Odd – Order Nonlinear Neutral Functional Differential Equations of H^*

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Abstract— In this present paper we exhibit some less amount of similar theorems and the for the odd – order nonlinear neutral functional differential equation of H^* oscillation.

$[x(t)+p(t)x(T(t))]^{(n)} + l(t)x(\mu(t)) + m(t)x(\tau(t)) + n(t)x(\psi(t)) = 0 \; (H^*)$

Where $0 \le p(t) \le p_o < \infty$.

Key words: Odd- Order Neutral Differential Equation of H^*, First Order Differential Inequality, Odd Nonlinear Neutral Functional Differential Equation of H^* Oscillation

I. Introduction

This article is primarily deals with the asymptotic and odd nonlinear neutral functional differential equation of H^* oscillation

$$[x(t)+p(t)x(T(t))]^{(n)} + l(t)x(\mu(t)) + m(t)x(\tau(t)) + n(t)x(\varphi(t)) = 0$$

Where $n \ge 3$ is an odd integer $p(t), l(t), m(t), n(t), T(t), \mu(t), \tau(t), \psi(t) \in C([t_0,\infty))$ and

$$(H_1^*) l(t) > 0, m(t) \ge 0, n(t) \ge 0,$$
$$0 \le p(t) \le p_0 < \infty;$$
$$(H_2^*) \lim_{t\to\infty} \mu(t) = \lim_{t\to\infty} \tau(t) = \lim_{t\to\infty} \psi(t) = \infty;$$
$$(H_3^*) T(t) = a + bt \text{, with } b > 0.$$

Then

$$T(t) \in C'([t_0,\infty)), T(t) \le t,$$
$$T'(t) = T_0 > 0,$$
$$T\circ\mu = \mu\circ T,\; T\circ\tau = \tau\circ T,$$
$$T\circ\psi = \psi\circ T;$$

We fixed $\omega(t) = [x(t) + p(t)xT(t)]$then we define a function$x(t) \in c([T_x,\infty))$, $T_x \ge t_0$ whichhas the property $\omega(t) \in C^n([T_x,\infty))$ and satisfies (H^*) on $[T_x,\infty)$.we mentioned only those solutions.$\omega(t)$ of (H^*) which deals with $\{|x(t)|: t \ge T\} > 0$ for all $T \ge T_x$.The equation (H^*) is saidto be oscillatory. Its solutions are oscillatory or asymptotically convergent to zero. The differential equations have a major results in population models, movement of electricity, modelling chemical reactions, to find optimum investment strategies.

Then there is a prevailing sufficient conditions for the oscillation of the solution of different kinds of odd order differential equations;

They researched the H^*oscillatory behavior of the odd – order neutral differential equations.

$[x(t)+p(t)x(T(t))]^{(n)} + l(t)x(\mu(t))$=0 and exhibited some standard oscillation results for the case when $0 \le p(t) \le p_0 < \infty; T'(t) > T_0$ and $T\circ\mu = \mu\circ T$.

To the finest of our principle the search of oscillatory behavior of odd – order neutral differential equations has not enough.

In this paper we try to accept the some of new oscillation results forH^*. To exhibit our response we utilise the upcoming definitions and remarks.

A. Definition .1

Consider the sets

$\mathbb{R}_0 = \{(u,v): u > v > u_0\}$ and

$\mathcal{R} = \{(u,v): u \ge v \ge t_0\}$.

Assume $H^* \in C(\mathcal{R},\mathcal{D})$ satisfies the following assumptions:

$(A_1) H^*(u,u) = 0, u \ge u_0; H^*(u,v) > 0, (u,v) \in \mathbb{R}_0$;

$(A_2) H^*$ has a negative (include zero) continuous partial derivative with respect to the second variable in $\mathbb{R}_0$.

Then the function H^* has the property$P$.

B. Definition .2

Let X be a convex set in a real vector space and let $f: X \longrightarrow R$ be a function. Then f is called convex, if $\forall\, x_1, x_2 \in X, \forall\, t \in [0,1]$

$$f(tx_1, tx_2) \le tf(x_1) + tf(x_2).$$

C. Lemma .3

Assume that $\gamma \ge 1$, $i, j, k \in \mathbb{R}$. If $i \ge 0, j \ge 0$ and $k \ge 0$, then

$$i^\gamma + j^\gamma + k^\gamma \ge \frac{1}{3^{\gamma-1}}(i+j+k)^\gamma \qquad (1)$$

1) Proof:

1) Suppose $i = 0$ or $j = 0$ or $k = 0$ then we have (1)
2) Suppose that $i > 0, j > 0$

and $k > 0$. Define the function f, by $f(x) = (x)^\gamma$, $x \in (0,\infty)$, then $f'(x)=\gamma\,(x)^{\gamma-1}$

$f''(x) = \gamma(\gamma-1)x^{\gamma-2} \ge 0$ for all $x > 0$.

Thus by the definition of convex function

$$f\left(\frac{i+j+k}{3}\right) \le \frac{f(i)+f(j)+f(k)}{3}$$

That is,

$$i^\gamma + j^\gamma + k^\gamma \ge \frac{1}{3^{\gamma-1}}(i+j+k)^\gamma.$$

II. Main Results

For our deliberate references, let us denote

$$L(t) = \min\{l(t), l(T(t))\}$$
$$M(t) = \min\{m(t), m(T(t))\}$$
$$N(t) = \min\{n(t), n(T(t))\} \qquad (2)$$

A. Theorem 2.1

Assume that

$$\int_{t_0}^{\infty}[L(t)+M(t)+N(t)]dt = \infty \qquad (3)$$

Then by assuming first order neutral differential inequality

$$\left(y(t) + \frac{p_0}{\lambda}\, y(T(t))\right)' + \frac{\beta}{(n-1)}\left[L(t)\mu^{n-1}(t)y(\mu(t))M(t)\tau^{n-1}(t)y(\tau(t)) + N(t)\psi^{n-1}y(\psi(t))\right] \le 0 \qquad (H_4{}^*)$$

Has negative solution for some $\beta \in (0,1)$. Then, H^* is oscillatory.

1) Proof:

Assume that x is non negative solution of H^*which does not tends to zero asymptotically.

Then the comparable function ω satisfies.

$$\omega(\mu(t)) = [x(\mu(t)) + p(\mu(t)x(T(\mu(t))] \leq x(\mu(t)) + p_0 x(\mu(T(t)) \quad (4)$$

Then by the hypothesis we get

$$\omega(\tau(t)) \leq x(\tau(t)) + p_0 x(\tau(T(t)) \quad (5)$$

$$\omega(\mu(t)) \leq x(\psi(t)) + p_0 x(\psi(T(t)) \quad (6)$$

Next it follows from H^* that

$$\omega^n(t) + l(t)x(\mu(t)) + m(t)x(\tau(t)) + n(t)x(\psi(t)) = 0 \quad (7)$$

By additionally we are taking (H_1^*) and (H_3^*) into account, we have

$$0 = \frac{p_0}{T'(t)}(\omega^{n-1}(T(t))' + p_0\, l\,(T(t)x(\mu(T(t)) + p_0\, m(T(t)x(\tau(T(t)) + p_0\, n\,(T(t)x(\psi(T(t)) = \frac{p_0}{T'(t)}(\omega^{n-1}(T(t))' + p_0\, l\,(T(t)x(\mu(T(t)) + p_0\, m(T(t)x(\tau(T(t)) + \quad (8)$$

By combining (7) and (8) we get

$$\left[\omega^{n-1}(t) + \frac{p_0}{T'(t)}\omega^{n-1}(T(t))\right]' + l(t)x(\mu(t)) + p_0\, l\,(T(t)x(\mu(T(t)) + m(t)x(\tau(t)) + p_0\, m(T(t)x(\tau(T(t)) + n(t)x(\psi(t)) + p_0\, n\,(T(t)x(\psi(T(t)) \leq 0$$

From the equation (2), (4). (5), (6)

$$\left[\omega^{n-1}(t) + \frac{p_0}{T'(t)}\omega^{n-1}(T(t))\right]' + L(t)\omega(\mu(t)) + M(t)\omega(\tau(t)) + N(t)\omega(\psi(t)) \leq 0 \quad (9)$$

Next we prove $\omega'(t) > 0$

If not,then $\lim_{t\to\infty}\omega(t) = a > 0$

(a is not infinite), then

$$\lim_{t\to\infty}\omega^{(k)}(t) = 0,$$

k = 1,2, … … . n. integrating (9) from t to ∞ for a total of (n-1) times and integrating the results from t_1 to ∞.

$$\int_{t_1}^{\infty}\frac{(v-t_1)^{n-1}}{(n-1)!}[L(v)\omega + M(v)\omega(\tau(v)) + N(v)\omega(\psi(v))]dv < \infty$$

Which yields

$$\int_{t_1}^{\infty} v^{n-1}[L(v) + M(v) + N(v)]dv < \infty$$

This contradicts to

$$\int_{t_0}^{\infty} t^{n-1}[L(t) + M(t) + N(t)]dt = \infty$$

Hence by the "let g be a function as in knesar theorem, if $g^n(t)g^{n-1}(t) \leq 0$ and if $\lim_{t\to\infty} g(t) \neq 0$, then every $\lambda \in (0,1)$, there exists $t_\lambda \in [t_1, \infty)$ such that

$$|g| \geq \frac{\lambda}{(n-1)!}t^{n-1}|g^{n-1}| \text{ holds on } [t_{\lambda}, \infty)\text{"}$$

and "If x is a positive solution of

$$[x(t) + p(t)x(T(t))]^{(n)} + l(t)x(\mu(t)) + m(t)x(\tau(t)) + n(t)x(\psi(t)) = 0$$

then the comparable function

$\omega(t) = x(t) + p(t)x(\tau(t))$ satisfies $\omega(t) > 0$, $\omega^{(n-1)}(t) > 0$, $\omega^{(n)}(t) < 0$."

We get $\omega(t) \geq \frac{\lambda}{(n-1)!}t^{(n-1)}\omega^{(n-1)}(t)$ for every $\lambda \in (0,1)$

By

$$\left(y(t) + \frac{p_0}{\lambda}y(T(t))\right)' + \frac{\beta}{(n-1)}[L(t)\mu^{n-1}(t)y(\mu(t)) + M(t)\tau^{n-1}(t)y(\tau(t))\, N(t)\psi^{n-1}y(\psi(t))] \leq 0$$

$y(t) = \omega^{(n-1)}$ in above equation

We get

$$\left[\omega^{(n-1)}(t) + \frac{p_0}{\lambda}\omega^{(n-1)}(T(t))\right]' + \frac{\beta}{(n-1)!}\left[[L(t)\mu^{n-1}(t)y(\mu(t)) + M(t)\tau^{n-1}(t)y(\tau(t)) + N(t)\psi^{n-1}y(\psi(t))]\right] \leq 0$$

We establish that y is a positive solution of $(H_4{}^*)$. This is negation to our assumption.At lastour proof is completed.

B. Corollary 2.2

Assume that $\int_{t_0}^{\infty}[L(t) + M(t) + N(t)]dt = \infty$ holds. If $\tau \leq \tau\psi$, $\mu\tau \geq \tau\psi$, $\mu\tau \leq \tau\psi$ and also

$$\liminf_{t\to\infty}\int_{T^{-1}(\mu(t))}^{t}(\mu^{n-1}(s)L(s)ds + \tau^{n-1}(s)M(s)ds + \psi^{n-1}N(s)ds) > \frac{(n-1)!(T_0+P_0)}{T_0^e} \quad (10)$$

$$\liminf_{t\to\infty}\int_{T^{-1}(\tau(t))}^{t}(\mu^{n-1}(s)L(s) + \tau^{n-1}(s)M(s)d + \psi^{n-1}N(s)ds) > \frac{(n-1)!\,(T_0+P_0)}{T_0^e}$$

$$\liminf_{t\to\infty}\int_{T^{-1}(\psi(t))}^{t}(\mu^{n-1}(s)L(s)ds + \tau^{n-1}(s)M(s)ds + \psi^{n-1}N(s)ds) > \frac{(n-1)!\,(T_0+P_0)}{T_0^e}$$

Then $H^*$is almost oscillatory.

1) Proof:

We admit that v is a positive solution of (9) for some$0 < \lambda < 1$. If $\mu\tau \leq \tau\mu$, then $v(T^{-1}(\mu(t)) \geq v(T^{-1}(\tau(t))$ and (10) gives that v is a solution of differential inequality.

$$v' + \frac{\lambda T_0}{(n-1)!(T_0+P_0}\mu^{n-1}(t)L(t) + \tau^{n-1}(t)M(t) + \psi^{n-1}N(t) \leq 0 \quad (11)$$

Similarly,$\mu\tau \geq \tau\mu$,

$$v'(t) + \frac{\beta T_0}{(n-1)!(T_0+P_0} \times [L(t)\mu^{n-1}(t)v(T^{-1}(\mu(t)) + M(t)\tau^{n-1}(t)v(T^{-1}(\tau(t)) + N(t)\psi^{n-1}v(T^{-1}(\psi(t))] \leq 0$$

$(H_5{}^*)$

By ([7], Theorem 2.11)Then the equation (10) showed that (11) as no positive solution.$H_5{}^*$ has no positive solution and H^* is oscillatory. The case $\mu\tau \geq \tau\mu$ be treated similarly.

C. Theorem 2.3

Assume that $\int_{t_0}^{\infty}[L(t) + M(t) + N(t)]dt = \infty$ holds. if the first order differential inequality

$$v'(t) + \frac{\beta T_0}{(n-1)!\,(T_0+P_0} \times [L(t)\mu^{n-1}(t)v(T^{-1}(\mu(t)) + M(t)\tau^{n-1}(t)v(T^{-1}(\tau(t)) + N(t)\psi^{n-1}v(T^{-1}(\psi(t))] \leq 0$$

Has negative solution for some $0 < \beta < 1$. Then the (H^*) is almost oscillatory.

1) Proof:

Assume thatx be a positive solution of H^* which does not tend to zero asymptotically.

Then$y(t) = \omega^{n-1}(t) > 0$ is non increasing solution of H_4^* we denote

$$v(t) = y(t) + \frac{P_0}{\lambda} y(T(t))$$

Then $v(t) \leq y(t) + y(t)\,(1+\frac{P_0}{\lambda})$ (12)

Substitute (12) in (H_4^*)

$(H_4^*)\ \left(y(t) + \frac{p_0}{\lambda}\, y\big(T(t)\big)\right)' + \frac{\beta}{(n-1)}\left[L(t)\mu^{n-1}(t)y\big(\mu(t)\big) + M(t)\tau^{n-1}(t)y\big(\tau(t)\big) + N(t)\psi^{n-1}y\big(\psi(t)\big)\right] \leq 0$

We get

$(v'(t)) + \frac{\beta}{(n-1)}\left[L(t)\mu^{n-1}(t)y\big(\mu(t)\big) + M(t)\tau^{n-1}(t)y\big(\tau(t)\big) + N(t)\psi^{n-1}y\big(\psi(t)\big)\right](v'(t)) + \frac{\beta}{(n-1)}\left[L(t)\mu^{n-1}(t)y\big(\mu(t)\big) + M(t)\tau^{n-1}(t)y\big(\tau(t)\big) + N(t)\psi^{n-1}y\big(\psi(t)\big)\right]$

By (H_4^*) we get v is a positive solution of $(H_5{}^*)$.

This is negation to our assumption.

D. *Epilogue*

The article especially exhibits about the new similarity theorems for examine of the principle oscillation of$(H)^*$. This similarity exhibits less oscillation of odd- order neutral equations to research the properties of different kinds of first order different inequalities which specially defines the hypothesis $(H)^*$**.** Our methodology provides to reduce prohibition is normally explored on the coefficient of $(H)^*$. So that our results are manually extrapolated. Acceptable results are casually reliable and are expressed on a comfortable model.

References

[1] R.P.Agarwal, S.R.Grace and D.O'Regan; Oscillation Theory for Difference and Functional Differential Equations, Kluwer Academic, Dordrecht, 2000.

[2] B.Baculikova, E.M.Elabbasy, S.H.Saker and J.Dzurina; Oscillation criteria for third order nonlinear differential equations, Math. Slovaca 58(2008). 1-20.

[3] T.Candan and R.S.Dahiya;Oscillatory and asymptotic behavior of odd order neutral differential equations, Dyn. Contin. Discrete Impulse. Syst .Ser. A Math .Anal. 14(2007)767-774

[4] P.Das; Oscillation in odd –order neutral delay differential equations, Proc. Indian Acad .Sci. Math .Sci 105 (1995) 219-225.

[5] J.Dzurina; Oscillation theorems for neutral differential equations of higher order, Czechoslovak Math. J. 54 (2004) 185-195.

[6] K. Gopalasamy, B.S.Lalli and B.G.Zhang; Oscillation of odd order neutral differential equations, Czechoslovak Math. J. 42 (1992) 313-323.

[7] I.Gyori and G.Ladas. Oscillatory theory of delay differential equation with Application ,Oxford Universitypress, New York,1991.

[8] B.Karpuz, O. Ocalan and S.Ozturk, Comparison theorems on the oscillation and asymptotic behavior of higher-order neutral differential equations, Glasgow Math .J. 52(2010)107-114.

[9] R. N.Rath;Oscillatory and asymptotic behaviourof solutions of higher order neutral equations, Bull. Inst .Math. Acad. Sinica 30 (2002) 219-228.

[10] X.H.Tang; Linearized 0scillation of odd order nonlinear neutral delay differential equations (I), J.Math. Anal .Appl. 332 (2006) 864-872.

[11] B.G.Zhang and W.T.Li ; on the Oscillation of odd order neutral differential equations, Fasc. Math. 29 (1999) 167-183.