Performance Analysis of DOA Estimation under Coherent Signal Conditions

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Abstract— We present an analysis of a "spatial smoothing" preprocessing scheme, suggested by Evans et al., to circumvent problems encountered in direction-of-arrival estimation of fully correlated signals. Simulation results shows the comparison of MUSIC with and without spatial smoothing.

Key words: ULA, conventional MUSIC, spatial smoothing, FOSS, FBSS

I. Introduction

Recently, there has been a growing interest in high resolution eigen structure techniques for direction-of-arrival estimation. These methods are Pisarenko, MUSIC, ESPRIT, Min norm are known to yield high resolution and asymptotically unbiased estimates, even in the case that the sources are uncorrelated or partially correlated. Theoretically, these methods encounter difficulties only when the signals are perfectly correlated. In practice, however, significant difficulties arise even when the signals are highly correlated as happens, for example, in multipath propagation or in military scenarios involving smart jammers. The perfect correlation case, referred to as the coherent case, serves as a good model for the highly correlated case [2].

The MUSIC algorithm works on the premise that the signals impinging on the array are not fully correlated or coherent. Only under uncorrelated conditions does the source covariance matrix Rss satisfy the full rank condition which is the basis of the MUSIC Eigen decomposition. The performance of MUSIC degrades severely in a coherent or highly correlated signal environment as encountered in multipath propagation where multiple versions of the same signal arrive within the resolvable chip or symbol duration. Many modifications to the MUSIC algorithm have been proposed to make it work in the presence of coherent signals. Many of these techniques involve modification of the covariance matrix through a preprocessing scheme called spatial smoothing [2].

One method of spatial smoothing proposed by Evans and further expanded by Shan is based on averaging the covariance matrix of identical overlapping arrays. This method requires an array of identical elements built with some form of periodic structure, such as the uniformly spaced linear array. An adaptive spatial smoothing technique was proposed by Takao and Kikuma, which is useful for interference cancellation in multipath environments. Another form of spatial smoothing proposed by Haber and Zoltowski involves moving the entire array structure during the time interval in which the covariances are estimated. A similar technique based on moving the array was proposed by Li and Compton. Spatial smoothing techniques always impose restrictions on the type and structure of the array. For the general case coherent signal detection involves employing a multidimensional search through all possible linear combination of steering vectors to find those orthogonal to the noise subspace [2].

II. Problem Statement

Consider a uniform linear array composed of N identical sensors. Let r (r < N) narrowband plane waves, centered at frequency $\omega 0$, impinge on the array from directions {$\theta 1$, $\theta 2$, …, θ }. Using complex (analytic) signal representation [10], the received signal at the ith sensor can be expressed as

$$x_i(t) = \sum_{k=1}^{r} a_k s_k(t) e^{-j\omega_0 (i-1)\sin\theta_k {}^{d}/_{c}} + n_i(t)$$

where, in fairly common notation, $s_k(.)$ is the signal of the kth wavefront, ak is the complex response of the sensor to the kth wavefront, d is the spacing between the sensors, c is the propagation speed of the wavefronts, and $n_i(.)$ is the additive noise at the ith sensor [8].

We assume that the noises are assumed to be uncorrelated with the signals and uncorrelated between themselves, and to have identical variance σ^2. Rewriting above equation in vector notation, assuming for simplicity that the sensors are omnidirectional, i.e., $a_k = 1$, we obtain

$$x(t) = \sum_{i=1}^{r} a(\theta_i) s_i(t) + n(t)$$

Where, x(t) is the $N \times 1$ vector and $a(\theta_i)$ is the "steering vetor" of the array in the direction θ_i.
To further simplify the notation,we rewrite equation as

$$x(t) = As(t) + n(t)$$

Where s(t) is the $r \times 1$ vector and A is the $N \times r$ matrix. Thus ,

$$E[x(t)x^H(t) = R = ASA^H + \sigma^2 I]$$
$$S = E[s(t)s^H(t)]$$

Where H denotes the conjugate transpose . Notice that S is the diagonal when the signals are uncorrelated, nondiagonal and non singular when the signals are partially correlated, and non diagonal but singular when some signals are fully correlated (or coherent). If s is nonsingular, then the rank of ASA^H is r. If $\{\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_N\}$ and $\{v_1, v_2 \ldots v_N\}$ are the eigenvalues and the corresponding eigenvectors of R, then the above rank properties imply that [8]
The minimal eigenvalue of R is equal to σ^2 with multiplicity N-r :$\lambda_{r+1} = \lambda_{r+2} \ldots = \lambda_N = \sigma^2$

The eigenvectors corresponding to the minimal eigenvalue are orthogonal to the columns of the matrix A, namely, to the "direction vectors" of the signals[8]

$$\{v_{r+1}, \ldots, v_N\} \perp \{a(\theta_1), \ldots, a(\theta_r)\}$$

We shall refer to the subspace spanned by the eigenvectors corresponding to the smallest eigenvalue as the "noise" subspace, and to its orthogonal complement, spanned by the “direction vectors” of the signals, as the “signal” subspace [10].

The high resolution eigenstructure techniques are based on the exploitation of properties 1) and 2) above. Unfortunately, these properties hold only when the covariance matrix of the sources S is nonsingular. Different relations hold when S is singular. Assume, for simplicity, that the rank of S is r - 1. This implies that two signals, say the first two, are coherent, i.e.,$s_2(t) = \alpha s_1(t)$, with α denoting a complex scalar describing the gain and phase relationship between the two coherent signals. In this case, we can rewrite as $(t) = As(t) + n(t)$
Where s(t) is the (r-1)×1 vector
$s(t) = [(1+\alpha) + s_1(t), s_3(t), \dots, s_r(t)]^T$ and A is the (r-1)×m matrix
A=$[a(\theta_1) + \alpha a(\theta_2), a(\theta_3), \dots, a(\theta_r)]$
Now the covariance matrix of r(t) also can be written as

$$R = ASA^H + \sigma^2 I$$

Now S, covariance matrix of the modified signals, is a (r -1) X (r-1) nonsingular matrix and A is of full column rank. Therefore, in complete analogy to properties 1) and 2) above, we have 1) the multiplicity of smallest eigenvalue is N - (r -1); 2) the eigenvectors corresponding to the minimal eigenvalue are orthogonal to the columns of the matrix A. Because of their Vandermonde structure, note that the first column of A is no longer a legitimate steering vector since no linear combination of two “direction vectors” can yield another steering vector.

The results of a straightforward application of the eigen structure technique to R can now be easily understood. First, because the multiplicity of the smallest eigenvalue of R is now r - 1, the detection step will give r - 1 as the number of signals. Second, since only the “direction vectors” corresponding to $\{\theta_3, \dots, \theta_r\}$ are included in the “signal” subspace, only these directions-of-arrival will be resolved in the estimation step.

In general, if m out of the r wavefronts are coherent, the application of the conventional eigenstructure technique will result in an inconsistency while the number of signals detected will be r–m+1, only r-m directions-of-arrival, corresponding to the incoherent wavefronts, will be resolved. Thus, if only one group of coherent signals exists, the difference between the number of signals detected and the number of signals resolved will be indicative of the size of the coherent group.

III. SPATIAL SMOOTHING TECHNIQUES

A. *Forward Averaging Spatial Smoothing(FOSS)*

The nonsingularity of the covariance matrix of the signals is the key to a successful application of the eigenstructure technique. Let a uniform linear array with N identical sensors {1,…,N} be divided into overlapping subarrays of size p, with sensors {1,…, p} forming the first forward subarray, sensors {2,…, p+1} forming the second forward subarray, etc. (see Fig. 1). Let $x_k(.)$ denote the vector of received signals at the kth subarray. Following the notation, we can write

$$x_k^f(t) = AF^{k-1}s(t) + n_k(t)$$

Where $F^{(k)}$ denotes the kth power of the $r \times r$ diagonal matrix

$$F = diag\{e^{-\frac{j\omega_0 \sin\theta_1 d}{c}}, \dots, e^{-\frac{j\omega_0 \sin\theta_r d}{c}}\}$$

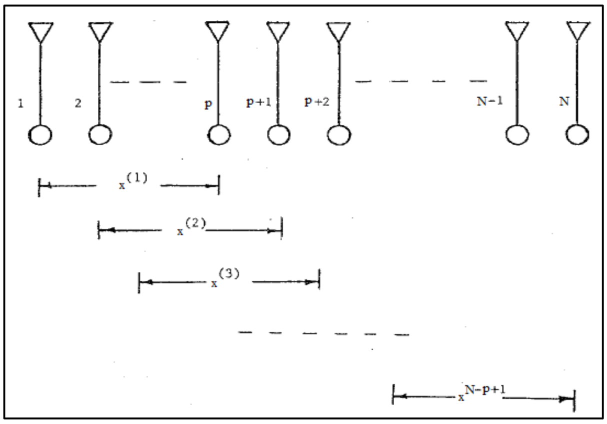

Fig. 1: Subarray Spatial Smoothing

The covariance matrix of the kth forward subarray is therefore given by

$$R_k^f(t) = AF^{(k-1)}R_{ss}F^{H(k-1)}A^H + \sigma^2 I$$

Where R_{ss} is the covariance matrix of the source.

The forward averaged spatially smoothed covariance matrix is defined as the sample means of the subarray covariances:

$$R^f = \frac{1}{M}\sum_{k=1}^{M} R_k^f$$

Where M=N-p+1 is the number of subarrays.We can rewrite above equation as

$$R^f = A\left(\frac{1}{M}\sum_{k=1}^{M} F^{(k-1)}R_{ss}F^{H(k-1)}\right)A^H + \sigma^2 I$$

$$R^f = AR_{ss}^f A^H + \sigma^2 I$$

Where R_{ss}^f is the modified covariance matrix of the signals, given by

$$R_{ss}^f = \frac{1}{M}\sum_{k=1}^{M} F^{(k-1)}R_{ss}F^{H(k-1)}$$

For M≥r the covariance matrix R_{ss}^f will be nonsingular regardless of the coherence of the signals. This robustness comes at the expense of a reduced effective aperture. To see this more quantitatively, consider the number of sensors needed to cope with r coherent wavefronts, Recalling that the number of subarrays, given by M = N - p + 1, must be greater than or equal to r, and that the size of each subarray p must be at least r + 1, it follows that the minimum number of sensors needed is N = 2r. Comparing this to N = r + 1 for the conventional case, it is clear that we trade off half the effective aperture [10].

The price paid for detection of coherent signals using forward averaging spatial smoothing is the reduction in the array aperture. An N element array can detect only M/2 coherent signals using MUSIC with forward averaging spatial smoothing as opposed to M-1 non coherent signals that can be detected by conventional MUSIC[8][10].

B. *Forward/Backward Spatial Smoothing(FBSS)*

Pillar and Kwon proved that by making use of a set of forward and conjugate backward subarrays simultaneously. It is possible to detect up to 2N/3 coherent signals [4]. In this scheme, in addition to splitting the array into overlapping forward subarrays. It is also split into overlapping backward arrays such that the first backward subarray is formed using elements {N, N-1… N-p+1}. The second subarray is formed using elements {N – 1, N- 2, ...,N–p},and so on.

Now, the complex conjugate of the received signal vector at the kth backward subarray can be expressed as

$$x_k^b = \left[x_{N-k+1}^*, x_{N-k}^*, \dots, x_{p-k+1}^*\right]^T$$

$$= AF^{(k-1)}(F^{N-1}s^*) + n_k^* \qquad 1 \le k \le M$$

The covariance matrix of the kth backward subarray is therefore given by

$$R_k^b = AF^{(k-1)}R_{ss}F^{H(k-1)}A^H + \sigma^2 I$$

Where,

$$R_{ss} = F^{-(N-1)}E[s^* s^T]\left(F^{-(N-1)}\right)^H$$

$$=F^{-(N-1)}R_{ss}^*\left(F^{-(N-1)}\right)^H$$

Now the spatially smoothed backward subarray matrix R^b can be defined as

$$R^b = \frac{1}{M}\sum_{k=1}^{M} R_k^b = AR_{ss}^b A^H + \sigma^2 I$$

It can be shown that the backward spatially smoothed covariance matrix R^b will be of full rank as long as R_{ss}^b is non-singular, and the non-singularity of R_{ss}^b is guaranteed whenever $M \ge r$.

Now the forward/conjugate backward smoothed covariance matrix R is defined as the mean of R^f and R^b , i.e.

$$\hat{R} = \frac{R^f + R^b}{2}$$

Using an N element array, applying MUSIC on $\hat{R}$, it is possible to detect up to 2M/3 coherent signals.

IV. Simulation Results

Figure 2 shows a comparison between conventional MUSIC and MUSIC with forward/backward spatial smoothing in a coherent multipath signal environment.

Simulations with three coherent signals impinging on a 6-element uniform linear array at 60, 70, and 120 degrees show that MUSIC fails almost completely, whereas with a spatial smoothing preprocessing scheme. All of the three multipath signals are detected clearly.

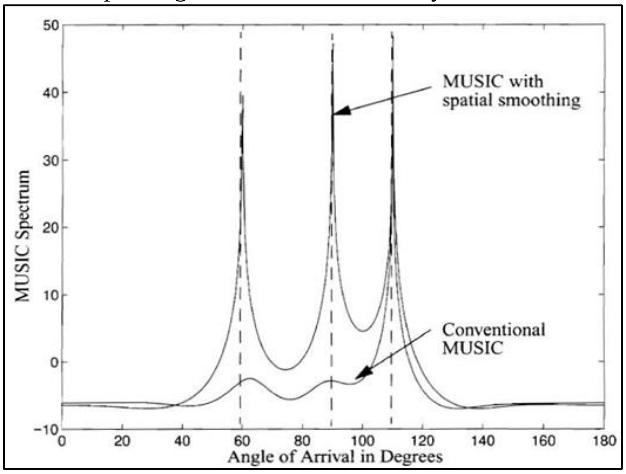

Fig. 2: Comparison of MUSIC with and without forward/backward averaging in coherent multipath. (Three coherent signals of equal power with SNRs = 20 dB, 6-element ULA, d=.5λ, θ = 60, 90, and 120 degrees respectively)

Another example we considered had three (r = 3) planar wavefronts at directions-of-arrival $-30°, 45°$ and $60°$. The first two signals were coherent, while the third signal was not correlated with the others. The array was uniform and linear, with 8-elements a half wavelength apart. The signal-to-noise ratio was 3 dB, and the number of samples ("snapshots") taken from the array was 1024. Applying the MUSIC eigenstructure method, we obtained the results shown in Fig. 3.

Only one dominant peak corresponding to the direction-of-arrival of the third signal is seen. The directions-of arrival of the two coherent signals were not resolved. However, first applying the spatial smoothing preprocessing scheme with three (M = 3) subarrays of four (p = 6) sensors each, and then applying the MUSIC eigenstructure method to the spatially smoothed covariance matrix yielded the results shown in Fig. 3. In this case, the three peaks corresponding to the directions-of-arrival of all the three signals are clearly seen.

Simulation parameter	Conventional MUSIC	Spatially Smoothed MUSIC
Array Geometry	ULA	ULA
Number of sensors	N=8	N=8
Number of sources	r=3	r=3
Number of snapshot	1024	1024
Spacing between elements	0.5λ	0.5λ
SNR	3dB	3dB
Angle(θ)	[-30 45 60]	[-30 45 60]
Number of coherent/Noncoherent signal	2/1	2/1
Number of Subarray	-	M=3
Each Subarray Size	-	p=6

Table 1: Simulation parameter and its value for MUSIC and Spatially Smoothed MUSIC

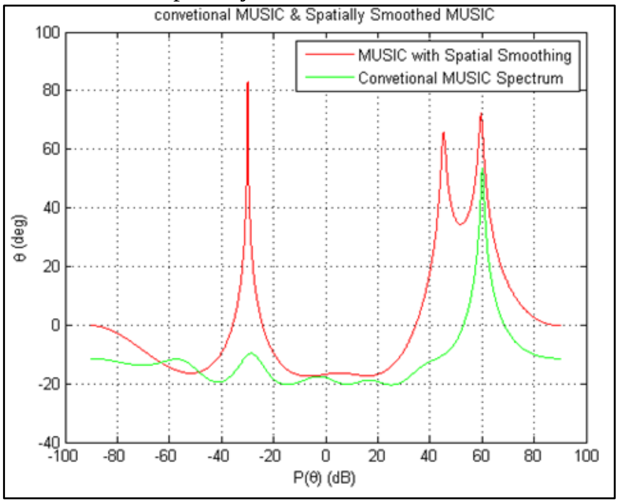

Fig. 3: comparison of conventional MUSIC spectrum and spatially smoothed MUSIC spectrum (8-sensors; subarray size=3; SNR = 3 dB; 1024 "snapshots"; two coherent narrow-band source from $-30°$, 45°; one incoherent source from 60°)

V. CONCLUSION

In these paper we have seen that MUSIC algorithm completely fails in the coherent signal conditions whereas it can detect coherent signals more clearly after applying spatial smoothing pre-processing schemes. Fig 2 and Fig 3 shows that conventional MUSIC can detect incoherent signals clearly, while applying FBSS and FOSS to MUSIC algorithm it can also be able to get the peaks in coherent signal conditions also.

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