# A Case Study on Applications of Trigonometry in Oceanography

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*Abstract*— This article introduces a trigonometric field that extends in the field of real numbers adding two elements: sin and cos satisfying an axiom $\sin^2+\cos^2=1$.It is shown that assigning meaningful names to particular elements to the field, All know trigonometric identities may be introduced and proved. The main objective of this study is about oceanography how the oceanographers guide everyone with the awareness of the Tsunami waves and protect the marine animals hitting the ships and cliffs.

***Key words:*** Trigonometric Identities, Trigonometric Ratios, Trigonometric Functions

## I. Introduction

### A. *Trigonometry*

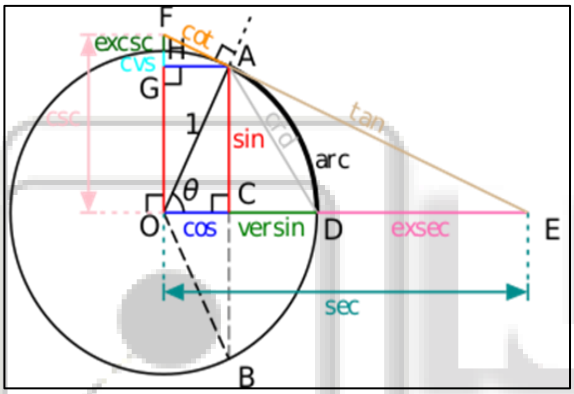



Fig. 1:

All of the trigonometric functions of an angle θ can be constructed geometrically in terms of a unit circle centered at O.

Trigonometry (from Greek trigonon, "triangle" and metron, "measure") is a branch of mathematics that studies relationships involving lengths and angles of triangles. The field emerged in the Hellenistic world during the 3rd century BC from applications of geometry to astronomical studies. The 3rd-century astronomers first noted that the lengths of the sides of a right-angle triangle and the angles between those sides have fixed relationships: that is, if at least the length of one side and the value of one angle is known, then all other angles and lengths can be determined algorithmically. These calculations soon came to be defined as the trigonometric functions and today are pervasive in both pure and applied mathematics: fundamental methods of analysis such as the Fourier transform, for example, or the wave equation, use trigonometric functions to understand cyclical phenomena across many applications in fields as diverse as physics, mechanical and electrical engineering, music and acoustics, astronomy, ecology, and biology. Trigonometry is also the foundation of surveying. Trigonometry is most simply associated with planar right-angle triangles (each of which is a two-dimensional triangle with one angle equal to 90 degrees). The applicability to non-right-angle triangles exists, but, since any non-right-angle triangle (on a flat plane) can be bisected to create two right-angle triangles, most problems can be reduced to calculations on right-angle triangles. Thus the majority of applications relate to right-angle triangles. One exception to this is spherical trigonometry, the study of triangles on spheres, surfaces of constant positive curvature, in elliptic geometry (a fundamental part of astronomy and navigation). Trigonometry on surfaces of negative curvature is part of hyperbolic geometry.

## II. History of Trigonometry

Fig. 2:

Hipparchus, credited with compiling the first trigonometric, is known as "the father of trigonometry".

A thick ring-like shell object found at the Indus Valley Civilization site of Lothal, with four slits each in two margins served as a compass to measure angles on plane surfaces or in the horizon in multiples of 40degrees,upto 360 degrees. Such shell instruments were probably invented to measure 8–12 whole sections of the horizon and sky, explaining the slits on the lower and upper margins. Archaeologists consider this as evidence that the Lothal experts had achieved an 8–12 fold division of horizon and sky, as well as an instrument for measuring angles and perhaps the position of stars, and for navigation. Sumerian astronomers studied angle measure, using a division of circles into 360 degrees. They, and later the Babylonians, studied the ratios of the sides of similar triangles and discovered some properties of these ratios but did not turn that into a systematic method for finding sides and angles of triangles. The ancient Nubians used the same method. In the 3rd century BC, Hellenistic mathematicians such as Euclid and Archimedes studied the properties of chords and inscribed angles in circles, and they proved theorems that are equivalent to modern trigonometric formulae, although they presented them geometrically rather than algebraically. In 140 BC, Hipparchus (from Nicaea, Asia Minor) gave the first tables of chords, analogous to modern tables of sine values, and used them to solve problems in trigonometry and spherical trigonometry. In the 2nd century AD, the Greco-Egyptian astronomer Ptolemy (from Alexandria, Egypt) printed detailed trigonometric tables (Ptolemy's table of chords, his

Almagest. Ptolemy used chordlength to define his trigonometric functions, a minor difference from the sine convention we use today. (The value we call sin(θ) can be found by looking up the chord length for twice the angle of interest (2θ) in Ptolemy's table, and then dividing that value by two.) Centuries passed before more detailed tables were produced, and Ptolemy's treatise remained in use for performing trigonometric calculations in astronomy throughout the next 1200 years in the medieval Byzantine, Islamic, and, later, Western European worlds.

The modern sine convention is first attested in the Surya Siddhanta, and its properties were further documented by the 5th century (AD) Indian mathematician and astronomer Aryabhata. These Greek and Indian works were translated and expanded by medieval Islamic mathematicians. By the 10th century, Islamic mathematicians were using all six trigonometric functions, had tabulated their values, and were applying them to problems in geometry. At about the same time, Chinese mathematicians developed trigonometry independently, although it was not a major field of study for them. Knowledge of trigonometric functions and methods reached Western Europe via Latin translations of Ptolemy's Greek Almagest as well as the works of Persian and Arabic astronomers such as Al Battani and Tusi. One of the earliest works on trigonometry by a northern European mathematician is De Triangulis by the 15th century German mathematician Regiomontanus, who was encouraged to write, and provided with a copy of the Almagest, by the Byzantine Greek scholar cardinal Basilios Bessarion with whom he lived for several years At the same time, another translation of the Almagest from Greek into Latin was completed by the Cretan Trebizond. Trigonometry was still so little known in 16th-century northern Europe that Nicolaus Copernicus devoted two chapters of De revolution bus orbiumcoelestium to explain its basic concepts Driven by the demands of navigation and the growing need for accurate maps of large geographic areas, trigonometry grew into a major branch of mathematics. Bartholomaeus was the first to use the word, publishing his Trigonometria in 1595.Gemma Frisius described for the first time the method of triangulation still used today in surveying. It was Leonhard Euler who fully incorporated complex numbers into trigonometry. The works of the Scottish mathematicians James Gregoryin the 17th century and Colin Maclaurin in the 18th century were influential in the development of series. Also in the 18th century, Brook Taylor defined the general Taylor series.

### A. *Trigonometric Functions*

If one angle of a triangle is 90 degrees and one of the other angles is known, the third is thereby fixed, because the three angles of any triangle add up to 180 degrees. The two acute angles therefore add up to 90 degrees: they are complementary. The shape of a triangle is completely determined, except for similarity, by the angles. Once the angles are known, the ratios of the sides are determined, regardless of the overall size of the triangle. If the length of one of the sides is known, the other two are determined. These ratios are given by the following trigonometric functions of the known angle A, where a, b and c refer to the lengths of the sides.

#### 1) *In this Right Triangle*

$$\text{Sin A} = \frac{a}{c};\ \cos A = \frac{b}{c};\ \tan A = \frac{a}{b}$$

– Sine function (sin), defined as the ratio of the side opposite the angle to the hypotenuse.

$$\text{SinA} = \frac{\text{Opposite}}{\text{Hypotenuse}} = \frac{a}{c}$$

– Cosine function (cos), defined as the ratio of the adjacent leg to the hypotenuse.

$$\text{CosA} = \frac{\text{Adjacent}}{\text{Hypotenuse}} = \frac{b}{c}$$

– Tangent function (tan), defined as the ratio of the opposite leg to the adjacent leg.

$$\text{TanA} = \frac{\text{Opposite}}{\text{Hypotenuse}} = \frac{a}{b} = \frac{a}{c} * \frac{c}{b} = \frac{a}{c} / \frac{b}{c} = \frac{\text{SinA}}{\text{cOSA}}$$

The hypotenuse is the side opposite to the 90 degree angle in a right triangle; it is the longest side of the triangle and one of the two sides adjacent to angle A. The adjacent leg is the other side that is adjacent to angle A. The opposite side is the side that is opposite to angle A. The terms perpendicular and base are sometimes used for the opposite and adjacent sides respectively. The reciprocals of these functions are named the cosecant (csc or cosec), secant (sec), and cotangent (cot), respectively

$$\text{CosecA} = \frac{1}{\text{SinA}} = \frac{\text{Hypotenuse}}{\text{opposite}} = \frac{c}{a}$$

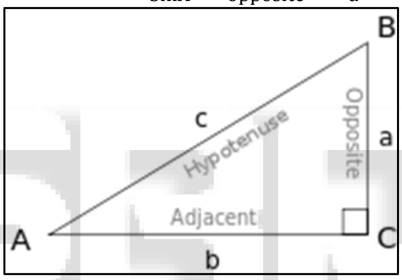



Fig. 3:

$$\text{SecA} = \frac{1}{\text{CosA}} = \frac{\text{Hypotenuse}}{\text{Adjacent}} = \frac{c}{b}$$

$$\text{CotA} = \frac{1}{\text{TanA}} = \frac{\text{Adjacent}}{\text{Opposite}} = \frac{\text{CosA}}{\text{SinA}} = \frac{b}{a}$$

The inverse functions are called the arcsine, arccosine, and arctangent, respectively. There are arithmetic relations between these functions, which are known as trigonometric identities. The cosine, cotangent, and cosecant are so named because they are respectively the sine, tangent, and secant of the complementary angle abbreviated to "co-".With these functions, one can answer virtually all questions about arbitrary triangles by using the law of sine’s and the law of cosines. These laws can be used to compute the remaining angles and sides of any triangle as soon as two sides and their included angle or two angles and a side or three sides are known. These laws are useful in all branches of geometry, since every polygon may be described as a finite combination of triangles.

## III. PYTHAGOREAN IDENTITIES

The following identities are related to the Pythagorean theorem and hold for any value.

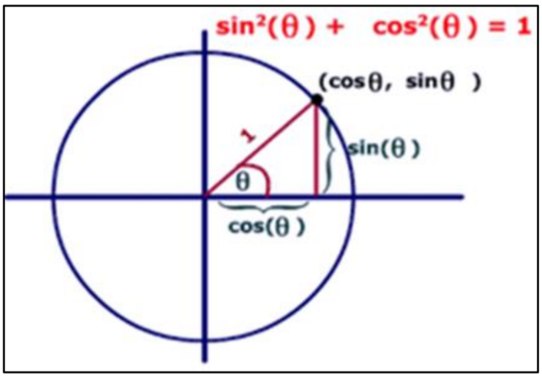



Fig. 4:
$\sin^2 A + \cos^2 B = 1$
$\tan^2 A + 1 = \sec^2 A$
$\cot^2 A + 1 = \mathrm{cosec}^2 A$

### A. *Trigonometric Ratio*

The trigonometric functions are functions of an angle. They relate the angles of a triangle to the lengths of its sides

| $\theta$ | 0° | 30° | 45° | 60° | 90° |
|---|---|---|---|---|---|
| $\sin\theta$ | 0 | $\frac{1}{2}$ | $\frac{1}{\sqrt{2}}$ | $\frac{\sqrt{3}}{2}$ | 1 |
| $\cos\theta$ | 1 | $\frac{\sqrt{3}}{2}$ | $\frac{1}{\sqrt{2}}$ | $\frac{1}{2}$ | 0 |
| $\tan\theta$ | 0 | $\frac{1}{\sqrt{3}}$ | 1 | $\sqrt{3}$ | Not defined |
| $\cot\theta$ | Not defined | $\sqrt{3}$ | 1 | $\frac{1}{\sqrt{3}}$ | 0 |
| $\sec\theta$ | 1 | $\frac{2}{\sqrt{3}}$ | $\sqrt{2}$ | 2 | Not defined |
| $\mathrm{cosec}\,\theta$ | Not defined | 2 | $\sqrt{2}$ | $\frac{2}{\sqrt{3}}$ | 1 |

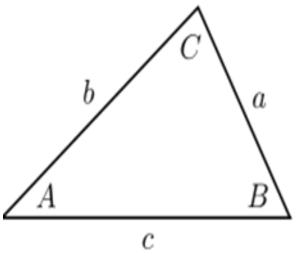
Table 1:

### B. *Angle Transformation Formula*

Sin(A±B) =sinAcosB ± cosAsinB
Cos(A±B)= cosAcosB ± sinAsinB
tan (A±B) =tanA ± tan/ 1±tanA tanB
cot(A±B) = cotAcotB ±1/ cotB ± cotA

### C. *Law of Sines*

Law of sines is a sine rule, it is an equation relating the lengths of the sides of a triangle to the sines of its angles.




$$\frac{a}{\mathrm{SinA}} = \frac{b}{\mathrm{SinB}} = \frac{C}{\mathrm{SinC}} = 2R = \frac{abc}{2\Delta}$$

Where Δ the area of the triangle and R is the radius of the circumscribed circle of the triangle.

$$R = \frac{abc}{\sqrt{(a+b+c)(a-b+c)(a+b-c)(b+c-a)}}$$

Another law involving sines can be used to calculate the area of a triangle. Given two sides a and b and the angle between the sides C, the area of the triangle is given by half the product of the lengths of two sides and the sine of the angle between the two sides.

$$\text{Area} = \Delta = \frac{1}{2} = ab\ \mathrm{SinC}$$

### D. *Law of Cosines*

The law of cosines (known as the cosine formula, or the "cos rule") is an extension of the Pythagorean Theorem to arbitrary triangles.

$$c^2 = a^2+b^2-2ab\mathrm{CosC}$$

Or Equivalently

$$\mathrm{CosC} = \frac{a^2+b^2-c^2}{2ab}$$

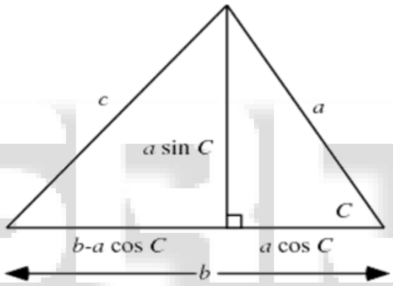
The law of cosines may be used to prove Heron's formula, which is another method that may be used to calculate the area of a triangle. This formula states that if a triangle has sides of lengths a, b, and c, and if the semi perimeter is

$$S = \tfrac{1}{2}(a+b+c)$$

Then the area of the triangle is

$$\text{Area} = \Delta = \sqrt{s(s-a)(s-b)(s-c)} = \frac{abc}{4R}$$

Where R is the radius of the circumcircle of the triangle.




## IV. APPLICATIONS IN TRIGONOMETRY

Fig. 5:

Sextants are used to measure the angle of the sun or stars with respect to the horizon. Using trigonometry and a marine chronometer, the position of the ship can be determined from such measurements.

There is an enormous number of uses of trigonometry and trigonometric functions. For instance, the technique of triangulation is used in astronomy to measure the distance to nearby stars, in geography to measure distances between landmarks, and in satellite navigation systems. The sine and cosine functions are fundamental to the theory of periodic, such as those that describe sound and light waves. Fields that use trigonometry or trigonometric functions include astronomy (especially for locating apparent positions of celestial objects, in which spherical trigonometry is essential) and hence navigation (on the oceans, in aircraft, and in space), music theory, audio synthesis, acoustics, optics, electronics, biology, medical imaging (CAT scans and ultrasound), pharmacy, chemistry, number theory (and hence cryptology),seismology, meteorology, oceanography, many physical sciences, land surveying and geodesy, architecture, image compression, phonetics, economics, electrical engineering, mechanical engineering, civil engineering, graphics, cartography, crystallography and game development.

### A. *Uses of Trigonometry in Astronomy*

Trigonometry is often used to find distances to nearby stars and other celestial objects using a method of parallax. Parallax can be defined as the apparent shift of a nearby star against the fixed background that can be noticed as the earth orbits the sun.

### B. *Uses of Trigonometry in Game Development*

Trigonometry is used extensively in game development in order for the game to function. Trigonometry is used in writing the program for games so that objects can move . Also used for designing object, characters and sets.

### C. *Uses of Trigonometry in Architecture*

Transcript of how trigonometry is used in architecture for example architects have to calculate exact angles of intersection for components of their structure to ensure stability and safety. Example of trigonometric use in architecture include arches, domes, support beams and suspension bridges.

### D. *Uses of Trigonometry in Music Theory*

Trigonometry plays a major role in musical theory and production. Sound waves travel in repeating wave pattern, which can be represented graphically by sine and cosine function. A single note can be modeled on sine curve, and chord can be modeled with multiple sine curves used in conjunction with one another.

### E. *Uses of Trigonometry in Navigation*

Trigonometry was developed for use in sailing as a navigation method used with astronomy. It is the branch of trigonometry concerned with the measurement of the angles and sides of spherical triangles. It is used for planning long distances routes around the world.

### F. *Uses of Trigonometry in Chemistry*

In chemistry, chemist use trigonometry when accurately describing the angles that are created when atoms bond together to form molecules geometry. Trigonometric functions such as sine cosines are essential to describe materials in their three dimensions.

### G. *Uses of Trigonometry in Crystallography*

Trigonometric functions, trigonometry comes up in many aspects of biology One example is x-ray crystallography, a technique used to determine the three dimensional structure of molecule etc.

## V. OCEANOGRAPHY

### A. *Introduction of Oceanography*

Oceanography also known as Oceanology is the study of the physical and biological aspects of the ocean. Oceanography covers a wide range of topics including marine life and ecosystems, ocean circulation, waves, plate tectonics and the geology of the sea floor, and the chemical and physical properties of the ocean. Oceanography is the application of all science to the phenomena of the ocean. To truly understand the ocean and how it works, one must know something about almost all fields of science and their relationship to the marine environment. Thus, oceanography is no single science but rather a combination of various sciences. The objective of oceanography, at least to the scientist, is to increase human understanding of all aspects of the world's ocean and of the processes. This is encompasses the subsidiary aim of describing as many marine features as possible of the many scientific disciplines that make up oceanography, one would be justified in calling the study of the oceans a very board science, and however in another sense oceanography is restrictive. It is not a universal science like physics or chemistry; where in the physical laws governing matter appear to have application throughout our universe.

### B. *Scope*

#### 1) *Chemical Oceanography*

Chemical reactions that occur both in the ocean and on the sea floor.

#### 2) *Biologcial Oceanography*

Biological oceanography deals with the distribution and environmental aspect of life in the ocean

#### 3) *Physical Oceanography*

Physical reactions such as changes and motion of the ocean.

#### 4) *Geological Oceanography*

Geological oceanography is used to study the sediments and topography of the ocean floor.

#### 5) *Ocean Engineering*

Ocean engineering concerns with the development of technology for oceanographic research and exploitation.

#### 6) *Marine Policy*

Marine policy considers the application of social and political sciences such as economics, laws and policy towards the use and management of the ocean.

#### 7) *Satellite Oceanography*

It deals with the measurements of the ocean color, which can be used assessments of phytoplankton biomass and are of great interest to marine biologists.

### C. *History of Oceanography*

Humans first acquired knowledge of the waves and currents of the seas and oceans in pre-historic times. Observations on tides were recorded by Aristotle and Strabo. Early exploration of the oceans was primarily for cartography and mainly limited to its surfaces and of the animals that fishermen brought up in nets, though depth soundings by lead line were taken. Although Juan Ponce de Leon in 1513 first identified the Gulf Stream, and the current was well known to mariners, Benjamin Franklin made the first scientific study of it and gave it its name. Franklin measured water temperatures during several Atlantic crossings and correctly explained the Gulf Stream's cause. Franklin and Timothy Folger printed the first map of the Gulf Stream in 1769-1770.


Fig. 6:

Map of the Gulf Stream by Benjamin Franklin, 1769-1770. Courtesy of the NOVAA photo library Information on the currents of the Pacific Ocean was gathered by explorers of the late 18th century including James Cook and Louis Antoine De Bougainville. James Rennell wrote the first scientific textbooks on oceanography, detailing the current flows of the Atlantic and Indian oceans. During a voyage around the Cape of Good Hope in 1777, he mapped "the banks and currents at the Lagullas". He was also the first to understand the nature of the intermittent current near the Isles of Scilly, (now known as Rennell"s Current).

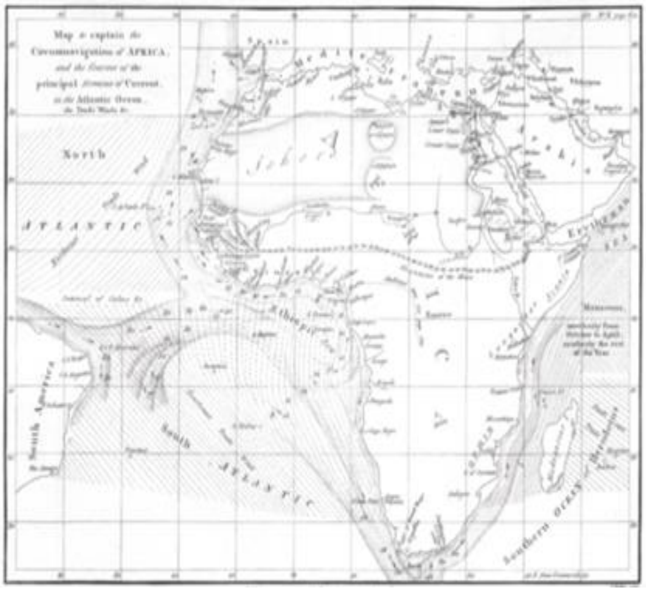



Fig. 7:

1799 Map of the currents in the Atlantic and Indian oceans, by James Rennell Sir James Clark Ross took the first modern sounding in deep sea in 1840, and Charles Darwin published a paper on reefs and the formation of atolls as a result of the second voyage of HMS Beagle in 1831-6. Robert FitzRoy Published a four-volume report of the Beagle's three voyages. In 1841-1842 Edward Forbes undertook dredging in the Aegean Sea that founded marine ecology. The first superintendent of the United States naval observatory (1842-1861), Matthew Fontaine Maury devoted his time to the study of marine meteorology, navigation, and charting prevailing winds and currents.

His 1855 textbook physical geography of the sea was one of the first comprehensive oceanography studies. Many nations sent oceanographic observations to Maury at the naval observatory, where he and his colleagues evaluated the information and distributed the results worldwide.

### *D. Modern Oceanography*

Human knowledge of the oceans remained confined to the topmost few fathoms of the water and a small amount of the bottom, mainly in shallow areas. Almost nothing was known of the ocean depths. The Royal Navy's efforts to chart all of the world's coastlines in the mid-19th century reinforced the vague idea that most of the ocean was very deep, although little more was known. As exploration ignited both popular and scientific interest in the polar regions and Africa, so too did the mysteries of the unexplored oceans. The seminal event in the founding of the modern science of oceanography was the 1872-76 Challenger expedition. As the first true oceanographic cruise, this expedition laid the groundwork for an entire academic and research discipline. In response to a recommendation from the Royal Society, Government announced in 1871 an expedition to explore world's oceans and conduct appropriate scientific investigation. Charles Wyville Thompson and Sir John Murray launched the Challenger expedition. The Challenger, leased from the Royal Navy, was modified for scientific work and equipped with separate laboratories for natural history and chemistry. Under the scientific supervision of Thomson, Challenger travelled nearly 70,000 nautical miles (130,000 km) surveying and exploring. On her journey circumnavigating the globe, 492 deep sea soundings, 133 bottom dredges, 151 open water trawls and 263 serial water temperature observations were taken. Around 4,700 new species of marine life were discovered. The result was the Report of the Scientific Results of the Exploring Voyage of H.M.S. Challenger during the years 1873-76. Murray, who supervised the publication, described the report as "the greatest advance in the knowledge of our planet since the celebrated discoveries of the fifteenth and sixteenth centuries". He went on to found the academic discipline of oceanography at the University of Edinburgh, which remained the centre for oceanographic research well into the 20th century. Murray was the first to study marine trenches and in particular the Mid-Atlantic Ridge, and map the sedimentary deposits in the oceans. He tried to map out the world's ocean currents based on salinity and temperature observations, and was the first to correctly understand the nature of coral reef development.

### *E. Ocean Waves*

#### *1) Wave Guides*

A Wave Guide is a structure that guides waves, such as electromagnetic waves or sound, with minimal loss of energy. There are different types of wave guides for each type of wave. The original and most common meaning is a hollow conductive metal pipe used to carry high frequency Radio Waves, particularly Micro Waves.

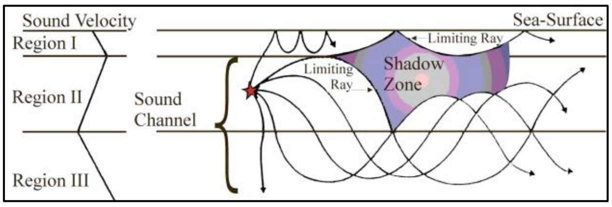



Fig. 8:

*2) Wave Interference*

Wave Interference is the phenomenon that occurs when in phase; the two lower waves create constructive interference (left), resulting in a wave of greater amplitude. When 180° out of phase, they create destructive interference (right).

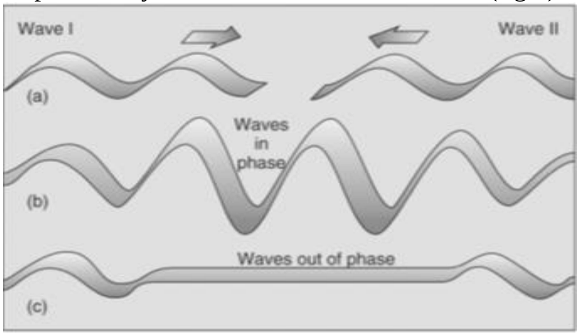



Fig. 9:

*3) Internal Waves*

Waves that occur at the boundaries of water layers with different densities are called internal waves.

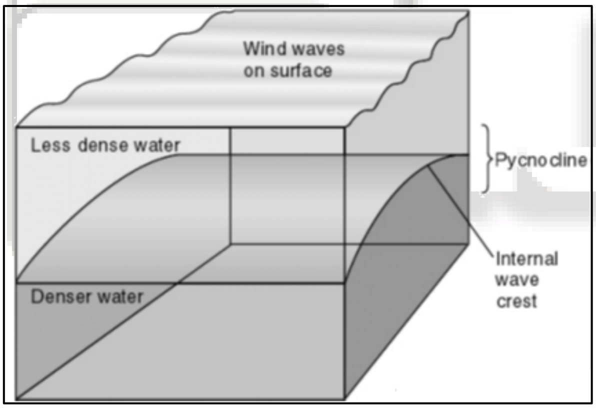



Fig. 10:

- Waves transmit energy, not water mass, across the ocean's surface.
- The behavior of a wave depends on the relation between the wave's size and the depth of water through which it is moving.
- Wind waves from when energy is transferred from wind to wave.
- Waves can change direction by refraction and diffraction, can interfere with one another, and reflect from solid objects.
- Wave speed = wave length/wave period

### *F. Uses of Trigonometry in Oceanography*

A Oceanographer uses trigonometry to measure distance. For instance if the oceanographer was tracking a sea creature and they needed to know how far they were from the animal they would use trigonometry to find out the distance from them and the animal. Oceanographer also use trigonometry to calculate the height of tides in oceans.

*1) Problem*

A Man on the deck of a ship, 14 km above the water level, observes that the angle of elevation of the top of a cliff is 60 degree and the angle of depression of the base of the cliff is 30 degree. Find the height of the cliff.

*2) Solution*

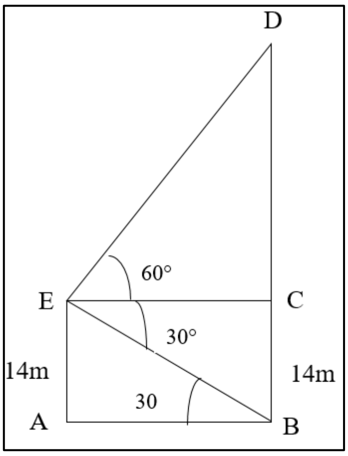



Fig. 11:

Let BD be the height of the cliff

Let A be the position of the ship and E be the point of observation so that AE = 14 m. Draw EC parallel to AB such that AB=EC.

Given that △ ABE =30°

△ DEC =60°

In the right angled ABE,

Tan 30° = $\frac{\mathbf{AE}}{\mathbf{AB}}$

AB = $\frac{\mathbf{AE}}{\mathbf{Tan\ 30°}}$

AB =14√3

Thus, EC =14√3 (AB=EC)

In right angled DEC,

Tan 60 ° = $\frac{\mathbf{CD}}{\mathbf{EC}}$

CD =EC Tan60°

CD = $(14\sqrt{\mathbf{3}})\sqrt{3}$ = 42 m

Thus, the height of the cliff,

BD = BC+CD = 14+42 = 56 m

## VI. Result

1) The height of the cliff is 56 m.
2) Suppose that you are on the dock at change point. At 1 pm on January 4, the tide is in (the water is at its deepest). At that time you find that the depth of the water at the end of the pier is 6 meters. At 8 pm the same day when the tide is out, you find that the depth of the water is 2 meters. Assume the depth of the water varies sinusoid ally with time.

a) Derive an equation expressing depth of the water in terms of the number of hours that have elapsed since 12noon on January 4.

b) use your mathematical model to predict the depth of the water at

1.3pm on January 4

2.7am on January 5

### A. *Solution*

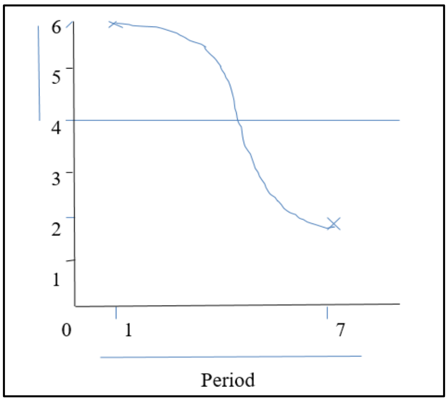



Fig. 12:

The graph is of cosine form so,

Y=Acos (BX+C)+D

Here, Period= $2\pi/B$

Period from 1 pm to 7pm is 6

$6= 2\pi/B$

$6*B=2\pi$

$B= \pi/3$

A=Amplitude

A= 2

C= the time water is at the deepest

C=1

D=2+6/2 = 4

Time elapsed since 12noon so the equation is,

$Y= 2\cos(\frac{\pi}{3}(t-1))+4$

1) 3pm on January 4
   $Y=2\cos(\frac{\pi}{3}(3-1))+4$
   $Y=2\cos(\frac{\pi}{3})+4$
   Y=2(0.49965) +4
   Y=0.9993+4
   Y=4.9993
   Y= 5 meters
2) 7am on January 5
   $Y=2\cos(\frac{\pi}{3}(20-1))+4$
   $Y=2\cos(\frac{18\pi}{3})+4$
   Y=2(0.9998) + 4
   Y=1.9998+4
   Y= 5.9998
   Y=6 meters.

### B. *Result*

1) The depth of water at 3am on January 4 is 5meters.
2) The depth of water at 7pm on January 5 is 6meters.

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