Coupling of the Magneto Plasmons-Phonons in a Polar Semiconducting Cylindrical Geometries

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Abstract— The Surface Polariton waves are electromagnetic waves that remains localized within a thin surface layer, or bound along the interface of two media. Thus these waves can be used exclusively for the study of surfaces and interfaces, which is of great scientific and practical importance, not only in the field of Physics, but also in the fields of Chemistry and Bio-Chemistry. Important physical phenomena, like surface enhanced Raman Effect, fractional quantum Hall Effect etc., applied fields like micro-electronics and in integrated optics are all directly related to the study of surface of solids. As surface Plasmon and surface optical Phonon may interact with each other in polar semiconductors if their frequencies are of the same order. The frequency behavior on the surface of condensed material can be studied with the help of dispersion relation which can be obtained by various methods. This work focuses on the coupling of surface modes in presence of magnetic field.

Key words: Coupling, Surface Modes, IR, Plasmons

I. INTRODUCTION

Surface phonon polaritons SPs are electromagnetic surface modes formed by the strong coupling of light and optical phonons in polar crystals, and are generally excited using infrared IR or terahertz THz radiation [1]. Generation and control of surface phonon polaritons are essential for realizing novel applications in microscopy [2, 3] data storage [4] thermal emission [2,5] or in the field of meta-materials [6]. Materials supporting SPs such as SiC, quartz, or GaN offer strong temperature stability, anisotropic properties, and also the ability to strongly couple phonon and plasmon polaritons by doping [7,8] which could be exploited in photonic applications. Controlling, guiding, and focusing surface polaritons by optical elements has been recently shown with surface plasmon polaritons at visible and near-infrared wavelengths [9–17] but comparatively little attention has been given to surface phonon polaritons despite certain advantages including their ability to be generated in a wide spectral range, from IR to terahertz wavelengths typically between 8 and 200 m at the surface of a large variety of semiconductors, insulators, and ferroelectrics [18, 19]. S-SNOM with pseudo heterodyne detection [20] to map spatially the surface polaritons with sub wavelength resolution in amplitude and phase, focusing of SPs could be essential for applications that require localized IR or terahertz fields including photo detectors and emitters.

Polariton propagation on a material's surface requires a negative dielectric constant, which may result from collective conduction electron oscillations plasmon polaritons [11, 14, 21, 22] or from lattice vibrations in polar crystals phonon polaritons.[1,23,24]. For SPs, the electric field *Ep* is strongly coupled to lattice vibrations and is confined to surface propagation with the electromagnetic energy density falling off evanescently with distance from the surface.

II. THEORETICAL INVESTIGATION

Let us consider a semi conducting cylinder of radius 'R' having frequency dependent dielectric function $\varepsilon_1 = \varepsilon_1(\omega)$, and bounded by medium of dielectric constant ε_2.

In the presence of D.C. magnetic field the dielectric function $\varepsilon_1(\omega)$ of polar semiconducting medium, no longer remains a scalar quantity but it becomes a tensor ε_{ij} with non-zero off diagonal elements and is given by

$$\varepsilon_{ij} = \varepsilon_L \delta_{ij} - \bar{\varepsilon} \frac{\omega_p^2}{\omega^2\left(\omega^2 - \omega_c^2\right)}\left[\omega^2 \delta_{ij} - \omega_{ci}.\omega_{cj} + i\delta_{\omega k}.\delta_{ck}\right] \quad (1)$$

Where

$$\omega_p^2 = \frac{4\pi n_0 e^2}{\bar{\varepsilon} m^\phi} \quad (2)$$

And

$$\omega_c = \frac{e\bar{B}}{m^\phi} \quad (3)$$

Are the plasma and cyclotron frequencies respectively. m^ϕ is the effective mass of electrons in solids. ε_L is background dielectric constant of the polar semiconducting medium, δ_{ij} is Kronecker's delta function and δ_{ijk} is the third rank anti-symmetric tensor defined by:

$\delta_{ijk} = 1$ if i, j, k is an even permutation.

$\delta_{ijk} = -1$ if i, j, k is an odd permutation.

$\delta_{ijk} = 0$ otherwise (4)

As the magnetic field has been applied in Z-direction we have

$$\omega_{cz} = \omega_c \text{ and } \omega_{cx} = \omega_{cy} = 0 \quad (5)$$

Using eqn. (4) and (5) the components of the dielectric tensor ε_{ij} with the help of equation (1) may be written as-

$$\varepsilon_{xx} = \varepsilon_{yy} = \varepsilon_L - \bar{\varepsilon}\frac{\omega_p^2}{\left(\omega^2 - \omega_c^2\right)}$$

$$\varepsilon_{zz} = \varepsilon_L - \varepsilon\frac{-\omega_p^2}{\omega^2}$$

$$\varepsilon_{xz} = \varepsilon_{zx} = \varepsilon_{yz} = \varepsilon_{zy} = 0$$

Thus we see that if the field is oriented in Z-direction the component ε_{zz} is independent of the field, the component of ε_{xy} and ε_{yx} are the imaginary and the components $\varepsilon_{xz}, \varepsilon_{zx}, \varepsilon_{yz}, \varepsilon_{zy}$ are zero. There are only two equal, non-zero and real components ε_{xx} and ε_{xy}. Hence, the dielectric function of the semiconducting medium ε_1 can be described by the component of ε_{xx} or ε_{yy} of the dielectric tensor ε_{ij}

i.e. $\varepsilon_1(\omega) = \varepsilon_{xx} = \varepsilon_{yy}$

$$\varepsilon_1(\omega) = \varepsilon_L - \bar{\varepsilon}\left(\frac{\omega_p^2}{\omega^2 - \omega_c^2}\right) \quad (6)$$

Where ε_1 is the background dielectric constant of the polar semiconducting medium and is given by equation as:

$$\varepsilon_L = \frac{\varepsilon_\infty \omega^2 - \varepsilon_\infty \omega_t^2}{\omega^2 - \omega_t^2} \quad (7)$$

If the bounding medium is vacuum i.e. $\varepsilon_2 = 1$, then the Dispersion relation for two mode coupling for cylindrical surface is given by as

$$\left[(\varepsilon_2 - \bar{\varepsilon})\omega_P + (\varepsilon_1 + \varepsilon_2)\omega^2 - \beta^2 k^2(\varepsilon_1 + \varepsilon_2)\right] I_{l+1}(pR) \quad (8)$$

$$= \left[(\bar{\varepsilon} + \varepsilon_2)\omega_P^2 - (\varepsilon_1 + \varepsilon_2)\omega^2 + \beta^2 k^2(\varepsilon_1 + \varepsilon_2)\right] I_{l-1}(pR)$$

$$\left[(\varepsilon_2 - \bar{\varepsilon})\omega_P^2 + \varepsilon_1\omega^2 + \varepsilon_2\omega^2 - \beta^2 k^2 \varepsilon_1 - \beta^2 k^2 \varepsilon_2\right] I_{l+1}(pR)$$

$$= \left[(\bar{\varepsilon} + \varepsilon_2)\omega_P^2 - \varepsilon_1\omega^2 - \varepsilon_2\omega^2 + \beta^2 k^2 \varepsilon_1 + \varepsilon_2 \beta^2 k^2\right] I_{l-1}(pR)$$

or

$$\left[(\varepsilon_2 - \bar{\varepsilon})\omega_P^2 + (\varepsilon_2\omega^2 - \beta^2 k^2 \varepsilon_2)\right] I_{l+1}(pR) + \varepsilon_1\left[\omega^2 - \beta^2 k^2\right] I_{l+1}(pR)$$

$$= \left[(\bar{\varepsilon} + \varepsilon_2)\omega_P^2 - \varepsilon_2\omega^2 + \beta^2 k^2 \varepsilon_2\right] I_{l-1}(pR) - (\omega^2 - \beta^2 k^2)\varepsilon_1 I_{l-1}(pR)$$

After simplifying we get

$$\varepsilon_1\left[I_{l+1}(pR)(\omega^2 - \beta^2 k^2) + I_{l+1}(pR)(\omega^2 - \beta^2 k^2)\right] \quad (9)$$

$$= \left[(\bar{\varepsilon} + \varepsilon_2)\omega_P^2 - \varepsilon_2\omega^2 + \beta^2 k^2 \varepsilon_2\right] I_{l-1}(pR) - \left[(\varepsilon_2 - \bar{\varepsilon}) + \varepsilon_2\omega^2 - \beta^2 k^2 \varepsilon_2\right]$$

Now we substitute the value of ε_1 and ε_L from equation (6 & 7) in eqn. (9)

$$\left(\varepsilon_L - \bar{\varepsilon}\left(\frac{\omega_P^2}{\omega^2 - \omega_c^2}\right)\right)\left[I_{l+1}(pR)(\omega^2 - \beta^2 k^2) + I_{l+1}(pR)(\omega^2 - \beta^2 k^2)\right]$$

$$= \left[(\bar{\varepsilon} + \varepsilon_2)\omega_P^2 - \varepsilon_2\omega^2 + \beta^2 k^2 \varepsilon_2\right] I_{l-1}(pR) - \left[(\varepsilon_2 - \bar{\varepsilon}) + \varepsilon_2\omega^2 - \beta^2 k^2 \varepsilon_2\right] I_{l+1}(pR)$$

Now substituting the values of ε_L in above eqn.

$$\left(\left(\frac{\varepsilon_\infty\omega^2 - \varepsilon_0\omega_t^2}{\omega^2 - \omega_t^2}\right) - \bar{\varepsilon}\left(\frac{\omega_P^2}{\omega^2 - \omega_c^2}\right)\right)\left[I_{l+1}(pR)(\omega^2 - \beta^2 k^2) + I_{l+1}(pR)(\omega^2 - \beta^2 k^2)\right]$$

$$= \left[(\bar{\varepsilon} + \varepsilon_2)\omega_P^2 - \varepsilon_2\omega^2 + \beta^2 k^2 \varepsilon_2\right] I_{l-1}(pR) - \left[(\varepsilon_2 - \bar{\varepsilon}) + \varepsilon_2\omega^2 - \beta^2 k^2 \varepsilon_2\right] I_{l+1}(pR)$$

$$\left(\left(\frac{\varepsilon_\infty(\omega^2/\omega_t^2) - \varepsilon_0}{(\omega^2/\omega_t^2) - 1}\right) - \bar{\varepsilon}\left(\frac{(\omega_p^2/\omega_t^2)}{(\omega^2/\omega_t^2) - (\omega_c^2/\omega_t^2)}\right)\right)\left[I_{l+1}(pR)\left(\frac{\omega^2}{\omega_t^2} - \frac{\beta^2 k^2}{\omega_t^2}\right) + I_{l+1}(pR)\left(\frac{\omega^2}{\omega_t^2} - \frac{\beta^2 k^2}{\omega_t^2}\right)\right]$$

$$= \left[(\bar{\varepsilon} + \varepsilon_2)\frac{\omega^2}{\omega_t^2} - \varepsilon_2\frac{\omega^2}{\omega_t^2} + \frac{\beta^2 k^2 \varepsilon_2}{\omega_t^2}\right] I_{l-1}(pR) - \left[\frac{(\varepsilon_2 - \bar{\varepsilon})}{\omega_t^2} + \frac{\varepsilon_2\omega^2}{\omega_t^2} - \frac{\beta^2 k^2 \varepsilon_2}{\omega_t^2}\right] I_{l+1}(pR)$$

(10)

Let X= $(\omega_c/\omega_t)^2$, Y= $(\omega/\omega_t)^2$ and Z= $(\omega_p/\omega_t)^2$

$$\left(\left(\frac{\varepsilon_\infty Y - \varepsilon_0}{Y - 1}\right) - \bar{\varepsilon}\left(\frac{Z}{Y - X}\right)\right)\left[I_{l+1}(pR)\left(Y - \frac{\beta^2 k^2}{\omega_t^2}\right) + I_{l-1}(pR)\left(Y - \frac{\beta^2 k^2}{\omega_t^2}\right)\right]$$

$$= \left[(\bar{\varepsilon} + \varepsilon_2)Y - \varepsilon_2 Y + \frac{\beta^2 k^2 \varepsilon_2}{\omega_t^2}\right] I_{l-1}(pR) - \left[\frac{(\varepsilon_2 - \bar{\varepsilon})}{\omega_t^2} + \varepsilon_2 Y - \frac{\beta^2 k^2 \varepsilon_2}{\omega_t^2}\right] I_{l+1}(pR)$$

$$\left[\frac{(\varepsilon_\infty Y - \varepsilon_0)(Y - 1) - \bar{\varepsilon}Z(Y - 1)}{(Y - 1)(Y - X)}\right]\left[I_{l+1}(pR)\left(Y - \frac{\beta^2 k^2}{\omega_t^2}\right) + I_{l-1}(pR)\left(Y - \frac{\beta^2 k^2}{\omega_t^2}\right)\right]$$

$$= \left[\bar{\varepsilon}Y + \frac{\beta^2 k^2 \varepsilon_2}{\omega_t^2}\right] I_{l-1}(pR) - \left[\frac{(\varepsilon_2 - \bar{\varepsilon})}{\omega_t^2} + \varepsilon_2 Y - \frac{\beta^2 k^2 \varepsilon_2}{\omega_t^2}\right] I_{l+1}(pR)$$

$$(\varepsilon_\infty Y^2 - \varepsilon_\infty YX - \varepsilon_0 Y + \varepsilon_0 X - \bar{\varepsilon}ZY + \bar{\varepsilon}Z)\left[I_{l+1}(pR)\left(Y - \frac{\beta^2 k^2}{\omega_t^2}\right) + I_{l-1}(pR)\left(Y - \frac{\beta^2 k^2}{\omega_t^2}\right)\right]$$

$$= \left[\bar{\varepsilon}Y + \frac{\beta^2 k^2 \varepsilon_2}{\omega_t^2}\right] I_{l-1}(pR) - \left[\frac{(\varepsilon_2 - \bar{\varepsilon})}{\omega_t^2} + \varepsilon_2 Y - \frac{\beta^2 k^2 \varepsilon_2}{\omega_t^2}\right](Y - 1)(Y - X) I_{l+1}(pR)$$

or

$$\left(\varepsilon_\infty Y^3 - \varepsilon_\infty Y^2 X - \varepsilon_0 Y^2 + \varepsilon_0 XY - \bar{\varepsilon}ZY^2 + \bar{\varepsilon}ZY - \varepsilon_\infty Y^2\frac{\beta^2 k^2}{\omega_t^2} + \varepsilon_\infty YX\frac{\beta^2 k^2}{\omega_t^2} + \varepsilon_0 Y\frac{\beta^2 k^2}{\omega_t^2} - \varepsilon_0 X\frac{\beta^2 k^2}{\omega_t^2} + \bar{\varepsilon}ZY\frac{\beta^2 k^2}{\omega_t^2} - \bar{\varepsilon}Z\frac{\beta^2 k^2}{\omega_t^2}\right) I_{l+1}(pR)$$

$$+\left(\varepsilon_\infty Y^3 - \varepsilon_\infty Y^2 X - \varepsilon_0 Y^2 + \varepsilon_0 XY - \bar{\varepsilon}ZY^2 + \bar{\varepsilon}ZY - \varepsilon_\infty Y^2\frac{\beta^2 k^2}{\omega_t^2} + \varepsilon_\infty YX\frac{\beta^2 k^2}{\omega_t^2} + \varepsilon_0 Y\frac{\beta^2 k^2}{\omega_t^2} - \varepsilon_0 X\frac{\beta^2 k^2}{\omega_t^2} + \bar{\varepsilon}ZY\frac{\beta^2 k^2}{\omega_t^2} - \bar{\varepsilon}Z\frac{\beta^2 k^2}{\omega_t^2}\right) I_{l-1}(pR)$$

$$= \left[\bar{\varepsilon}Y + \frac{\beta^2 k^2 \varepsilon_2}{\omega_t^2}\right] I_{l-1}(pR) - \left[\frac{(\varepsilon_2 - \bar{\varepsilon})}{\omega_t^2} + \varepsilon_2 Y - \frac{\beta^2 k^2 \varepsilon_2}{\omega_t^2}\right](Y^2 - YX - Y + X) I_{l+1}(pR) \quad (11)$$

On Simplifying we get

$$Y^3\left\{(\varepsilon_\infty - \bar{\varepsilon}) I_{l+1}(pR) + (\varepsilon_\infty + \varepsilon_2) I_{l-1}(pR)\right\}$$

$$+Y^2\left\{\begin{array}{l}\left(-\varepsilon_0 X - \varepsilon_\infty - \bar{\varepsilon}Z - \varepsilon_\infty\frac{\beta^2 k^2}{\omega_t^2} + \bar{\varepsilon}X + \bar{\varepsilon} - \frac{\beta^2 k^2 \varepsilon_2}{\omega_t^2}\right) I_{l+1}(pR) \\ +\left(-\varepsilon_\infty X - \varepsilon_0 - \bar{\varepsilon}Z + \varepsilon_\infty\frac{\beta^2 k^2}{\omega_t^2} + \frac{\varepsilon_2 - \bar{\varepsilon}}{\omega_t^2} - \varepsilon_2 X - \varepsilon_2 - \frac{\beta^2 k^2 \varepsilon_2}{\omega_t^2}\right) I_{l-1}(pR)\end{array}\right\}$$

$$+Y\left\{\begin{array}{l}\left(\varepsilon_0 X + \varepsilon_0 X + \varepsilon_\infty\frac{\beta^2 k^2 X}{\omega_t^2} + \varepsilon_0\frac{\beta^2 k^2}{\omega_t^2} + \bar{\varepsilon}Z\frac{\beta^2 k^2}{\omega_t^2} - \bar{\varepsilon}X + \varepsilon_2\frac{\beta^2 k^2 X}{\omega_t^2} + \varepsilon_2\frac{\beta^2 k^2}{\omega_t^2}\right) I_{l+1}(pR) \\ +\left(\varepsilon_0 X + \varepsilon_0 X - \varepsilon_\infty\frac{\beta^2 k^2 X}{\omega_t^2} - \varepsilon_0\frac{\beta^2 k^2}{\omega_t^2} - \bar{\varepsilon}Z\frac{\beta^2 k^2}{\omega_t^2} - \frac{(\varepsilon_2 - \bar{\varepsilon})X}{\omega_t^2} - \frac{(\varepsilon_2 - \bar{\varepsilon})}{\omega_t^2} + \varepsilon_2 X + \frac{\beta^2 k^2 \varepsilon_2 X}{\omega_t^2} + \frac{\beta^2 k^2 \varepsilon_2}{\omega_t^2}\right) I_{l-1}(pR)\end{array}\right\}$$

$$+\left\{\left(-\varepsilon_0 X\frac{\beta^2 k^2}{\omega_t^2} - \bar{\varepsilon}Z\frac{\beta^2 k^2}{\omega_t^2} - \frac{\beta^2 k^2 X}{\omega_t^2}\right) I_{l+1} + \left(\frac{\varepsilon_0 X\beta^2 k^2}{\omega_t^2} - \frac{\bar{\varepsilon}Z\beta^2 k^2}{\omega_t^2} - \frac{\beta^2 k^2 \varepsilon_2 X}{\omega_t^2}\right) I_{l-1}\right\}$$

Or

$$Y^3\left[(\varepsilon_\infty - \bar{\varepsilon}) I_{l+1}(pR) + (\varepsilon_\infty + \varepsilon_2) I_{l-1}(pR)\right]$$

$$+Y^2\left[\begin{array}{l}\left\{(\bar{\varepsilon} - \varepsilon_0)X + \bar{\varepsilon}Z + \varepsilon_\infty + \bar{\varepsilon} - (\varepsilon_\infty + \varepsilon_2)\frac{\beta^2 k^2}{\omega_t^2}\right\} I_{l+1}(pR) \\ \left\{(-\varepsilon_\infty - \varepsilon_2)X - \bar{\varepsilon}Z - \varepsilon_0 - \varepsilon_2 + (\varepsilon_\infty - \varepsilon_2)\frac{\beta^2 k^2}{\omega_t^2} + \frac{\varepsilon_2 - \bar{\varepsilon}}{\omega_t^2}\right\} I_{l-1}(pR)\end{array}\right]$$

$$+Y\left[\begin{array}{l}\left\{(\varepsilon_0 - \bar{\varepsilon}) + \left(\bar{\varepsilon} + \bar{\varepsilon}\frac{\beta^2 k^2}{\omega_t^2}\right)Z + (\varepsilon_\infty + \varepsilon_2)\frac{\beta^2 k^2 X}{\omega_t^2} + \frac{(\varepsilon_0 + \varepsilon_2)\beta^2 k^2}{\omega_t^2}\right\} I_{l+1}(pR) \\ +\left\{(\varepsilon_0 - \varepsilon_2)X + \left(\bar{\varepsilon} - \bar{\varepsilon}\frac{\beta^2 k^2}{\omega_t^2}\right)Z + (\varepsilon_2 - \varepsilon_\infty)\frac{\beta^2 k^2 X}{\omega_t^2} + \frac{(\varepsilon_2 - \varepsilon_0)\beta^2 k^2}{\omega_t^2} - (X + 1)\frac{(\varepsilon_2 - \bar{\varepsilon})}{\omega_t^2}\right\} I_{l-1}(pR)\end{array}\right]$$

$$+\left[\begin{array}{l}\left\{(-\varepsilon_0 - 1)\frac{\beta^2 k^2 X}{\omega_t^2} - \bar{\varepsilon}\frac{\beta^2 k^2 Z}{\omega_t^2}\right\} I_{l+1}(pR) \\ +\left\{(\varepsilon_0 - \varepsilon_2)\frac{\beta^2 k^2 X}{\omega_t^2} + \bar{\varepsilon}\frac{\beta^2 k^2 Z}{\omega_t^2}\right\} I_{l-1}(pR)\end{array}\right] \quad (12)$$

Eqn. (12) is the dispersion relation for the coupling of the magneto plasmons-phonons in a polar semiconducting cylindrical. This equation is the cubic in 'Y' and hence give three roots after Y for each mode 'l' and for the given values of a,b,c,x and z the quantity x determine the strength of the applied magnetic field. If the magnetic field is reduced to zero i.e. (ω_c/ω_t)=0 then eqn. (12) become as eqn.

$$\phi_{li}(r) = \mathrm{B}r^l + \frac{\mathrm{A}_1}{p^2}\mathrm{I}_l(pr) \qquad r<R$$

Thus eqn. (12) is correct. Hence it is concluded that the application of magnetic-field leads to the appearance of the three coupled modes instead of two which appear in the absence of magnetic field.

Using the same data, calculation of the frequencies of all the three coupled modes the three frequencies of all the three coupled modes for InSb cylinder of radius R=100Å (for l=1 mode) has been done.. The frequencies of the three coupled mode have been listed in the table (1)-(3)

InSb ϵ_0=17.7 ϵ_∞=15.6 $\bar{\varepsilon} = 16.65$ X=400 $\omega_t = 3.5\times10^{13}$ $\beta = 1.62\times10^{14}$ $\varepsilon_2 = 1$ Z=19.89

k	Y1	Y2	Y3
0	0	20.7461	1.0858

1	1.057509	20.767	5.645
2	1.058211	20.9862	11.7488
3	1.058359	21.8613	17.4534
4	1.058416	26.1701	19.812

Table 1: Value

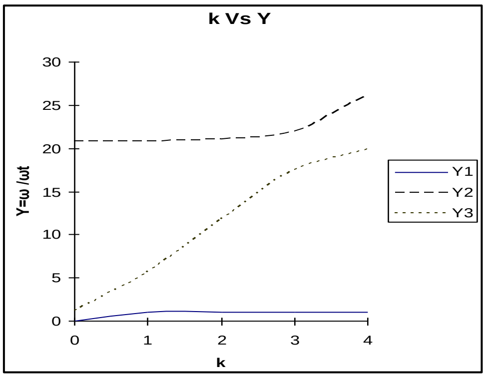

Fig. 1: Graph

Fig (1) shows that the dispersion curves for the coupled SP-SOP modes. From the dispersion curves we observe that the frequency of the upper and lower modes vary slowly with wave vector k. till k=3Å^{-1}. The frequency of middle modes varies sharply with wave vector k and becomes constant after the wave vector k= 3Å^{-1} and is nearly equal to the frequency of the pure SOP mode.

Further it is observed that the frequency of upper mode increase after the value of k=3Å^{-1}. Thus it is seen that the dispersion curve consist of a band gap lying between low and upper mode and another band gap lying between upper modes and lower modes. In the region of these two band gap, the coupling is possible only for k=3Å^{-1}.

Z	Y1	Y2	Y3
0	1.061015	20.00075	8.816921
1	1.060867	20.04483911	8.810306
2	1.06043	20.176285900	8.785639
3	1.059715	20.392743110	8.746094
4	1.058741	20.690574000	8.693768
5	1.057535	21.06520200	8.631177
6	1.056126	21.571470500	8.560972

Table 2: Value

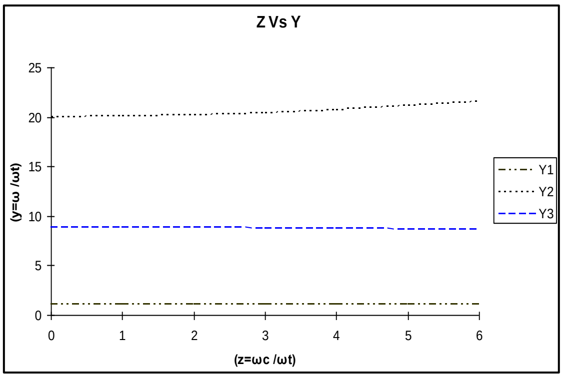

Fig. 2: Graph

III. Results

Plotted curves for all the two coupled SP-SOP modes is shown and also the uncoupled SP-SOP modes as a function of (ω_p/ω_t) as shown in fig (2). It is being observed that at higher concentrations the lower mode becomes like pure SOP mode. The lower and upper modes are almost constant with respect to plasmon frequency and due to D.C. magnetic field third mode (middle) created between them and which is almost constant. The coupling between them is not possible as the variation of plasmon frequencies because the polariton frequency do not depend upon plasmon frequency. Further, at very high concentration the middle and upper modes tend to attain the character of pure SP and SOP modes.

x	Y1	Y2	Y3
0	4.232203	9.652094	0.138922
10	5.029675	9.769925	0.613116
20	5.684763	9.913535	0.749326
30	6.225629	10.08736	0.821046
40	6.670134	10.29458	0.865787
50	7.031375	10.53594	0.896396
60	7.321232	10.8091	0.918683

Table 3: Value

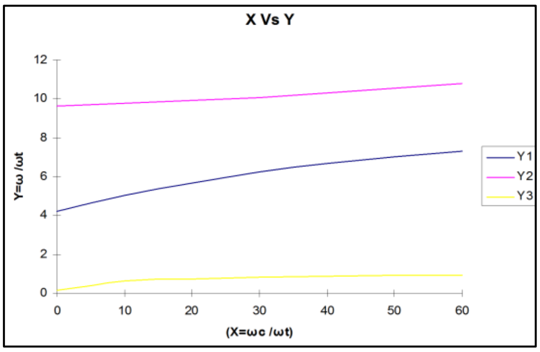

Fig. 3: Graph

IV. Conclusion

Fig (3) shows the variation of the frequencies of coupled modes with the strength of applied magnetic field from this figure, observations are as under:

1) The frequency of the lower mode increases upto (frequency of magneto plasmon $3.5\times10^{14}\sec^{-1}$ (ω_c)) remain almost constant at high fields and is very close to the pure phonon frequency.
2) The frequency of the middle mode, increases with the applied magnetic field upto $\omega_{c2}(1.4\times10^{15}\sec^{-1})$ but at higher field it becomes almost constant, i.e. the frequency of this mode reaches the saturation stage. The frequency of this mode at saturation matches very closely with the bulk plasma frequency.
3) The frequency of upper mode slightly increases and is proportional to the strength of applied field which is the frequency of magneto plasmon frequency.

The above conclusions are of great scientific and practical importance and may provide useful information for the study of the coupling between SP-SOP waves on the cylindrical surfaces for higher value of k.

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