Super Fibonacci Graceful Labelling of Some Cycle Related Graphs

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Abstract— This paper discusses about the Fibonacci number and its functionf: V(G) →{0,F1,F2,……..Fq} is said to be Super Fibonacci graceful , ifthe induced edge labeling f *:E (G) →F1,F2,F3,…….Fq} and it is defined by f*(uv) =|f(u)-f(v)| is bijective. Some examples are given to illustrate the theory. In this paper Friendship graph and (k4-e) ^{n} graph are investigated.

Key words: Friendship Graph, The Graph (k4-e) ^{n}, Fibonacci Graceful Labelling and Super Fibonacci Graceful Labelling

I. Introduction

In this graph labeling vertices or edges or both assigned their values based on some conditions .Rosa (1967)[7] introduced "Graceful Labeling".A complete survey on graph labeling was studied by J.A.Gallian[4]. The cycle structure of Fibonacci graceful mapping was investigated by David.W. and Anthony .E.Baraaukas [2].In 2006 Kathirasen and Amutha[5] introduced a Fibonacci graceful labeling and super Fibonacci graceful labeling.Friendship graphs are harmonious was proved by dushyant tanna[4].The windmill graphs are strong edge graceful was proved by Dr.M.subbiah.

II. Definitions

A. Definition 2.1

If the vertices or edges or both assigned values subject to certain conditions then it is known as graph labeling.

B. Definition 2.2

The function f is called a graceful labeling of a graph G if f: →V {0,1,2,…..q} is injective and the induced function f*:E →{1,2,3,…q}defined as f*(e=uv)=|f(u)-f(v)| is bijective.

C. Definition 2.3

The function f:V(G)→{0,1,2…….Fq}(where Fq is the q th Fibonacci number) is said to be Fibonacci graceful if the induced edge labeling f*:E(G)→{F1,F2,F3,…..Fq}defined by f*(uv)=|f(u)-f(v)| is bijective.

D. Definition 2.4

The function f:V(G)→{ 0,F1,F2,F3,….. Fq}(where Fq is the qthFibonacci number is said to be super Fibonacci graceful if the induced edge labeling f*:E(G)→{F1,F2,F3,…..Fq}defined by f*(uv)=|f(u)-f(v)| is bijective.

E. Definition 2.5

A friendship graph Fn is a graph which consists of a triangle with a common vertex.

F. Definition 2.6

The wind mill graph (km) ^{n} to be the family of graph consisting of n copies of km with a vertex in common.

G. Definition 2.7

The graph (k4-e) ^{n} is obtained from the windmill graph (k4) ^{n}, removing an edge in each k4.

III. Results

A. Theorem 3.1

Friendship graphs are super Fibonacci graceful.

1) Proof:

Let Fn be the friendship graph. The order of Fn is p=2n+1 and the size of Fn is q=3n. By the definition of Fn ,the vertex set V={$u_1,u_2,\ldots,u_n,v_0,v_1,v_2,\ldots v_n$}. Let $u_1,u_2,\ldots,u_n$ be the second vertices of the triangles and let $v_1,v_2,\ldots v_n$ be the third vertices of the triangles, and let the apex vertex v0 be the first vertex of all the triangles. The edge set E= {e_i, e_{ii}, e*}where e_i= (v0, v_1),e_{ii}= (v_i, u_i) and e_i*=(v_0,u_i),

Now, let us define the function

f:V→{ 0,F1,F2,F3,….. Fq} as follows

$f(v_0)=0$

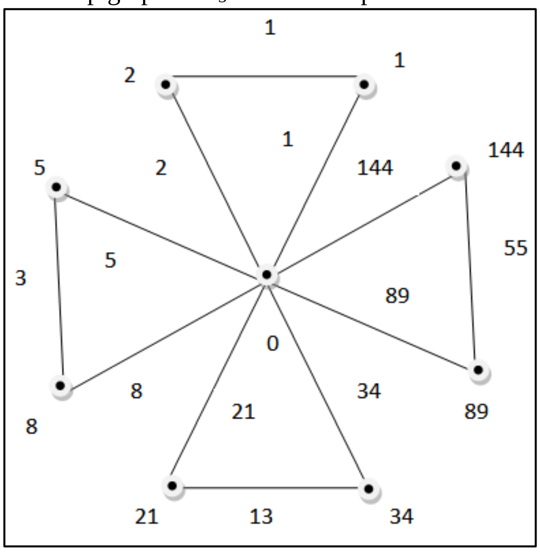
$f(u_i)= F_{3i-1}$ for i =1,2,3,…,.n

$f(v_i) = F_{3i}$ for i = 1,2,3,…..,n

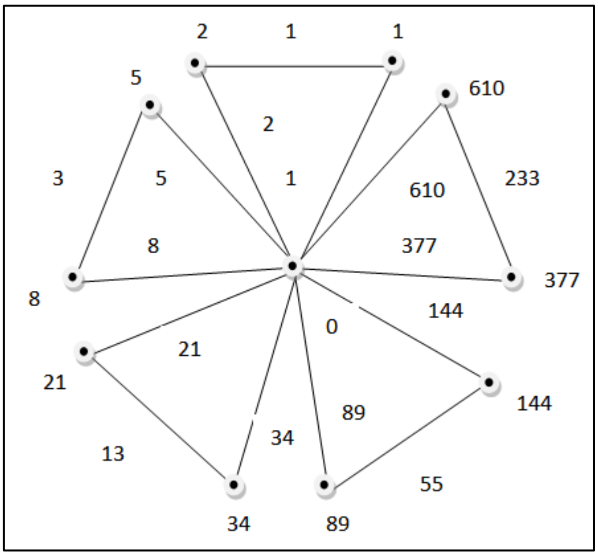
Then the above defined function f admits Super Fibonacci graceful labeling. Hence Friendship graphs are Super Fibonacci graceful. The example for Friendship graph F_n are shown below

2) Example:

The friendship graph of F_3 the order is p=7and the size q=9



The friendship graph for F_5 the order is p=11 and the size q=15



B. Theorem 3.2:

The graph $(k4\text{-}e)^n$ are super Fibonacci graceful.

1) Proof:

Let G= $(k4\text{-}e)^n$ be a graph. The order of the graph G is p=3n+1and the size of the graph G is q=5n.Then the vertex setV={x,u1,u2,….,un,v1,v2,…,vn,w1,w2,….,wn}.Let u1, u2,…., un be the second vertices of the $(k4\text{-}e)^n$, let v1,v2,…,vn be the third vertices of the $(k4\text{-}e)^n$,letw1,w2,….,wn be the fourth vertices of the $(k4\text{-}e)^n$.And let the central vertex x be the first vertex of all the $(k4\text{-}e)^n$.The edge set $E=\{e_i,g_i,h_i,e_{ii},e^*_{ii}\}$ where $e_i=(x,u_i),e_{ii}=(u_i,v_i),g_i=(x,v_i),h_i=(x,w_i), e^*_{ii}=(v_i,w_i)$.

Now, let us define the function

f:V→{ 0,F1,F2,F3,….. Fq} as follows

$f(x) = 0$

$f(u_{i+1}) = F_{5i+1}$ for i = 0,1,2,3,……,n-1

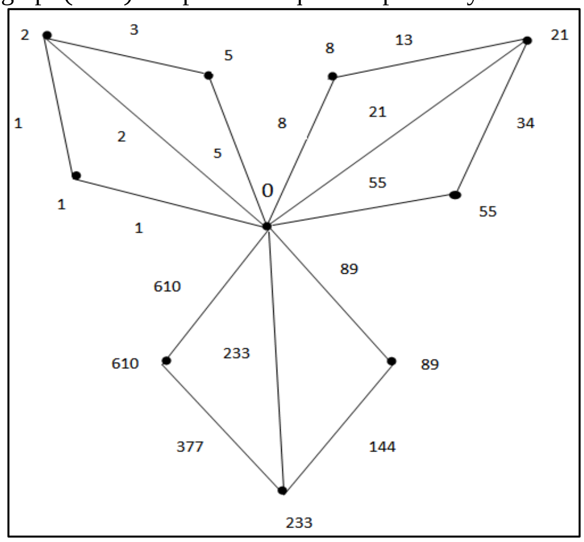
$f(v_i) = F_{5i-2}$ for i = 1,2,3,………,n

$f(v_i) = F_{5i}$ for i = 1,2,3,…….,n

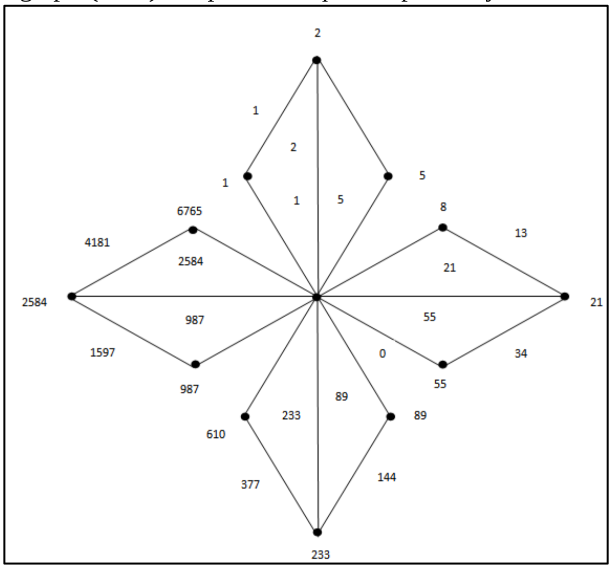
Then the above defined function f admits Super Fibonacci graceful labeling.

2) Example:

The graph $(k4\text{-}e)^3$ is shown below .The order and size of thegraph$(k4\text{-}e)^3$ is p=10 and q=15 respectively.



The graph $(k4\text{-}e)^4$ is shown below .The order and size of the graph $(k4\text{-}e)^3$ is p=13 and q=20respectively.



IV. Conclusion

In this paper we observed that the super Fibonacci graceful, if the induced edge labelling f *:E (G) →{F1,F2,F3,…….Fq} and it is defined by f*(uv) =|f(u)-f(v)| is bijective when the function is f: V(G) →{0,F1,F2,……..Fq}. An analysis is made on friendship graph F^3, F^4, F^5 and the graph $(k4\text{-}e)^3$, $(k4\text{-}e)^4$ by using super Fibonacci graceful labelling.

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