Some Theorems on Sum Divisor Cordial Graphs

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Abstract— A sum divisor cordial labeling of a graph G with vertex set V is a bijective function f from V(G) to {1, 2,…,|V (G)|} such that if 2 divides f (u) +f (v) then an edge uv is assigned the label 1 and 0 otherwise, then the number of edges labeled with 0 and 1 differ by at most 1. A graph is said to be sum divisor cordial graph if it has sum divisor labeling. This study proves that path, comb, star, complete bipartite, bistar, jewel, gear, crown are sum divisor cordial graphs.

Key words: Sum Divisor Cordial Graphs

I. Introduction

Here we considered all graphs are simple, finite, connected and undirected. In this paper we follow the basic notations and terminologies of graph theory as in [7]. A labeling of a graph is a map that carries the graph elements to the set of numbers, usually to the non-negative or positive integers. In a labeling graph if the domain is the set of vertices is called a vertex labeling and if the domain is the set of edges then the labeling graph is called an edge labeling. The labeling graph is said to be total labeling if the labels are assigned to both vertices and edges. We refer Gallian[3], for a dynamic survey of various graph labeling.

A. Definition 1.1

Let G=(V(G),E(G)) be a simple graph and f:V(G)→{1,2,3,…,|V(G)|} be a bijection. For each edge uv, assign the label 1 if either f(u)|f(v) or f(v)|f(u) and the label 0 otherwise. The function f is called a divisor cordial labeling if $|e_f(i)-e_f(j)| \le 1$, i,j∈{0,1}. A graph which admits a divisor cordial labeling then the graphis called a divisor cordial graph.

B. Definition 1.2

Let G=(V(G),E(G)) be a simple graph and if f:V(G)→{1,2,3,…,|V(G)|} be a bijection. For each edge uv, assign the label 1 if 2|(f(u)+f(v)) and the label 0 otherwise. If $|e_f(i)-e_f(j)| \le 1$, i,j∈{0,1} then the function f is called a divisor cordial labeling. A graph admits a sum divisor cordial labeling then the graph is called a sum divisor cordial labeling.

C. Definition 1.3

The graphobtained from a path by attaching a pendent edge to the vertex of the path, then the graph is called a comb and it is denoted by P_n.

D. Definition 1.4

The graph obtained by attaching the apex vertices of two copies of bipartite graph ($K_{1,n}$) by an edge is called a bistar and it is denoted by$B_{n,n}$.

E. Definition 1.5

A simple bipartite graph such that every vertex in one of the bipartition subsets is joined to every vertex in the other bipartition subset is called a complete bipartite graph. The complete bipartite graph one of the subset has mvertices and other has n vertices andit is denoted by$k_{n,m}$.

F. Definition 1.6

The graph with vertex $V(J_n)=\{u,v,x,y,u_j : 1\le j \le n\}$ and the edge set $E(J_n)=\{ux,uy,xv,yv,uu_j,vu_j: 1\le j \le n\}$ then the graphis called a jewel graphand it is denoted byJ_n.

G. Definition 1.7

The graph obtained by an extra vertex between each pair of adjacent vertices on the perimeter of a wheel graphW_n, then the graph is called gear and it is denoted byG_n.

H. Definition 1.8

The graph obtained by attaching a single edge and node to each node of the outer circuit of a wheel graphW_n, then the graph is called helm and it is denoted byH_n.

I. Definition 1.9

The graph obtained from a cycle by attaching a pendant edge to each vertex of the cycle then the graph is called a crown and it is denoted byC_n.

1) Theorem 2.1

The graph P_n is sum divisor cordial graph.

a) Proof:

Let be $\{v_1,v_2,\dots v_n\}$ a vertices ofP_n. The order of $P_n$is n and size of $P_n$is n-1. We define f: V (P_n)→ {1, 2,…, n}as follows:

Case: 1 *n* is odd

J if *j*≡ 0, 1(mod 4)

$$f(v_j) = \begin{cases} j+1 & \text{if } j \equiv 2 \pmod 2 \text{ for } 1\le j \le n \\ j-1 & \text{if } j\equiv 3 \pmod 4 \end{cases}$$

Case: 2 n is even

$$f(v_j) = \begin{cases} j & \text{if } j \equiv 1, 2 \pmod 4 \\ j+1 & \text{if } j \equiv 3 \pmod 4 \\ j-1 & \text{if } j \equiv 0 \pmod 4 \end{cases}$$

The edge labels for both cases are,

$$f^*(v_jv_{j+1}) = \begin{cases} 1 \text{if } 2|\ (f(v_j) + f(v_{j+1})) \\ 0 \quad \text{otherwise} \end{cases}$$

We note that,

$$\frac{n-1}{2} \text{ if } n \text{ is odd}$$

$$ef(0) = \begin{cases} \frac{n}{2} \text{ if } n \text{ is even} \end{cases}$$

$$ef(1) = \begin{cases} \frac{n-1}{2} & \text{if } n \text{ is odd} \\ \frac{n-2}{2} & \text{if } n \text{ is even} \end{cases}$$

Thus satisfies,

| ef(0) – ef(1) | ≤ 1.

Hence, the path P_n is sum divisor cordial graph.

2) Example 2.2

A sum divisor cordial labeling of P5 and P8 is shown in figure 2.1

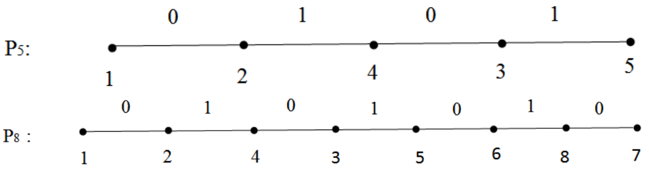

Fig. 2.1

3) Theorem: 2.3

The comb is sum divisor cordial graph.

a) Proof:

Let the path be $\{v_1, v_2, \ldots v_n\}$ vertices by joining the vertex uj to vj for each

j =1,2,…,nto the graph G obtaining a comb. The order of the graph G is 2n and the size of G is 2n-1. We define f :V(G) → {1,2,…,2n} as follows:

$f(v_j) = 2j;\ 1 \le j \le n$

$f(u_j) = 2j-1;\ 1 \le j \le n$

The edge labels are,

$f^*(v_j v_{j+1}) = 1\ ;\ 1 \le j \le n-1$

$f^*(v_j u_j) = 0\ ;\ 1 \le j \le n$

We note that,

ef(0) =nand ef(1) = n-1

Thus satisfies,

|ef(0) – ef(1)| ≤ 1

Hence, the comb is sum divisor cordial graph.

4) Example: 2.4

A sum divisor cordial labeling of comb is shown in figure 2.2

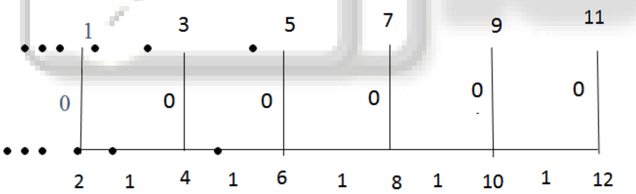

Fig. 2.2

5) Theorem: 2.5

The star $K_{1,n}$ is sum divisor cordial graph.

a) Proof:

Let V_1 and V_2 be two subsets of bipartition graph $K_{1,n}$. Let V_1={u} and

$V_2=\{v_1, v_2, \ldots v_n\}$ and let $E(K_{1,n})=\{uv_i, uv_i: 1 \le i \le n\}$. The $K_{1,n}$ is of order n+1 and size n. We define f :V($K_{1,n}$) →{1,2,…,n+1} as follows:

f (u) = 2 ;

$f(u_i) = j+2;\ 1 \le j \le n$

The edge labels are,

$f^*(uu_{2j-1}) = 0\ ; 1 \le j \le [\frac{n}{2}]$

$f^*(uu_{2j}) = 1\ ;\ 1 \le j \le [\frac{n}{2}]$

We note that,

$$e_f(0) = \begin{cases} \frac{n+1}{2} \text{ if } n \text{ is odd} \\ \frac{n}{2} \text{ if } n \text{ is even} \end{cases}$$

$$e_f(1) = \begin{cases} \frac{n+1}{2} \text{ if } n \text{ is odd} \\ \frac{n}{2} \text{ if } n \text{ is even} \end{cases}$$

Thus satisfies,

$|e_f(0) - e_f(1)| \le 1$

6) Example: 2.6

A sum divisor cordial labeling of $K_{1,7}$ is shown in figure 2.3

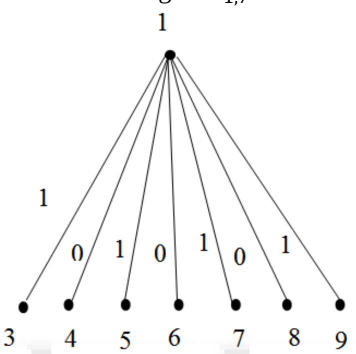

Fig. 2.3

7) Theorem: 2.7

The graph $K_{2,n}$ is sum divisor cordial graph.

a) Proof:

Let V_1 and V_2 be the bipartition of $K_{2,n}$. Let V_1={u,v} and $V_2=\{v_1, v_2, \ldots v_n\}$ and let $E(K_{2,n})=\{uv_i, uv_i: 1 \le i \le n\}$. The $K_{2,n}$ is of order n+2 and size 2n. We define

f :V($K_{2,n}$)→{1,2,…,n+2} as follows:

f(u)=2;

f(v)=3;

$f(v_j)=j+3\ ; 1 \le j \le n$

The edge labels are,

$f^*(uv_{2j-1}) = 1; 1 \le j \le [\frac{n}{2}]$

$f^*(uv_{2j}) = 0;\ 1 \le j \le [\frac{n}{2}]$

$f^*(vv_{2j-1}) = 0; 1 \le j \le [\frac{n}{2}]$

$f^*(vv_{2j}) = 1;\ 1 \le j \le [\frac{n}{2}]$

We note that,

$e_f(0) = n e_f(1) = n$

Thus satisfies,

$|e_f(0) - e_f(1)| \le 1$

Hence, $K_{2,n}$ is sum divisor cordial graph.

8) Example: 2.8

A sum divisor cordial labeling of $K_{2,6}$ is shown in figure 2.3

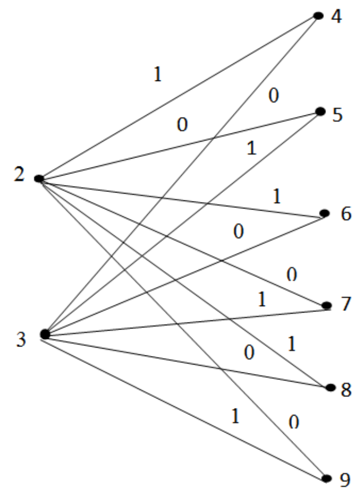

Fig. 2.3

9) Theorem: 2.9

Thebistar $B_{n,n}$ is sum divisor cordial graph.

a) Proof:

Let V(G)={u,v,u_i,v_i : 1≤i ≤ n} and E(G)={uv,vv_i,uu_i : 1≤i ≤ n}. Then the graph $B_{n,n}$ is of order 2n+2 and the size 2n+1. We define f:V(G) → {1,2,…,2n+2} as follows:

f(u) = 3;

f (v) = 2;

f(u_{2j-1}) = 2j-1 ;1≤ j ≤$[\frac{n}{2}]$

f(u_{2j}) = 2j+2 ; 1≤ j ≤$[\frac{n}{2}]$

f(v_{2j-1}) = 2j ; 1≤ j ≤$[\frac{n}{2}]$

f(v_{2j}) = 2j+1 ; 1≤ j ≤$[\frac{n}{2}]$

The edge labels are,

f*(uv) = 0

f*(uu_{2j-1}) = 1;1≤ j ≤$[\frac{n}{2}]$

f*(uu_{2j}) = 0; 1≤ j ≤$[\frac{n}{2}]$

f*(vv_{2j-1}) = 0;1≤ j ≤$[\frac{n}{2}]$

f*(vv_{2j}) = 1; 1≤ j ≤$[\frac{n}{2}]$

We note that,

$$e_f(0) = \begin{cases} n & \text{if n is odd} \\ n+1 & \text{if n is even} \end{cases}$$

$$e_f(1) = \begin{cases} n & \text{if n is even} \\ n+1 & \text{if n is odd} \end{cases}$$

Thus satisfies,

|$e_f(0)$ -$e_f(1)$|≤ 1.

Hence, the bistar $B_{n,n}$ is sum divisor cordial graph.

10) Example: 2.8

A sum divisor cordial labeling of $B_{7,7}$ is shown in figure 2.4

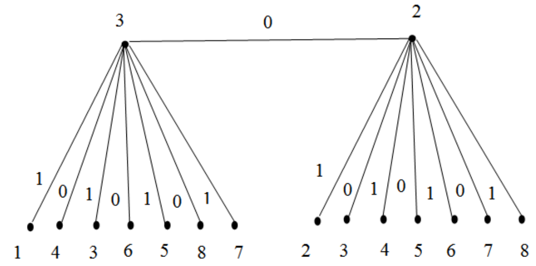

Fig. 2.4

11) Theorem: 2.9

A jewel J_n is sum divisor cordial graph.

a) Proof:

Let the vertices ofJ_n be V(G)={u,v,x,y,u_j : 1≤ j ≤ n} and the edges

E (G) = {ux,uy,xv,yv,uu_j,vu_j : 1≤ j ≤ n}. Then the graph J_n is of order n +4 and the size 2n+5. We define f: V (G) → {1, 2,…, n+4}.

f(u) = 3 ;

f(v) = 4 ;

f(x) = 1 ;

f (y) = 2 ;

f (u_i) = i + 4 ; 1≤i ≤ n .

The edge labels are,

f *(ux) = 1 ;

f *(vx) = 0 ;

f * (xy) = 0 ;

f * (vy) = 1 ;

f *(uy) = 0 ;

f*(uu_{2j-1}) = 1;1≤ j ≤$[\frac{n}{2}]$

f*(uu_{2j}) = 0 ;1≤ j ≤$[\frac{n}{2}]$

f*(vu_{2j}) = 1 ;1≤ j ≤$[\frac{n}{2}]$

f*(vu_{2j-1}) = 0 ;1≤ j ≤$[\frac{n}{2}]$

We note that,

$e_f(0)$ = n + 3 and $e_f(1)$ = n + 2

Thus satisfies,

|$e_f(0)$ -$e_f(1)$|≤ 1.

Hence, J_n is sum divisor cordial graph.

12) Example: 2.10

A sum divisor cordial labeling of J_4 is shown in figure 2.5

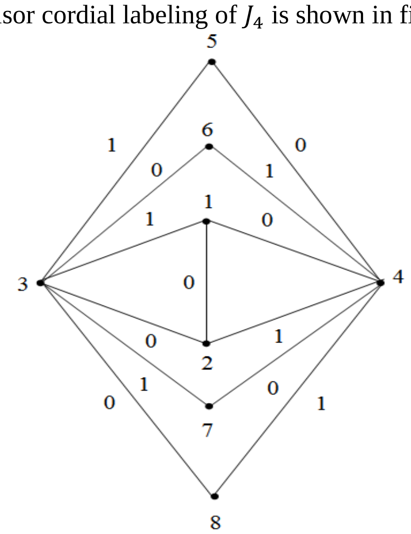

Fig. 2.5

13) Theorem: 2.11

The crown $C_n \odot K_1$ is sum divisor cordial graph.

a) Proof:

Let the vertices of the graph $C_n \odot K_1$ be V(G) ={u_j,v_j: 1≤ j ≤ n} and the edges E(G)={$u_j u_{j+1}$: 1≤ j ≤ n-1; $u_n u_1$,$u_j v_j$: 1≤ j ≤ n }. Then the graph $C_n \odot K_1$ is of order 2n andsize 2n. We define f: V (G) → {1, 2,…, 2n} as follows:

f(u_j)= 2j-1 ; 1≤ j≤n

f(u_j)= 2j;1≤ j≤n

The edge labels are,

f*($u_j u_{j+1}$)= 1 ; 1≤ j≤n-1

f*($u_1 u_n$) = 0 ;

f*($u_j v_j$) = 0 ; 1≤ j≤n

We note that,

$e_f(0)$ = nande$_f$(1)= n.

Thus satisfies,

|$e_f(0)$ -$e_f(1)$| ≤ 1.

Hence, $C_n \odot K_1$ is sum divisor cordial graph.

14) Example: 2.12

A sum divisor cordial labeling of $C_n \odot K_1$ is shown in figure 2.6

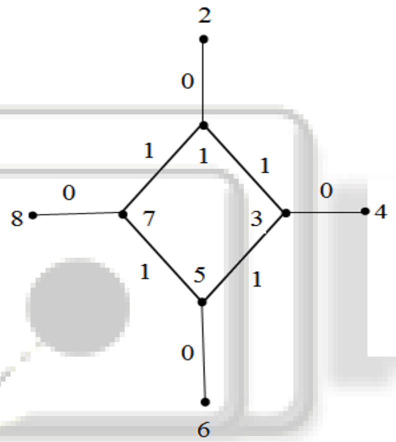

Fig. 2.6

15) Theorem: 2.13

The gear G_n is a sum divisor cordial graph.

a) Proof:

Let the vertices of the graph G_n be V(G)={v,u_i, v_i: 1≤ i ≤ n} and the edges E(G)={vv_j,vu_j : 1≤ j≤ n;$u_j v_{j+1}$: 1≤ j≤ n-1 ; $u_n v_1$}. Then the graph G_n is of order 2n+1 and size 3n.We define f: V(G) → {1, 2,…, 2n+1} as follows:

f (v) = 1;

f (v_{2j-1}) = 4j-11≤ j ≤[$\frac{n}{2}$]

f (v_{2j}) = 4j ;1≤ j ≤[$\frac{n}{2}$]

f (u_{2j-1}) = 2j-1 ; 1≤ j ≤[$\frac{n}{2}$]

f (u_{2j}) =2j+1 ;1≤ j ≤[$\frac{n}{2}$]

The edge labels are,

sf *(vv_{2j-1}) = 1 ; 1≤ j ≤[$\frac{n}{2}$]

f *(vv_{2j}) = 0 ; 1≤ j ≤[$\frac{n}{2}$]

f *($v_j u_j$) = 1 ; 1≤ j≤n

f *($u_j v_{j+1}$) = 0 ; 1≤ j≤n-1

We note that,

$e_f(0)$ = n+2 and $e_f(1)$ = n+2

Thus satisfies,

|$e_f(0)$ -$e_f(1)$| ≤ 1.

Hence, G_n is sum divisor cordial graph.

16) Example: 2.14

A sum divisor cordial labeling ofG_4 is shown in figure 2.7

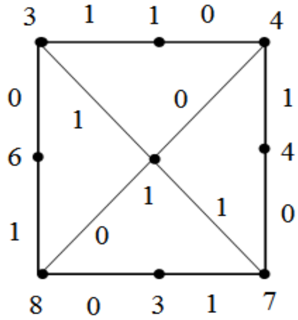

Fig. 2.7

II. CONCLUSION

All the graphs are not sum divisor cordial graphs. It is appealing and challenging to check out the sum divisor cordial graphs. The graph families which concede sum divisor cordial labeling. The sum divisor cordial graphs path, comb, star, complete bipartite, bistar, jewel, gear, crown are proved by the condition, sum divisor cordial labeling of a graph G with vertex set V is a bijective function f from V(G) to {1, 2,…,|V (G)|} such that if 2 divides f (u)+f(v) then an edge uv is assigned the label 1 and 0 otherwise, then the number of edges labeled with 0 and 1 differ by at most 1.

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