IEEE Trans.Image Process, vol. 14, no. 12, 2005, pp. 2091-2106.
[6] Glenn R. Easley, Demetrio Labate, “Image Processing using Shearlets,” Shearlets Applied and Numerical Harmonic Analysis, 2012, pp 283-325.
[7] P. Grohs et al, “α-Molecules: Curvelets, Shearlets, Ridgelets, and Beyond,” Seminar for Applied Mathematics, Swiss Federal Institute of Technology, Zurich, 2014.
[8] D.Labate et al, “Sparse multidimensional representation using Shearlet,” Wavelets XI, SPIE Bellingham, 2005, pp.254-262.
[9] J.L. Starck, E.J. Candes and D.L. Donoho, “The Curvelet Transform for Image Denoising,” IEEE Trans. on Image Proc., vol. 11, no. 6 ,2002, pp. 670-684.
[10] E.J.Candes, D.L. Donoho and L. Demanet, “Fast discrete Curvelet transforms,” Multiscale Model, Simu, vol.5, 2006, pp. 861-899.
[11] M.N. Do and M.Vetterli, “The Contourlet transform: An efficient directional multiresolution image representation,” IEEE Transactions on Image Processing, Dec 2005, pp. 2091–2096,.
[12] Rafael, C.Gonzalez.and Richard E. Woods, “Digital image processing” Second Edition, Beijing: Publishing House of Electronics Industry, Jul.2005.
[13] http://www.mr-tip.com/serv1.php?type=isimg
[14] D. Labate et al, “Sparse multidimensional representation using Shearlet,” Wavelets XI, SPIE Bellingham, 2015, pp.254-262..
[15] A.M Rufai, G. Anbarjafari and H. Demirel, "Lossy medical image compression using Huffman coding and singular value decomposition,"21st Signal Processing and Communications Applications Conference (SIU), 2013, pp.1-4.
[16] V.S.Vora et al, “Analysis of Compressed Image quality Assessments”,Interntional Journal of Advanced Engineering and Application, January 2010.
[17] Öz, İbrahim, Cemil Öz, and Nejat Yumuşak. "Image Compression Using 2-D Multiple-Level Discrete Wavelet Transform (DWT)." Sakarya University, Faculty of Engineering, Department of Electrical & Electronics Engineering, Sakarya, Turkey.
[18] Negar Riazifar and Mehran Yazd, “Effectiveness of Contourlet vs Wavelet Transform on Medical Image Compression: a Comparative Study,” World Academy of Science, Engineering and Technology, vol 3, 2009, pp. 714-719.