The Fundamentals of Fluid Motion Theory

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Abstract— In the present paper, I have discussed the fundamentals of Fluid Motion Theory and different types of wave motion. Motion of air in tunnel inlet, in aerodynamic problems, equations of motion, streamline maps for ideal fluids, irrotational motions and harmonic waves are discussed in detail.

Key words: Irrotational Motion, Streamline Map, Aerodynamic, Harmonic

I. INTRODUCTION

The most common and fundamental phenomena that occur in nature is the phenomena of wave motion and motion of air particles. One thing is common in various physical phenomena that energy is propagated with a finite velocity to distant points and disturbance travels through medium without giving any permanent displacement. It is still difficult to calculate exactly the resultant force experienced by a body of any given size and shape moving at a given speed through a fluid. Exact solutions for bodies of simple geometrical shape moving at very low speeds, and the mathematical theory has succeeded in predicting many of the outstanding features of forces experienced by bodies of simple shape moving at very high speeds through glasses, but apart from these examples, theory must be supplemented by experimental data in any realistic analysis. There is still a large element of empiricism in the treatment of problems of real aircraft. Different types of motions are discussed in detail.

A. Harmonics waves:

Harmonic wave is a simple example of a wave travelling with a constant velocity through any medium without change in profile. When harmonic waves are superimposed whose wave propagation velocities are the same, even though their wave numbers and angular frequencies differ, then the profile of the wave does not change with time. The superposition of harmonic waves with different propagation velocities results in a wave with changing profile such a wave is said to be dispersed. When the waves described by two wave functions travel with equal speeds but in opposite directions, the result wave thus does not appear to travel at all, but simply to pulsate, expanding and contracting it's amplitude with a frequency such a wave is called standing wave.

B. Wave motion in Tunnel Inlet:

It is seen when a body of simple shape (such as sphere) is suspended in a wind tunnel and the motion of the air is made visible by the introduction of smoke near the tunnel inlet. If the speed of air-stream is kept fairly low, the picture is one of continuous motion, when the paths of the smoke particles adapting themselves to the contours of the body. A closer examination shows that the individual particles have a wide range of speeds and that there is a tendency to form distinct and enduring patterns of flow.

In problems of aerodynamics if we adopt the hypothesis that air is a continuous medium, capable of being divided without limit into infinitesimal 'Fluid Particles' (Elementary volumes) which may be supposed to have all the physical properties of fluids in bulk.

C. Methods to solve Equations of Motion:

Fluid particles are free to move in the field and their properties such as density may change, this change is because the particle changes its position in the field and because the field itself is changing with time. The statements of the laws of motion of fluid dynamics necessarily take the form of particle differential equations because there are four independent variables, three spatial coordinates and time. In ordinary dynamics time is the sole independent variable and the equations of motion involve only ordinary derivatives.

There are two methods to find the solution. The first method, generally known by the name of Lagrange, starts with the concept of a typical fluid particle, subject to the ordinary laws of Newtonian dynamics, moving in the fluid field. The aim of this method is to find a mathematical expression which describes the movements of the particles during a prescribed interval of time. The second method given by Euler, tries to find an expression giving the magnitude and direction of the velocity at every point in the fluid at any time. In principal, the two methods are equivalent, but in practice Lagrange's method usually involves greater technical difficulties than Euler's.

The tendency of fluid motion to form stable patterns emphasizes the important part played by purely kinematical considerations in aerodynamics. Euler's method aims at finding the expression for the velocity vector V as a single-valued function of space and time. At any fixed time it is possible to form an instantaneous picture of the entire motion by constructing curves which have the same direction as V at all points. Such curves are called streamlines. If the field is steady, that is, if properties measured at a point are independent of time, the streamline pattern is unchanging and may be found experimentally by injecting colouring matter or smoke from a number of points equally spaced across the stream.

D. Streamline Maps for ideal fluids:

Any fluid, whether liquid or gas, is subject to internal stress. When a fluid is at rest the stress between contiguous parts is pressure, which always acts perpendicularly to the interface between the parts. When a fluid moves there is relative motion between contiguous parts, and tangential stresses, resembling friction between solids, are set up. The existence of such stresses expressed by the term viscosity, greatly complicates the mathematical treatment, and the study of fluid dynamics, both historically and practically, begins with hypothetical ideal or inviscid fluid, which cannot support ant tangential stress however small.

At first sight this particular idealization appears to entail no great break with reality, since the dynamic viscosity of air is very small (about 10^{-4} g/cm sec), and it seems reasonable to assume that such small forces cannot affect the motion seriously. To some extent this is true, but the persistent neglect of viscosity is fatal for the development of fluid dynamics. The reason is that although at some distance from a solid surface the air behaves exactly as if it were devoid of internal friction, in regions adjacent to the body the viscous forces are very important, and the ultimate pattern of flow of the fluid depends largely on what happens in the so-called boundary layer, a very thin sheath of slowly moving air which envelops the entire body.

If the circumstances of the motion are such that variations in the density of a moving fluid particle are unimportant, the fluid is said to be incompressible. Since matter cannot be created or destroyed by motion, the mass of fluid which enters any length of streamtube must be the same as that which leaves, and therefore, the speed of an incompressible fluid is inversely proportional to the spacing of the streamlines. A properly constructed streamline map thus indicates how the direction of flow changes, and also reveals changes in speed.

E. Patterns of Irrotational Motions:

Fluid motion is characterized by an embarrassingly wide range of possible modes, and some restriction must be imposed if the analysis is to remain simple. If we consider the general motion of a fluid particle typified in two dimensions by, say, a square, the following changes are possible during a brief interval of time:

1) The element is translated, that is it's centre has moved a certain distance;
2) The element is rotated, that is, an axis of symmetry, such as diagonal, has turned about the centre;
3) The element has undergone strain, that is, the length of sides have changed, the area remaining the same.

The simplest change of motion is one in which the elements undergo translation and perhaps distortion (strain) but do not rotate. A motion of this type is called irrotational. This definition does not affect in any way the ability of fluid particles to follow curved paths, and it is thus quite logical to speak of a finite mass of fluid rotating irrotationaly around a body. With the restriction to steady irrotational motion of an incompressible inviscid fluid, the two dimensional kinematics of fluid takes a very simple form. In the first place, it can be proved that in these circumstances there always exists a function $\phi(x,y)$, called the velocity potential, such that if u and v are the components of velocity in the x and v direction, respectively

u= , v=

Conversely, if a velocity potential exists, the motion must be irrotational. So far we have dealt entirely with real variables. The complex variable

$$z = x+iy$$

may also be used to specify position in the (x,y) plane. Among the possible functions of z there is an important class distinguished by the name holomorphic; these are single-valued functions which are well behaved in the sense that there are no discontinuities, such as infinities. Thus z, z^2 ,z^3, …….., sin z, e^x, or any finite combination of them, are holomorphic, the function 1/z, log z, are holomorphic except at z = 0, where they become infinite.

II. References

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