A Study on Epidemic Model

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Abstract— In this paper, we consider an epidemic model in which all disease transmission is through shedding of virus by infectives and acquisition by susceptibles, rather than by direct contact. This leads to a susceptible-infectious-virus-removed (SIVR) model for which we can determine the basic reproduction number and the final size relation. We extend the model to an age of infection model with virus shedding a function of the age of infection.

Key words: Epidemic Models, Indirect Transmission, Virus Shedding

I. Introduction

Mathematical models can be defined as a method of emulating real life situations with mathematical equations to expect their future behavior. In epidemiology, mathematical models play role as a tool in analyzing the spread and control of infectious diseases. One of the famous principles of ecology is "two species competing for the same resources cannot coexist indefinitely with the same ecological niche" [1, 2].

Vito Volterra was an Italian mathematician and physicist, known for his contributions to mathematical biology, integral equations and showed that the indefinite coexistence of two or more species limited by the same resource is impossible [3]. Ackleh and Allen [4] were the first who used the competitive exclusion principle of the infectious disease with different levels in single host population. Kermack and Mckendrick initiated a famous SIR type of compartmental model in 1927.

It can be specific to each strain of an epidemic model. In fact the basic reproduction number of the model is defined as the maximum reproduction numbers of other strains [5–7]. Diekmann et al. [8] had studied epidemic models with one strain, while Martcheva in [9] studied the SIS -type of disease with multistrain. However, Ackleh and Allen [10] studied SIR -type of disease with n strain and vertical transmission.

The declaration of an epidemic usually requires a good understanding of a baseline rate of incidence; epidemics for certain diseases, such as influenza, are defined as reaching some defined increase in incidence above this baseline. A few cases of a very rare disease may be classified as an epidemic, while many cases of a common disease (such as the common cold) would not.

II. Preliminary

An epidemic is a disease that spreads over a large area and affects many people at the same time. Epidemics are when a disease spreads from person to person faster than doctors can control. If the disease spreads over the whole world, sometimes it is called a pandemic.

Such diseases must be treated quickly and properly otherwise they can infect people on large scale and pose a danger to the world. It is very important that timely vaccination and treatment is provided. The study of epidemics is part of epidemiology.

An epidemic is the rapid spread of infectious disease to a large number of people in a given population within a short period of time, usually two weeks or less. Epidemics of infectious disease are generally caused by several factors including a change in the ecology of the host population (e.g. increased stress or increase in the density of a vector species), a genetic change in the pathogen reservoir or the introduction of an emerging pathogen to a host population.

Generally, an epidemic occurs when host immunity to either an established pathogen or newly emerging novel pathogen is suddenly reduced below that found in the endemic equilibrium and the transmission threshold is exceeded. An epidemic may be restricted to one location however, if it spreads to other countries or continents and affects a substantial number of people, it may be termed a pandemic.

There are several changes that may occur in an infectious agent that may trigger an epidemic. These include:

- Increased virulence
- Introduction into a novel setting
- Changes in host susceptibility to the infectious agent

III. Transmission

A. Airborne Transmission

Airborne transmission is the spread of infection by droplet nuclei or dust in the air. Without the intervention of winds or drafts the distance over which airborne infection takes place is short, say 10 to 20 feet.

B. Cyclopropagative Transmission

The agent undergoes both development and multiplication in the transmitting vehicle.

C. Developmental Transmission

The agent undergoes some development in the transmission vehicle.

D. Fecal-Oral Transmission

The infectious agent is shed by the infected host in feces and acquired by the susceptible host through ingestion of contaminated material.

E. Horizontal Transmission

Lateral spread to others in the same group and at the same time; spread to contemporaries.

F. Mechanical Transmission

The transmitter is not infected in that tissues are not invaded and the agent does not multiply.

G. Propagative Transmission

The agent multiplies in the transmission vehicle.

H. Biological Transmission

Involving a biological process, e.g. passing a stage of development of the infecting agent in an intermediate host. Opposite to mechanical transmission.

IV. THE SIR MODEL

The SIR model is one of the simplest compartmental models. There are three compartments– S for the number Susceptible, I for the number of Infectious, and R for the number Recovered (or immune). Recovered individuals may vary over time. The precise numbers over a function of t (time): S(t), I(t) and R(t). The variable function of t, may fluctuate over time.

In 1927, W. O. Kermack and A. G. McKendrick created a model in which they considered a fixed population with only three compartments: susceptible,S(t); infected, I(t); and removed, R(t). The compartments used for this model consist of three classes S(t) is used to represent the number of individuals not yet infected with the disease at time t, or those susceptible to the disease.

I(t) denote the number of individuals who have been infected with the disease and are capable of spreading the disease to those in the susceptible category.

R(t) is the compartment used for those individuals who have been infected and then removed from the disease, either due to immunization or due to death. Those in this category are not able to be infected again or to transmit the infection to others.

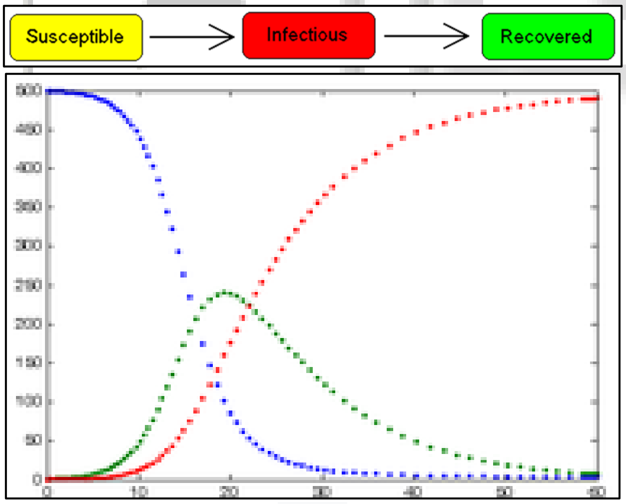

Fig. 1:
Blue=Susceptible, Green=Infected, and Red=Recovered

The SIR system without so-called vital dynamics (birth and death, sometimes called demography) described above can be expressed by the following set of ordinary differential equations

$$\frac{dS}{dt} = -\frac{\beta SI}{N}$$

$$\frac{dI}{dt} = \frac{\beta SI}{N} - \gamma I$$

$$\frac{dR}{dt} = \gamma I$$

Firstly note that from:

$$\frac{dS}{dt} + \frac{dI}{dt} + \frac{dR}{dt} = 0,$$

it follows that:

S(t)+I(t)+R(t) = Constant = N

expressing in mathematical terms the constancy of population N. Note that the above relationship implies that one need only study the equation for two of the three variables.

Secondly, we note that the dynamics of the infectious class depends on the following ratio:

$$Ro = \frac{\beta}{\gamma},$$

So-called basic reproduction number.

V. THE SIS MODEL

Some infections, for example those from the common cold and influenza, do not confer any long lasting immunity. Such infections do not give immunization upon recovery from infection, and individuals become susceptible again.

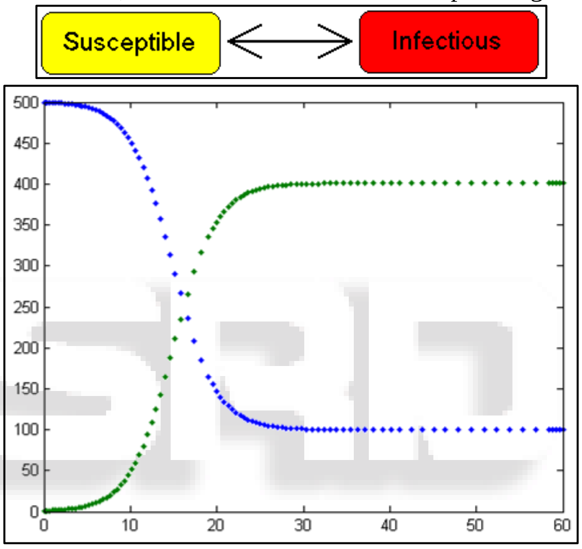

Fig. 2:
Susceptibles and Infected Get Equilibrated

Removing the equation representing the recovered population from the SIR model and adding those removed from the infected population into the susceptible population gives the following differential equations:

$$\frac{dS}{dt} = -\frac{\beta SI}{N} + \mu(N - S) + \gamma I$$

$$\frac{dI}{dt} = \frac{\beta SI}{N} - \gamma I - \mu I$$

Note that denoting with N the total population it holds that:

$$\frac{dS}{dt} + \frac{dI}{dt} = 0, \Rightarrow S(t)+I(t) = N$$

VI. REAL LIFE EXAMPLE

1) In an isolated community of 800 susceptible children, one child is diagnosed with chickenpox. Suppose the spread of the disease can be modeled by the system of differential equations

$$dS/dt=-0.001SI$$

$$dI/dt=0.001SI-0.3I$$

a) What is the maximum number of children sick at any one moment?

b) According to our model, how many susceptible children will avoid getting chicken pox while the epidemic runs its course?

A. Solution

We know I maximum when dI/dS=0 and I is maximum at s therefore S=300

$$dI/dS=(dI/dt)/(dS/dt)$$
$$=0.001SI-0.3I/-0.001SI$$
$$=-1+\{0.3/0.001S\}$$
$$=-1+\{300/S\}$$

Separate variables and integrate to obtain as a function of f.

$$dI=\{-1+(300/S)\}dS$$
$$\int dI=\int(-1+\frac{300}{s})dS$$
$$\int dI=\int(-1)dS+300\int\frac{ds}{s}$$
$$I=-S+300\log(800)+c$$

when I=1,S=800 and t=0 we get c

$$1=-800+300\log(800)+c$$
$$c=801-300\log(800)\approx-1204.38$$
$$I(S)\approx-S+300\log S-1204$$
$$I(S)=-300+300\log(300)-1204$$
$$\approx 207$$

- There were at most about 207 children sick at one time.
- Susceptible children will avoid getting chicken pox while the epidemic runs its course Therefore, we set I= 0 and solve for S. Using a calculator, we find S≈70. At the end of the epidemic about 70 children will still be susceptible to chicken pox; 730 children will caught the disease.

VII. Conclusion

Using epidemic model we can find the rate of population growth and calculate the numbers of susceptibles, infectives and removed at any time. It is possible to find infectives S,R mathematically and graphically by using MATLAB. This information is used to make precautions in order to control the spreading of disease.

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