

Design & Analysis of Composite Drive Shaft Made of Different Materials for Application in Automobiles

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Abstract— Substituting composite structures for conventional metallic structures has many advantages because of higher specific stiffness and strength of composite materials. Advanced composite materials seem ideally suited for long, power driver shaft applications. Their elastic properties can be tailored to increase the torque and the rotational speed at which they operate. The drive shafts are used in automotive, aircraft and aerospace applications for power transmission. This work deals with the replacement of conventional two-piece steel drive shafts with a single-piece E-Glass/ Epoxy, High Strength Carbon/Epoxy composite drive shaft for an automotive application. The design parameters were optimized for E-Glass/ Epoxy, High Strength Carbon/Epoxy composite drive shafts of an automobile using FEA. The design parameters such as thickness optimized with the objective of minimizing the weight of composite drive shafts which is subjected to the constraints such as torque transmission, torsional buckling capacities and fundamental lateral natural frequency. The weight savings of the E-Glass/ Epoxy, High Strength Carbon/Epoxy shaft were 48.36 %, 86.90 % of the steel shaft respectively. The torque transmission capacity of the composite drive shafts have been calculated by neglecting and considering the effect of centrifugal forces and it was observed that centrifugal forces will reduce the torque transmission capacity of the shaft. The variation of the stress along the thickness of the Steel, E-Glass/Epoxy, and High Strength Carbon/Epoxy composite drive shafts are observed. It is also observed that all stresses are within in the allowable limit. Static, Modal and Buckling analysis are carried out on the finite element Analysis of the composite drive shaft.

Key words: Composite Drive Shaft Made, Automobiles

I. INTRODUCTION

The advanced composite materials such as Graphite, Carbon, Kevlar and Glass with suitable resins are widely used because of their high specific strength (strength/density) and high specific modulus (modulus/density). Advanced composite materials seem ideally suited for long, power driver shaft (propeller shaft) applications. Their elastic properties can be tailored to increase the torque they can carry as well as the rotational speed at which they operate. The drive shafts are used in automotive, aircraft and aerospace applications. The automotive industry is exploiting composite material technology for structural components construction in order to obtain the reduction of the weight without decrease in vehicle quality and reliability. It is known that energy conservation is one of the most important objectives in vehicle design and reduction of weight is one of the most effective measures to obtain this result. Actually, there is almost a direct proportionality between the weight of a vehicle and its fuel consumption, particularly in city driving.

Composites consist of two or more materials or material phases that are combined to produce a material that has superior properties to those of its individual constituents. The constituents are combined at a macroscopic level and or not soluble in each other. The main difference between composite and an alloy are constituent materials which are insoluble in each other and the individual constituents retain those properties in the case of composites, where as in alloys, constituent materials are soluble in each other and forms a new material which has different properties from their constituents.

II. LITERATURE SURVEY

A. Composite

The theoretical details of composite materials and composite structures are extensively reviewed. The Spicer U-Joint Division of Dana Corporation for the Ford Economize van models developed the first composite propeller shaft in 1985. The General Motors pickup trucks, which adopted the Spicer product, enjoyed a demand three times that of projected sales in its first year. John. W. Weeton et al. briefly described the application possibilities of composites in the field of automotive industry to manufacture composite elliptic springs, drive shafts and leaf springs. Beard more and Johnson discussed the potential for composites in structural automotive applications from a structural point of view. Pollard studied the possibility of the polymer Matrix composites usage in driveline applications. Faust et.al, described the considerable interest on the part of both the helicopter and automobile industries in the development of lightweight drive shafts. Procedure for finding the elastic moduli of anisotropic laminated composites is explained by Azzi.V.D et.al, Azzi.V.D .et.al, discussed about anisotropic strength of composites.

B. Torsional Buckling

The problem of general instability under torsional load has been studied by many investigators. Greenhill obtained a solution for the torsional stability of a long shaft. The first analysis of buckling of thin-walled tubes under torsion made by Schwerin, but his analysis did not agree with his experimental data. However, all these papers were limited to isotropic materials.

As far as orthotropic materials are concerned, general theories of orthotropic shells were developed by Ambartsumyan and Dong et al. Cheng and Ho analyzed more generally, the buckling problems of non-homogeneous anisotropic cylindrical shells under combined axial, radial and torsional loads with all four boundary conditions at each end of the cylinder. Lien-Wen Chen et.al. analyzed the stability behavior of rotating composite shafts under axial compressive loads. A theoretical analysis was presented for determining the buckling torque of a cylindrical hollow

shaft with layers of arbitrarily laminated composite materials by means of various thin-shell theories. Bauchau et al., measured the torsional buckling loads of graphite/epoxy shafts, which were in good agreement with theoretical predictions based on a general shell theory including elastic coupling effects and transverse shearing deformations.

C. Lateral Vibrations

Bauchau developed procedure for optimum design of high-speed composite drive shaft made of laminates to increase the first natural frequency of the shaft and to decrease the bending stress. Shell theory based on critical speed analyses of drive shafts are presented by Dos Reis et al. Patricia L.Hetherington investigated the dynamic behavior of supercritical-composite drive shafts for helicopter applications. Ganapathi.et.al extensively studied the nonlinear free flexural vibrations of laminated circular cylindrical shells. A method of analysis involving Love’s first approximation theory and Ritz’s procedure is used to study the influence of boundary conditions and fiber orientation on the natural frequencies of thin orthotropic laminated cylindrical shells was presented. A first order theory was presented by Lee to determine natural frequencies of orthotropic shell. Nowinski. J.L. investigated the nonlinear transverse vibrations of elastic orthotropic shells using Von-Karman-Tsien equations.

D. Optimization

The optimum design of laminated plates and shells subjected to constraints on strength, stiffness, buckling loads, and fundamental natural frequencies were examined. Methods were proposed for the determination of the optimal ply angle variation through the thickness of symmetric angle-ply shells of uniform thickness. The main features of GAs and several ways in which they can solve difficult design problems were discussed by Gabor Renner et.al. Raphael T.Haftka discussed extensively about stacking-sequence optimization for buckling of laminated plates by integer programming. The use of a GA to optimize the stacking sequence of a composite laminates for buckling load maximization was studied. Various genetic parameters including the population size, the probability of mutation, and the probability of crossover were optimized by numerical experiments. The use of GAs for the optimal design of symmetric composite laminates subject to various loading and boundary conditions were explained. Kim et.al. minimized the weight of composite laminates with ply drop under a strength constraint. The working of Simple Genetic Algorithm was explained by Goldberg.

III. OBJECTIVES OF THE WORK

This work deals with the replacement of a conventional steel drive shaft with E-Glass/ Epoxy, High Strength Carbon/Epoxy composite drive shafts for an automobile application.

A. Analysis

- 1) Modeling of the High Strength Carbon/Epoxy,E-Glass/Epoxy composite drive shaft using ANSYS
- 2) Static, Modal and Buckling analysis are to be carried out on the finite element model of the E-Glass/Epoxy,

High Strength Carbon/Epoxy composite drive shaft using ANSYS

3) To investigate

- a) The stress and strain distributions in E-Glass/ Epoxy, High Strength Carbon/Epoxy composite drive shafts using ansys.
- b) The effect of centrifugal forces on the torque transmission capacity of the composite drive shafts.
- c) The effect of transverse shear and rotary inertia on the fundamental lateral natural frequency of the shaft.

IV. SPECIFICATION OF THE PROBLEM

The torque transmission capability of the drive shaft for passenger cars, small trucks, and vans should be larger than 3,500 Nm and fundamental natural bending frequency of the propeller shaft should be higher than 6,500 rpm to avoid whirling vibration. The drive shaft outer diameter should not exceed 100 mm due to space limitations. Here outer diameter of the shaft is taken as 90 mm. The drive shaft of transmission system is to be designed optimally for following specified design requirements as shown in Table 1.

Sl. No.	Name	Notation	Unit	Value
1.	Ultimate Torque	T _{max}	Nm	3500
2.	Max. Speed of shaft	N _{max}	rpm	6500
3.	Length of shaft	L	mm	1250
Parameters	Steel	E-Glass / Epoxy	HS Carbon/Epoxy	
do (mm)	90	90	90	90
L (mm)	1250	1250	1250	1250
tk (mm)	3.318	0.4	0.12	0.12
Optimum no. of Layers	1	17	17	17
t (mm)	3.318	6.8	2.04	2.04

Table 1: Design requirements and specifications

Steel (SM45C) used for automotive drive shaft applications. The material properties of the steel (SM45C) are given in Table 2. The steel drive shaft should satisfy three design specifications such as torque transmission capability, buckling torque capability and bending natural frequency.

Mechanical properties	Symbol	Units	Steel
Young’s Modulus	E	GPa	207.0
Shear modulus	G	GPa	80.0
Poisson’s ratio	ν	-----	0.3
Density	ρ	Kg/m ³	7600
Yield Strength	S _y	MPa	370
Shear Strength	S _s	MPa	--

Table 2: Mechanical properties of Steel (SM45C)

A. Torque Transmission Capacity Of The Drive Shaft

$$T = S_s \frac{\pi(d_o^4 - d_i^4)}{16 T d_o}$$

B. Torsional Buckling Capacity of the Drive Shaft

$$\text{If } \frac{1}{\sqrt{1-v^2}} \frac{L^2 t}{(2r)^3} > 5.5,$$

it is called as Long shaft otherwise it is called as Short & Medium shaft.

For long shaft, the critical stress is given by

$$\tau_{cr} = \frac{E}{3\sqrt{2}(1-v^2)^{3/4}} (t/r)^{3/2}$$

For short & medium shaft, the critical stress is given by

$$\tau_{cr} = \frac{4.39 E}{(1-v^2)} (t/r)^2 \sqrt{1 + 0.0257 (1-v^2)^{3/4}} \frac{L^3}{(rt)^{1.5}}$$

The relation between the torsional buckling capacity and critical stress is given by

$$T_{cr} = \tau_{cr} 2\pi r^2 t$$

C. Lateral or Bending Vibration

The shaft is considered as simply supported beam undergoing transverse vibration or can be idealized as a pinned-pinned beam. Natural frequency can be found using the following two theories.

1) Bernoulli-Euler Beam Theory- N_{crbe}

It neglects the both transverse shear deformation as well as rotary inertia effects. Natural frequency based on the Bernoulli-Euler beam theory is given by,

$$f_{nbe} = \frac{\pi p^2}{2L^2} \sqrt{\frac{EI_x}{m_1}}$$

Where $p = 1, 2, \dots$

$$N_{crbe} = 60 f_{nbe}$$

Timoshenko Beam Theory- N_{crt}

It considers both transverse shear deformation as well as rotary inertia effects. Natural frequency based on the Timoshenko beam theory is given by,

$$f_{nt} = K_s \frac{30 \pi p^2}{L^2} \sqrt{\frac{E_r^2}{2\rho}}$$

$$N_{crt} = 60 f_{nt}$$

$$\frac{1}{K_s^2} = 1 + \frac{n^2 \pi^2 r^2}{2L^2} \left[1 + \frac{f_s E}{G} \right]$$

$f_s = 2$ for hollow circular cross-sections

The relation between Timoshenko and Bernoulli-Euler Beam Theories

The relation between Timoshenko and Bernoulli-Euler beam theories is given by,

$$f_{nt} = K_s f_{nbe}$$

Sl.No.	Name	Notation	Unit	Value
1.	Ultimate Torque	T_{max}	Nm	3500
2.	Max. Speed of shaft	N_{max}	rpm	6500
3.	Length of shaft	L	mm	1250
Parameters	Steel	E-Glass / Epoxy	HS Carbon/Epoxy	

do (mm)	90	90	90	
L (mm)	1250	1250	1250	
tk (mm)	3.318	0.4	0.12	
Optimum no. of Layers	1	17	17	
t (mm)	3.318	6.8	2.04	

Table 3: Design requirements and specifications

V. DESIGN OF COMPOSITE DRIVE SHAFT

A. Specification of the Problem

The specifications of the composite drive shaft of an automotive transmission are same as that of the steel drive shaft for optimal design.

B. Selection of Cross-Section

The drive shaft can be solid circular or hollow circular. Here hollow circular cross-section was chosen because:

- The hollow circular shafts are stronger in per kg weight than solid circular.
- The stress distribution in case of solid shaft is zero at the center and maximum at the outer surface while in hollow shaft stress variation is smaller. In solid shafts the material close to the center are not fully utilized.

C. Selection of Reinforcement Fiber

Fibers are available with widely differing properties. Review of the design and performance requirements usually dictate the fiber/fibers to be used.

- Carbon/Graphite fibers its advantages include high specific strength and modulus, low coefficient of thermal expansion, and high fatigue strength. Graphite, when used alone has low impact resistance. Its drawbacks include high cost, low impact resistance, and high electrical conductivity.
- Glass fibers Its advantages include its low cost, high strength, high chemical resistance, and good insulating properties. The disadvantages are low elastic modulus, poor adhesion to polymers, low fatigue strength, and high density, which increase shaft size and weight. Also crack detection becomes difficult.
- Kevlar fibers Its advantages are low density, high tensile strength, low cost, and higher impact resistance. The disadvantages are very low compressive strength, marginal shear strength, and high water absorption. Kevlar is not recommended for use in torque carrying application because of its low strength in compression and shear.

Here, both glass and carbon fibers are selected as potential materials for the design of shaft.

D. Selection of Resin System

The important considerations in selecting resin are cost, temperature capability, elongation to failure and resistance to impact (a function of modulus of elongation). The resins selected for most of the drive shafts are either epoxies or vinyl esters. Here, epoxy resin was selected due to its high strength, good wetting of fibers, lower curing shrinkage, and better dimensional stability.

VI. SELECTION OF MATERIALS

Based on the advantages discussed earlier, the E-Glass/Epoxy, High Strength Carbon/Epoxy materials are selected for composite drive shaft. The Table 4 shows the properties of the E-Glass/Epoxy, High Strength Carbon/Epoxy materials used for composite drive shafts.

Sl.No	Property	Units	E-Glass/Epoxy	HS Carbon/Epoxy
1.	E ₁₁	GPa	50.0	134.0
2.	E ₂₂	GPa	12.0	7.0
3.	G ₁₂	GPa	5.6	5.8
4.	ν ₁₂	-	0.3	0.3
5.	S ₁ ^t =S ₁ ^c	MPa	800.0	880.0
6.	S ₂ ^t =S ₂ ^c	MPa	40.0	60.0
7.	S ₁₂	MPa	72.0	97.0
8.	P	Kg/m ³	2000.0	1600.0

Table 4: Properties of E-Glass/Epoxy, HS Carbon/Epoxy and HS Carbon/Epoxy

A. Factor of Safety

The designer must take into account the factor of safety when designing a structure. Since, composites are highly orthotropic and their fractures were not fully studied the factor of safety was taken as 2.

B. Torque Transmission Capacity of the Shaft

Stress-Strain Relationship for Unidirectional Lamina

The lamina is thin and if no out-of-plane loads are applied, it is considered as the plane stress problem. Hence, it is possible to reduce the 3-D problem into 2-D problem. For unidirectional 2-D lamina, the stress-strain relationship is given by,

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{Bmatrix} \epsilon_1 \\ \epsilon_2 \\ \gamma_{12} \end{Bmatrix}$$

$$Q_{11} = \frac{E_{11}}{1 - \nu_{12}\nu_{21}} \quad Q_{22} = \frac{E_{22}}{1 - \nu_{12}\nu_{21}}$$

$$Q_{12} = \frac{\nu_{12}E_{22}}{1 - \nu_{12}\nu_{21}} \quad Q_{66} = G_{12}$$

$$Q_{21} = Q_{12}$$

Stress-Strain Relationship for Angle-ply Lamina

The relation between material coordinate system and X-Y-Z coordinate system is shown in Fig 1. Coordinates 1, 2, 3 are principal material directions and coordinates X, Y, Z are transformed or laminate axes.

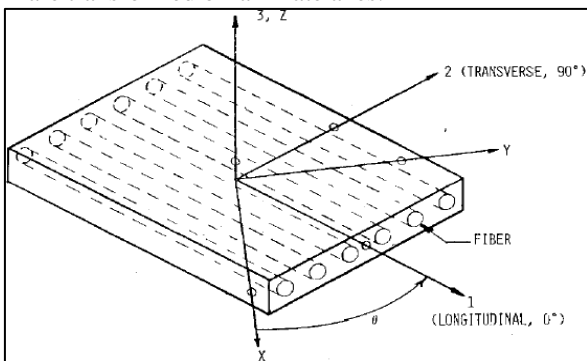


Fig. 1: Relation between material coordinate system and X-Y coordinate system

For an angle-ply lamina where fibers are oriented at an angle with the positive X-axis (Longitudinal axis of shaft), the effective elastic properties are given by

$$\frac{1}{E_{x \text{ lamina}}} = \frac{1}{E_{11}} C^4 + \left[\frac{1}{G_{12}} - \frac{2\nu_{12}}{E_{11}} \right] S^2 C^2 + \frac{1}{E_{22}} S^4$$

$$\frac{1}{E_{y \text{ lamina}}} = \frac{1}{E_{11}} S^4 + \left[\frac{1}{G_{12}} - \frac{2\nu_{12}}{E_{11}} \right] S^2 C^2 + \frac{1}{E_{22}} C^4$$

$$\frac{1}{G_{xy \text{ lamina}}} = 2 \left[\frac{2}{E_{11}} + \frac{2}{E_{22}} + \frac{2\nu_{12}}{E_{11}} - \frac{1}{G_{12}} \right] S^2 C^2 + \frac{1}{G_{12}} [C^4 + S^4]$$

The variation of the E_{xlamina}, E_{ylamina} and G_{xylamina} with ply orientation is shown in Fig 2 and 3 respectively.

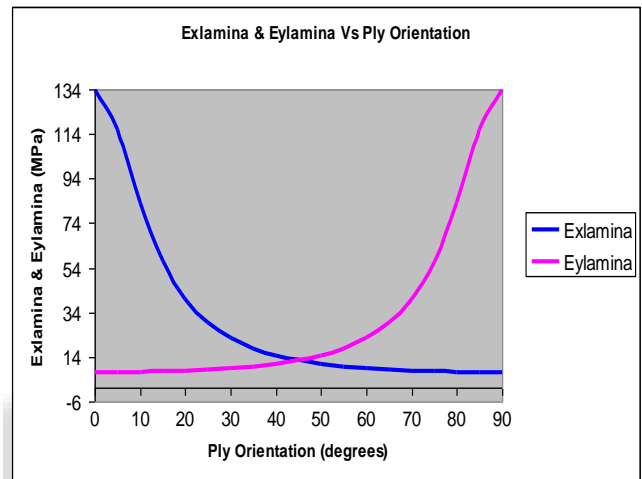


Fig. 2: The variation of the E_{xlamina} and E_{ylamina} with ply orientation for HS Carbon/Epoxy

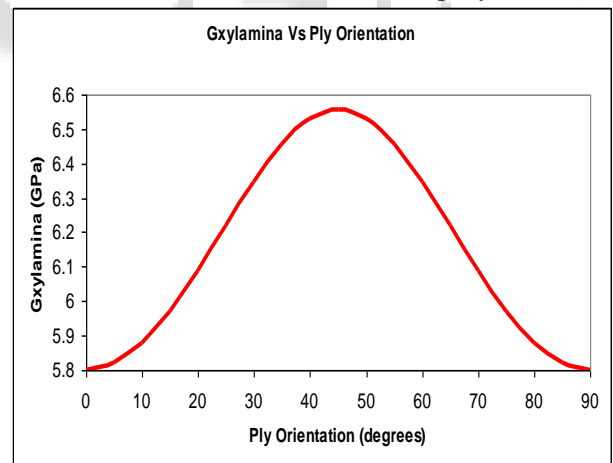


Fig. 3: The variation of the G_{xylamina} with ply orientation for HS Carbon/Epoxy

The stress strain relationship for an angle-ply lamina is given by,

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix} \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix}$$

$$\begin{aligned} \overline{Q_{11}} &= Q_{11}C^4 + Q_{22}S^4 + 2(Q_{12} + 2Q_{66})S^2C^2 \\ \overline{Q_{12}} &= (Q_{11} + Q_{22} - 4Q_{66})S^2C^2 + Q_{12}(C^4 + S^4) \\ \overline{Q_{16}} &= (Q_{11} - Q_{12} - 2Q_{66})C^3S - (Q_{22} - Q_{12} - 2Q_{66})CS^3 \\ \overline{Q_{22}} &= Q_{11}S^4 + Q_{22}C^4 + 2(Q_{11} + 2Q_{66})S^2C^2 \\ \overline{Q_{26}} &= (Q_{11} - Q_{12} - 2Q_{66})CS^3 - (Q_{22} - Q_{12} - 2Q_{66})C^3S \\ \overline{Q_{66}} &= (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66})S^2C^2 + Q_{66}(S^4 + C^4) \end{aligned}$$

$$\begin{Bmatrix} \kappa_x^o \\ \kappa_y^o \\ \kappa_{xy}^o \end{Bmatrix} = \begin{bmatrix} d_{11} & d_{12} & d_{16} \\ d_{12} & d_{22} & d_{26} \\ d_{16} & d_{26} & d_{66} \end{bmatrix} \begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix}$$

Where

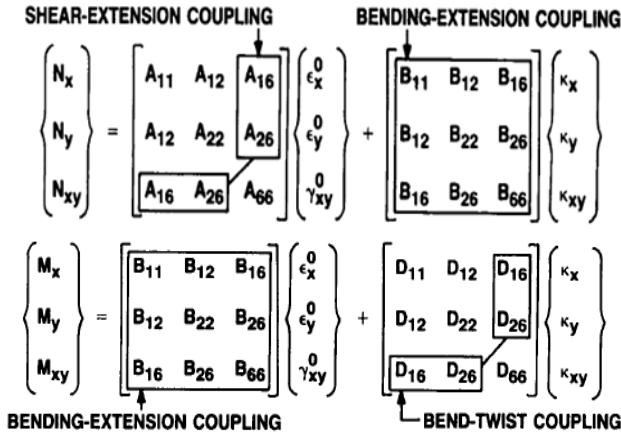
$$\begin{bmatrix} a_{11} & a_{12} & a_{16} \\ a_{12} & a_{22} & a_{26} \\ a_{16} & a_{26} & a_{66} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix}^{-1}$$

$$\begin{bmatrix} d_{11} & d_{12} & d_{16} \\ d_{12} & d_{22} & d_{26} \\ d_{16} & d_{26} & d_{66} \end{bmatrix} = \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix}^{-1}$$

$E_x = \frac{1}{a_{11}t}$ = Young's Modulus of the Shaft in axial direction

$E_y = \frac{1}{a_{22}t}$ = Young's Modulus of the Shaft in hoop direction

$G_{xy} = \frac{1}{a_{66}t}$ = Rigidity Modulus of the Shaft in xy plane



$$\begin{aligned} A_{ij} &= \sum_{k=1}^n (\overline{Q_{ij}})_k (h_k - h_{k-1}) & A_{ij} &= \sum_{k=1}^n (\overline{Q_{ij}})_k t_k \\ D_{ij} &= \frac{1}{3} \sum_{k=1}^n (\overline{Q_{ij}})_k (h_k^3 - h_{k-1}^3) \\ B_{ij} &= \frac{1}{2} \sum_{k=1}^n (\overline{Q_{ij}})_k (h_k^2 - h_{k-1}^2) \end{aligned}$$

[A], [B], [D] matrices are called the extensional, coupling, and bending stiffness matrices respectively. By combining the equations we get,

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} \epsilon_x^o \\ \epsilon_y^o \\ \gamma_{xy}^o \\ \kappa_x^o \\ \kappa_y^o \\ \kappa_{xy}^o \end{Bmatrix}$$

For symmetric laminates, the B matrix vanishes and the in plane and bending stiffness are uncoupled. For a symmetric laminate,

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{Bmatrix} \epsilon_x^o \\ \epsilon_y^o \\ \gamma_{xy}^o \end{Bmatrix}$$

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} \kappa_x^o \\ \kappa_y^o \\ \kappa_{xy}^o \end{Bmatrix}$$

$$\begin{Bmatrix} \epsilon_x^o \\ \epsilon_y^o \\ \gamma_{xy}^o \end{Bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{16} \\ a_{12} & a_{22} & a_{26} \\ a_{16} & a_{26} & a_{66} \end{bmatrix} \begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix}$$

When a shaft is subjected to torque T, the resultant forces in the laminate by considering the effect of centrifugal forces are

$$N_x = 0 \quad N_y = 2\rho r^2 \omega^2$$

Stresses in the Kth ply are given by,

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}_k = \begin{bmatrix} \overline{Q_{11}} & \overline{Q_{12}} & \overline{Q_{16}} \\ \overline{Q_{12}} & \overline{Q_{22}} & \overline{Q_{26}} \\ \overline{Q_{16}} & \overline{Q_{26}} & \overline{Q_{66}} \end{bmatrix}_k \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix}$$

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix}_k = \begin{bmatrix} C^2 & S^2 & 2CS \\ S^2 & C^2 & -2CS \\ -CS & CS & C^2 - S^2 \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}_k$$

Knowing the stresses in each ply, the failure of the laminate is determined by using the First Ply Failure criteria. That is, the laminate is assumed to fail when the first ply fails. Here maximum stress theory is used to find the torque transmitting capacity.

C. Torsional Buckling Capacity (TCR)

Since long thin hollow shafts are vulnerable to torsional buckling, the possibility of the torsional buckling of the composite shaft was checked by the expression for the torsional buckling load T_{cr} of a thin walled orthotropic tube, which was expressed below.

$$T_{cr} = (2\pi r^2 t)(0.272)(E_x E_y^3)^{0.25}(t/r)^{1.5}$$

This equation has been generated from the equation of isotropic cylindrical shell and has been used for the design of drive shafts. From the equation, the torsional buckling capability of composite shaft is strongly dependent on the thickness of composite shaft and the average modulus in the hoop direction.

D. Lateral or Bending Vibration

The shaft is considered as simply supported beam undergoing transverse vibration or can be idealized as a pinned-pinned beam. Natural frequency can be found using the following two theories.

$$N_{xy} = \frac{T}{2\pi r^2}$$

1) Bernoulli-Euler Beam Theory- N_{crbe}

It neglects the both transverse shear deformation as well as rotary inertia effects. Natural frequency based on the Bernoulli-Euler beam theory is given by,

$$f_{nbe} = \frac{\pi p^2}{2L^2} \sqrt{\frac{E_x I_x}{m_i}}$$

Where $p=1, 2, \dots$

$$N_{crbe} = 60 f_{nbe}$$

2) Timoshenko Beam Theory- N_{crt}

It considers both transverse shear deformation as well as rotary inertia effects. Natural frequency based on the Timoshenko beam theory is given by,

$$f_{nt} = K_s \frac{30\pi p^2}{L^2} \sqrt{\frac{E_x r^2}{2\rho}}$$

$$N_{crt} = 60 f_{nt}$$

Where K_s =shear coefficient of the lateral natural frequency (<1)

$$\frac{1}{K_s^2} = 1 + \frac{n^2 \pi^2 r^2}{2L^2} \left[1 + \frac{f_s E_x}{G_{xy}} \right]$$

$f_s = 2$ for hollow circular cross-sections

The relation between Timoshenko and Bernoulli-Euler Beam Theories

The relation between Timoshenko and Bernoulli-Euler beam theories is given by,

$$f_{nt} = K_s f_{nbe}$$

VII. RESULTS

A. Deflection

The deflection of E-Glass/Epoxy, HS Carbon/Epoxy and Steel drive shafts are shown in Table 5.

Material	Deflection(mm)
Steel	1.038
E-Glass/Epoxy	0.71547
HS Carbon/Epoxy	0.862118

Table 5: Deflection of Drive Shafts

B. VON MISES stress

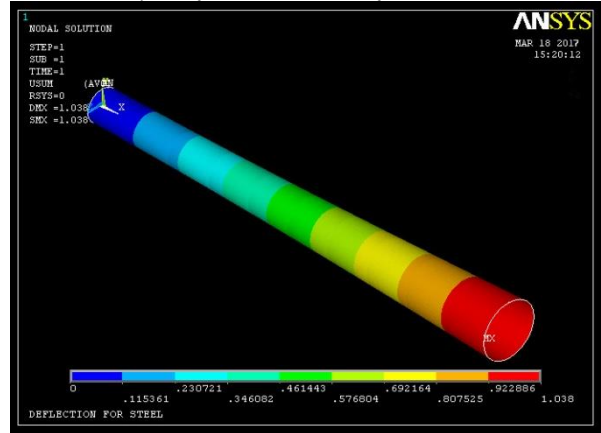
Material	STRESS(N/mm2)
Steel	54.186
E-Glass/Epoxy	25.582

HS Carbon/Epoxy	36.464
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Table 6: Deflection of Drive Shafts

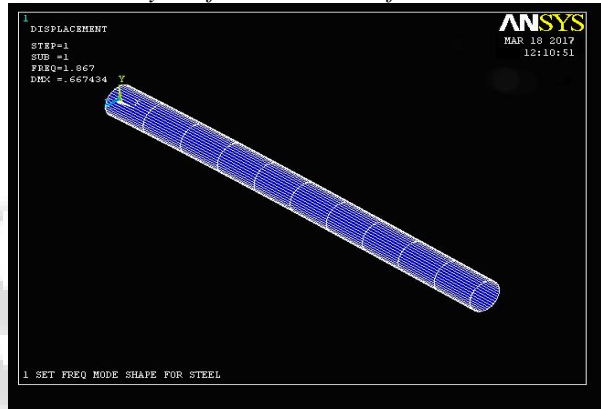
C. Steel Drive Shaft

1) Static Analysis of Steel Drive Shaft



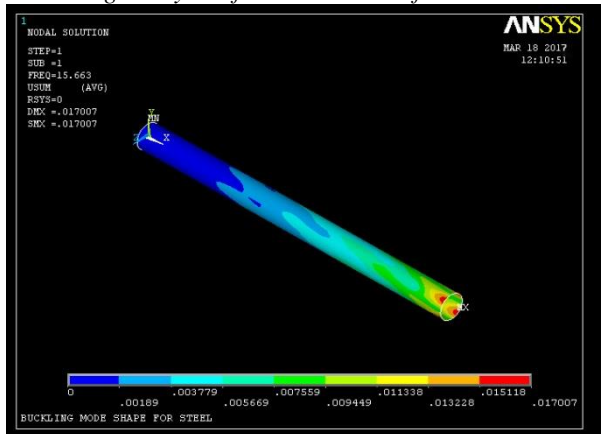
Von Mises Stress for Steel Drive Shaft

2) Modal Analysis of Steel Drive Shaft



1st Frequency Mode Shape for Steel Drive Shaft

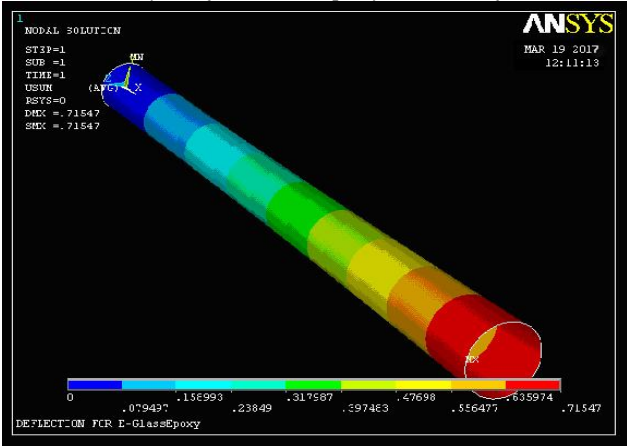
3) Buckling Analysis of Steel Drive Shaft



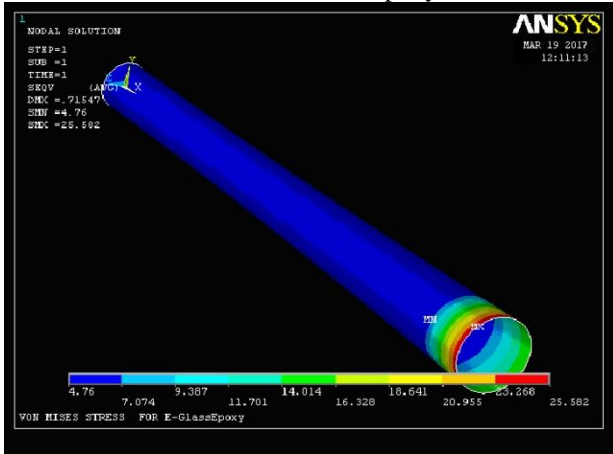
1st Buckling Mode Shape for Steel Drive Shaft

D. E-Glass/Epoxy Drive Shaft

1) Static Analysis of E-Glass/Epoxy Drive Shaft

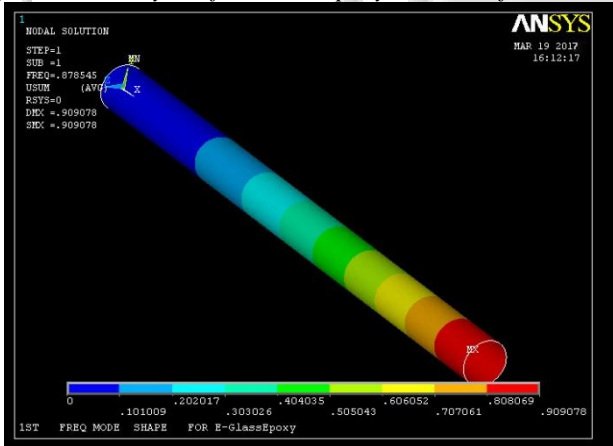


Deflection for E-Glass/Epoxy Shaft



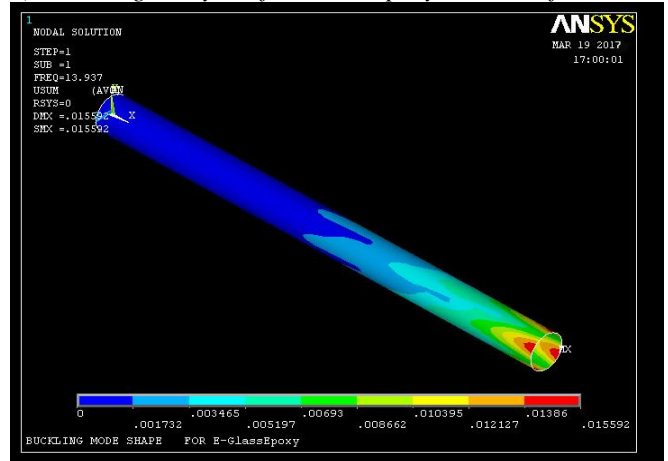
Von Mises Stress for E-Glass/Epoxy Shaft

2) Modal Analysis of E-Glass/Epoxy Drive Shaft



1st Frequency Mode Shape for E-Glass/Epoxy Shaft

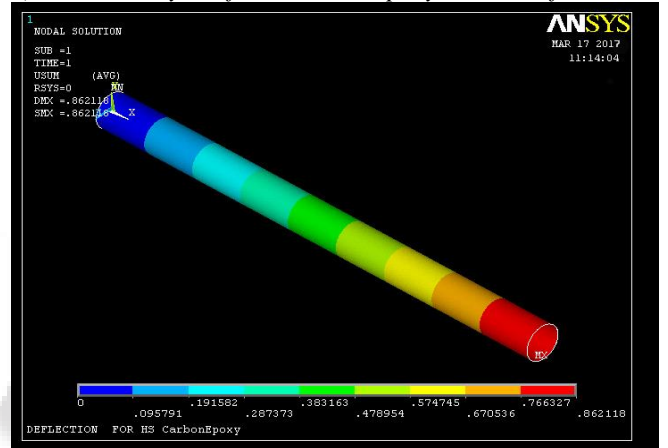
3) Buckling Analysis of E-Glass/Epoxy Drive Shaft



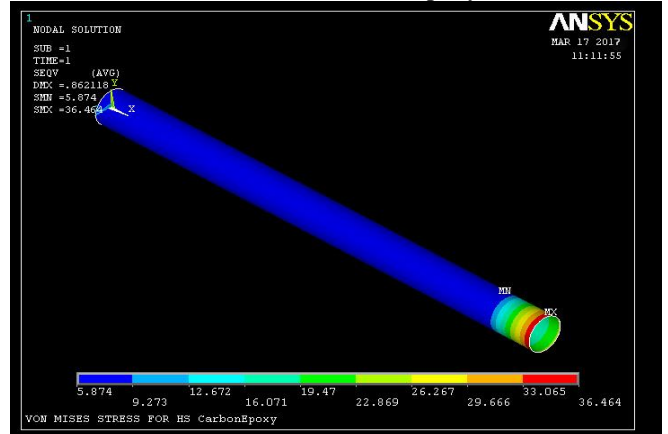
1st Buckling Mode Shape for E-Glass/Epoxy

E. HS Carbon/Epoxy Drive Shaft

1) Static Analysis of HS Carbon/Epoxy Drive Shaft

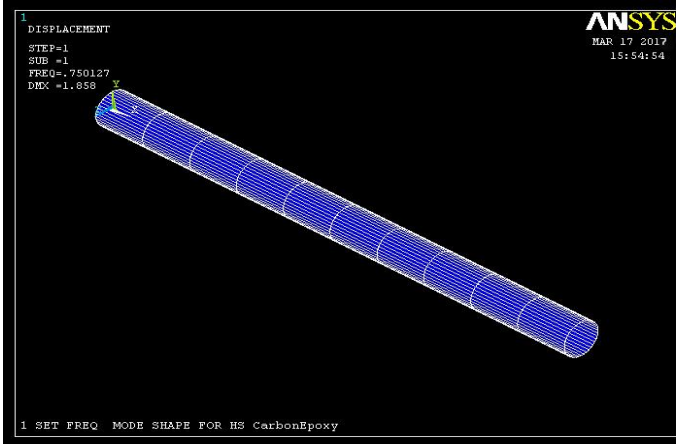


Deflection for HS Carbon/Epoxy Shaft



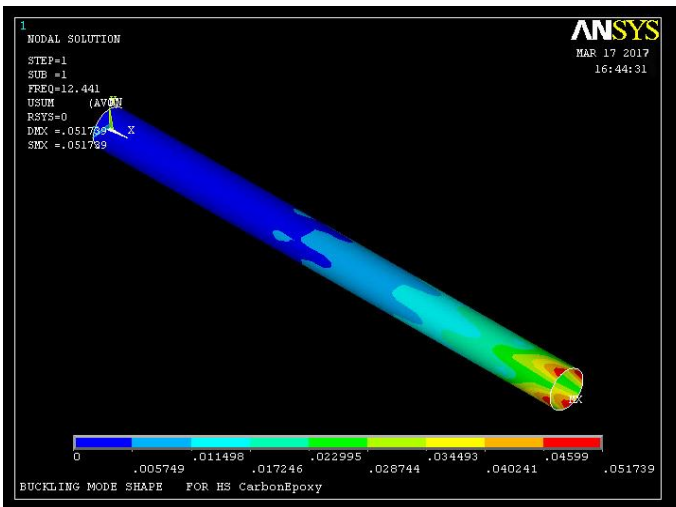
Von Mises Stress for HS Carbon/Epoxy Drive Shaft

2) Modal Analysis of HS Carbon/Epoxy Drive Shaft



1st Frequency Mode Shape for HS Carbon/Epoxy Shaft

F. Buckling Analysis of HS Carbon/Epoxy Drive Shaft



1st Buckling Mode Shape of HS Carbon/Epoxy Drive Shaft

VIII. CONCLUSION

- 1) The E-Glass/ Epoxy, High Strength Carbon/Epoxy composite drive shafts have been designed to replace the steel drive shaft of an automobile.
- 2) A one-piece composite drive shaft for rear wheel drive automobile has been designed optimally by using FEA for E-Glass/ Epoxy, High Strength Carbon/Epoxy and composites with the objective of minimization of weight of the shaft which was subjected to the constraints such as torque transmission, torsional buckling capacities and natural bending frequency.
- 3) The weight savings of the E-Glass/ Epoxy, High Strength Carbon/Epoxy and shafts were equal to 48.36%, 86.90% of the weight of steel shaft respectively.
- 4) The deflections of Steel, E-Glass/Epoxy, and High Strength Carbon/Epoxy shafts are equal to 1.038, 0.71547, and 0.862118 mm respectively.
- 5) The fundamental natural frequency of Steel, E-Glass/ Epoxy, High Strength Carbon/Epoxy shafts were 20.483, 10.615, 12.044 rpm respectively.
- 6) The torsional buckling capacity of Steel, E-Glass/ Epoxy, High Strength Carbon/Epoxy shafts were 54820.5, 48775.5, 43543.5 N-m respectively.

- 7) The torque transmission capacity of the composite drive shafts has been calculated by neglecting and considering the effect of centrifugal forces and it has been observed that centrifugal forces will reduce the torque transmission capacity of the shaft.

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