Super Mean Labeling and Square Difference Labeling of Some Graphs

J. Ravi[1] S. Dickson[2] H. Sabareesh[3] J. Mohan[4] G. Sathish Kumar[5]
[1,2,3,4,5]Assistant Professor
[1,2,3,4,5]Department of Mathematics and Statistics
[1,2,3,4,5]Vivekanandha College For Women, Tiruchengode, Namakkal, Tamilnadu, India

Abstract— In this paper a study on super mean labeling and square difference labeling of some graphs. Let G be a (p,q) graph and f:v(G)→{1,2,3…p+q} be an injection for each edge e=uv ,let f*(e)=((f(u)+f(v)))/2 if f(u)+f(v) is even and f*(e)=(f(u)+f(v)+1)/2 if f(u)+f(v) is odd then f is called a super mean labeling if f(v)∪{f*(e):e϶E(G)}={1,2,3….P+q}. A graph that admits a super mean labeling is called a super mean graph. The prove that s(pn,k1),s(p2*p4),s(Bn,n),<Bn,n;pm>,Cn.k2,n≥3.Generalizd ant prism {An} and the double triangular snake D(Tn) are super mean graph. We proved that some new graphs admit square difference labeling. A function f is called a square difference labeling if there exists a bijection f: V(G)→{0,1,2,….p-1} such that the induced Function f*:E(G) → N given by f*(u v) = |[f(u)]2 - [f(v)]2| for every uv∈E(G) are all distinct. any graph which satisfies the square difference labeling is called a square difference graph .it is investigated that the central graphs of path, square graphs of path some path related graphs fan and gear graphs are all square difference graph.

Key words: Super Mean Labeling, Square Difference Labeling, Graph and Bijection

I. INTRODUCTION

Graph theory is an important branch of mathematics with relevance and application to the entire field. There are several reasons for the acceleration of interest in graph theory. The fact is that graph servers as a mathematical model for any system involving binary relation. Because of their diagrammatic representation, graphs have an intuitive and aesthetic appeal. Labeling of a graph is an assignment of labels to its vertices (or) and edges (or) faces, which satisfy some condition. In thirteenth century, Chinese mathematician Yang Hui and Oyhers (1275) have studied labeling of geometric figures which are later called plane graphs. Later Change Choo (1670), PaoChhi – Shou (1880), Li Nien also contributed to this area. Most graph labeling methods trace their origin to one introduced by Rosa (875) in 1967 or one given by Graham and Sloane (445) in 1980. Labeling graphs serves as useful modes for a broad range of applications such as coding theory, Radar astronomy, circuit designs and communication network. All graphs in this dissertation are finite, simple and undirected. The symbols V (G), E (G) will denote the vertex set and edge set of the graph (G). The cardinality of the edge set is called the size of G, A Graph with p vertices and q edges is called a (p,q) graph.

II. PRELIMINARIES

A. *Definition: 2.1*

A graph $G = \{V, E\}$ consists of a set of objects $V = \{v_1, v_2, \ldots, v_n\}$ called vertices and another set $E = \{e_1, e_2, \ldots \ldots e_n\}$ whose elements are called edges such that each ek is identified with an unordered pair of vertices(v_i, v_j). The vertices v_i, v_j associated with edge $e_k$are called end vertices ofe_k.

1) *Example: 2.1*

Let $V = \{v_1, v_2, v_3, v_4, v_5\}$ and $x = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9, e_{10}\}$then $G = (V, X)$is a (5, 10) graph.

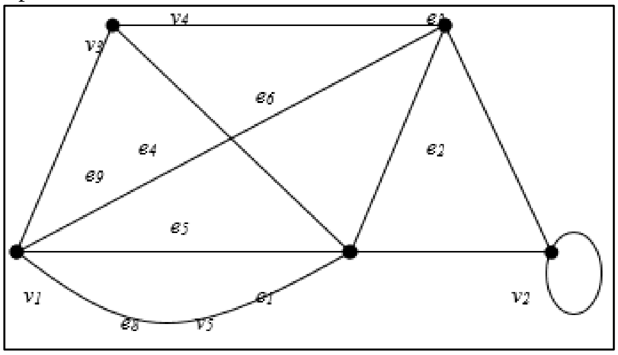


Fig. 2.1: Simple Graph

B. *Definition: 2.2*

If more than one edge associated with given pair of vertices, then such edges are referred to parallel edges or multiple edges.

1) *Example: 2.2*

In figure 1.1, e_1 and e_7 are parallel edges.

C. *Definition: 2.3*

A simple graph is a graph having no loops and multiple edges.

D. *Definition: 2.4*

A walk of a graph G is an alternating sequence of vertices and edges $v_0, x_1, v_1 \ldots . v_{n-1}, x_n, v_n$ beginning and ending vertices such that each edge is incident with the vertices preceding and following it.

1) *Example: 2.4*

In figure 1.1, $v_1e_1v_2e_2v_3e_3v_4e_4v_1e_5v_3$ is a walk.

E. *Definition: 2.5*

A walk which begins and ends at the same vertex is called a closed walk.

1) *Example: 2.5*

In figure 1.1, $v_1e_1v_2e_2v_3e_3v_4e_4v_1$ is closed walk.

F. *Definition: 2.6*

The degree of a vertex v_i of a graph, denoted by deg (v_i), is the number of edges incident with that vertex.

1) *Example: 2.6*

In figure 1.1, degree$(v_1) = 4$.

G. *Definition: 2.7*

A graph G is said to be connected if there is atleast one path between every pair of vertices in G, otherwise G is disconnected.

1) Example: 2.7

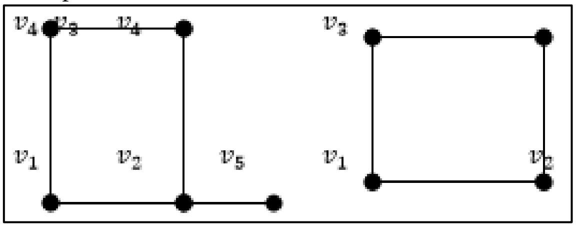


Fig. 2.2: Disconnected Graph

H. Definition: 2.8

A simple graph in which there exists an edge between every pair of vertices is called a complete graph or universal graph. The complete graph with p points is denoted by k_p.

1) Definition: 2.9

A bi-graph (or) bipartite graph is one whose vertex set can be partitioned into 2 subsets $v_1$and v_2 such that each edge has one end in v_1 and another end in v_2.

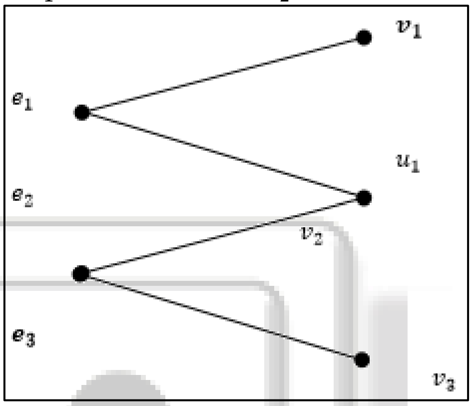


Fig. 2.3: Bipartite Graph

I. Definition: 2.10

A graph is said to be labeled graph is its end vertices and its edges are distinguished from one another by some labels, such as letters (or) numbers.

Example: 2.10

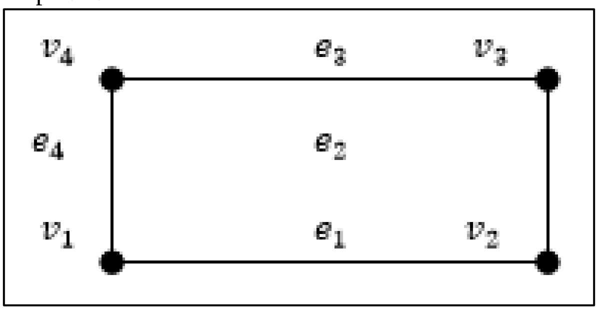


Fig. 2.4: Labeled Graph

J. Definition: 2.11

Let G_1 = (v_1,x_1) and G_2= (v_2,x_2) be 2 graph with v_1∪ v_2 =Φ.

We define, The union G_1∪ G_2to be (V,X) where V=V_1 ∪ V_2and X= X_1∪ 〖X〗_2.

The sum G1+G2 as G_1 ∪ G_2 together with all the edges joining vertices of v_1 to vertices of v_2.

1) Example: 2.11

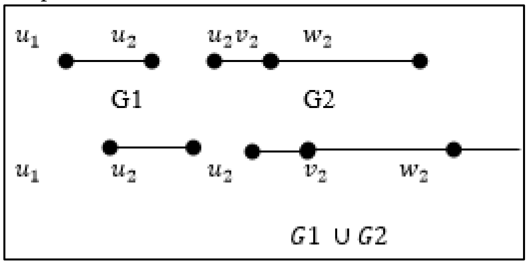


Fig. 2.5: G1 ∪G2

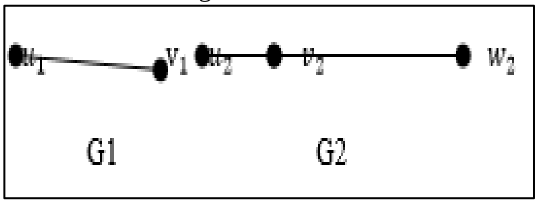


Fig. 2.6: G1 ∪ G2

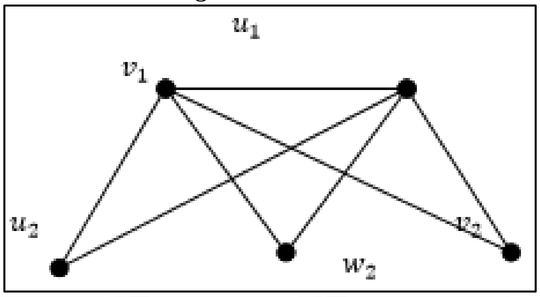
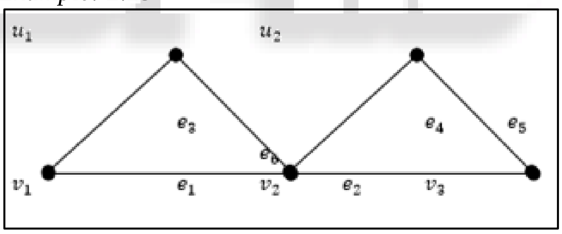


Fig. 2.7: G1 + G2

K. Definition: 2.12

A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions.

1) Example: 2.13



L. Definition: 1.13

A triangular snake is a connected graph in which each block is a triangle and the block cut point is path.

III. SUPER MEAN LABELING OF SOME GRAPHS

A. Definition: 3.1

A function f is called a mean labeling of G if f:V (G) {0,1,2,3….,q} is injective and the induced function. f * : E(G) ⟶ {1,2,……..q} defined as Let $f^*(e)=\frac{f(u)+f(v)}{2}$ if f (U) + f(v) is even $f^*(e) = \frac{f(u)+f(v)+1}{2}$ if f(U) + f (v)is odd is bijective. The graph which admits mean labeling is called a mean graph.

B. Definition: 3.2

Let G be a (p, q) graph and f: V(G)⟶{1, 2, 3,……., p + q } be an injection. For each edge e=uv.Let $f^*(e)=\frac{f(u)+f(v)}{2}$ if f (U) + f(v) is even $f^*(e) = \frac{f(u)+f(v)+1}{2}$ if f (U) + f (v)is odd.

Then f is called super mean labeling if f (V(G)) U { f*(e) : e€E (G)} = {1, 2, 3,………., p + q}. A graph that admits a super mean labeling is called super mean graph.

1) Example 3.1

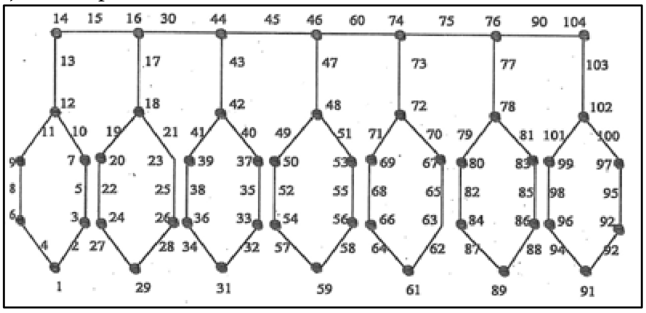


Fig. 3.1: Super mean labeling. (P_7: C_6)

C. *Definition: 3.3*

Let u_1, u_2,……….,u_nu_i be the cycle C_m. Add vertices V_1such that V_i is adjacent to u_i, $1 \leq i \leq m$. The resultant graph is C_m □K_1. Let w_1, w_2,………,w_n be the path P_n. Let G = (C_m□ K_1) UP_n whose edge set is E= { U_1 U_{i+1}, U_mU_1 /$1\leq i \leq m$ -1 } U {w_1 w_{i+1}/$1\leq i \leq n$-1} U. { U_i V_i / $1 \leq i \leq m - 1$ }

1) Theorem: 3.1

(C3 □ K1) U Pn is a super mean graph for all $n \geq 2$

a) Proof

(C3 □ K1) U Pn is a graph with the vertex set

V[(C3 □ K1) U Pn] = { ui ; ≤ i ≤ n}and the edge set

E[(C3 □ K1) U Pn] = {ei ; 1 ≤ i ≤ 6 } U{éi: 1≤ i ≤n}

The ordinary labeling of (C3 □ K1) U Pn

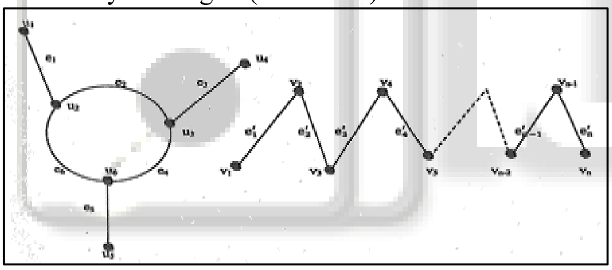


Fig. 3.2: ordinary labeling of (C3 □ K1) U Pn

We label the vertices of (C3 □ K1) U Pn, f (u1) = 1;

f (u2) = 3;

f (u3) = 5; f (u4) = 7;

f (u5) = 10

f(u6) = 12; f (Vi) = 2i + 11, i = 1 to n

Now the induced edge tables are as follows

f* (e1) = 2 ; f* (e2) = 4; f* (e3) = 6 ; f* (e4) = 9; f* (e5) = 11; f* (e6) = 8; f* (e1') = 2i + 12, i = 1 to n and Clearly, f (v) U { f* (e): e € E(G)} = { 1, 2, 3, …….., p + q }.

So, f is a super mean labeling. Hence, (C3 □ K1) U Pn is a super mean graph for all $n \geq 2$

1) Example: 2.3

Super mean labeling of (C3 □ K1) UP10 is give in figure 3.4

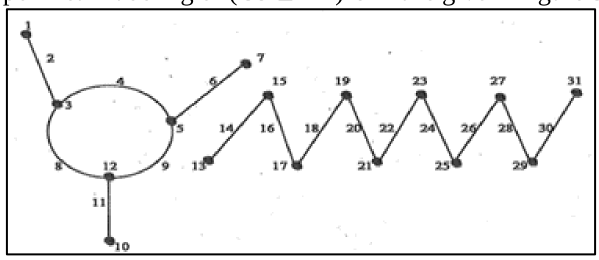


Fig. 3.3: Super Labeling of (C3 □ K1) UP10

2) Example: 3.4

Super mean labeling of (C3 □ K1)

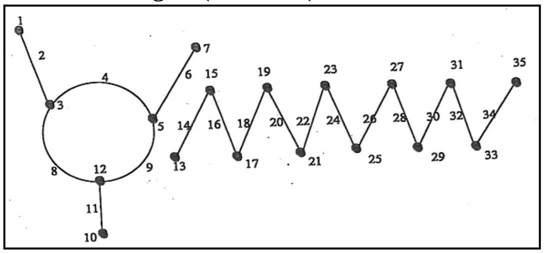


Fig. 3.4: Super labeling of (C_3 □ K_1) UP_{12}

Example: 3.5

Super labeling of (C_3 □ K_1) UP_{14} is given in figure 2.6

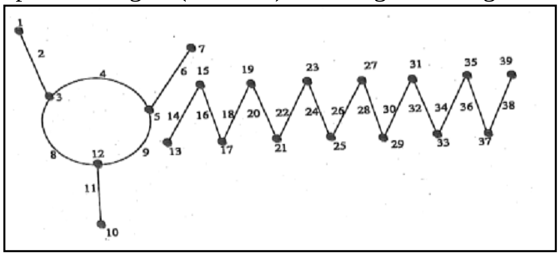


Fig. 3.5: Super labeling of (C_3 □ K_1) UP_{14}

D. *Definition: 3.4*

The graph (P_n ; C_3); the splitting graph is obtained by adding to each vertex v a new vertex V. So that V' is a adjacent to every vertex is adjacent to V in (P_n ; C_3). The graph obtained by joining two copies of (P_n ; C_3) splitting graph.

1) Theorem: 3.2

(P_n ; C_3) is a super mean graph for all $n \geq 2$.

a) Proof:

(P_n ; C_3) is a graph with vertex set

V (P_n ; C_3) = { v_i : $1 \leq i \leq n$} U { v_i': $1 \leq i \leq n$} U { u_i : $1 \leq i \leq n$}U { u_1': $1 \leq i \leq n$} and the edge set

E (P_n; C_3) = {e_i: $1 \leq i \leq n - 1$} U { e_i': $1 \leq i \leq n$} U { e_1'': $1 \leq i \leq n$} U{ e_1''': $1 \leq i \leq n$} U { e_1^{IV}: $1 \leq i \leq n$ }

The ordinary labeling of (P_n ; C_3) is given in figure 2.7

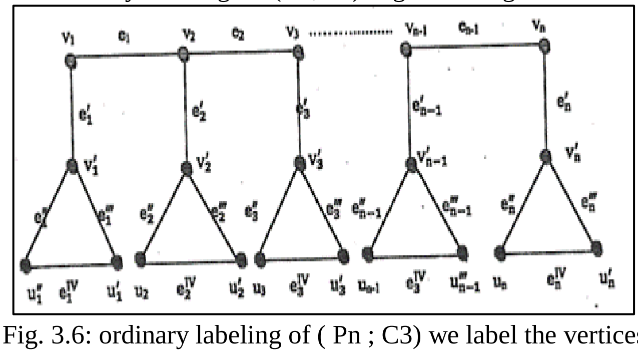


Fig. 3.6: ordinary labeling of (Pn ; C3) we label the vertices of (Pn ; C3) as follows.

$$f(v_i) = \begin{cases} 9i - 1, & i \text{ is odd, } 1 \leq i \leq n - 1 \\ 9i - 8, & i \text{ is even, } 2 \leq i \leq n \end{cases}$$

$$f(v_i') = \begin{cases} 9i - 3, & i \text{ is odd, } 1 \leq i \leq n - 1 \\ 9i - 6, & i \text{ is even, } 2 \leq i \leq n \end{cases}$$

$$f(u_i) = \begin{cases} 9i - 8, & i \text{ is odd, } 1 \leq i \leq n - 1 \\ 9i - 4, & i \text{ is even, } 2 \leq i \leq n \end{cases}$$

$$f(u_i'') = \begin{cases} 9i - 6, & i \text{ is odd, } 1 \leq i \leq n - 1 \\ 9i - 1, & i \text{ is even, } 2 \leq i \leq n \end{cases}$$

Now the induced edge labels are as follows

$f^*(e_i) = 9i,\ i = 1 \text{ to } n$

$$f^*(e_i') = \begin{cases} 9i-2, & i \text{ is odd},\ 1 \le i \le n-1 \\ 9i-7, & i \text{ is even},\ 2 \le i \le n \end{cases}$$

$$f^*(e_i'') = \begin{cases} 9i-2, & i \text{ is odd},\ 1 \le i \le n-1 \\ 9i-5, & i \text{ is even},\ 2 \le i \le n \end{cases}$$

$$f^*(e_i''') = \begin{cases} 9i-4, & i \text{ is odd},\ 1 \le i \le n-1 \\ 9i-3, & i \text{ is even},\ 2 \le i \le n \end{cases}$$

$$f^*(e_i^{IV}) = \begin{cases} 9i-7, & i \text{ is odd},\ 1 \le i \le n-1 \\ 9i-2, & i \text{ is even},\ 2 \le i \le n \end{cases}$$

Clearly, f (v) U { f * (e); e € E (G)} = { 1, 2, 3, …….. p + q}
So f is a super mean labeling.
Hence, ($P_n : C_3$) is a super mean graph for all $n \ge 2$.

2) *Example: 3.6*

$$f(v_i) = \begin{cases} 9i-1, & i \text{ is odd},\ 1 \le i \le n-1 \\ 9i-8, & i \text{ is even},\ 2 \le i \le n \end{cases}$$

$$f(v_i') = \begin{cases} 9i-3, & i \text{ is odd},\ 1 \le i \le n-1 \\ 9i-6, & i \text{ is even},\ 2 \le i \le n \end{cases}$$

$$f(u_i) = \begin{cases} 9i-8, & i \text{ is odd},\ 1 \le i \le n-1 \\ 9i-4, & i \text{ is even},\ 2 \le i \le n \end{cases}$$

$$f(u_i'') = \begin{cases} 9i-6, & i \text{ is odd},\ 1 \le i \le n-1 \\ 9i-1, & i \text{ is even},\ 2 \le i \le n \end{cases}$$

$f^*(e_i) = 9i,\ i = 1 \text{ to } n$

$$f^*(e_i') = \begin{cases} 9i-2, & i \text{ is odd},\ 1 \le i \le n-1 \\ 9i-7, & i \text{ is even},\ 2 \le i \le n \end{cases}$$

$$f^*(e_i'') = \begin{cases} 9i-2, & i \text{ is odd},\ 1 \le i \le n-1 \\ 9i-5, & i \text{ is even},\ 2 \le i \le n \end{cases}$$

$$f^*(e_i''') = \begin{cases} 9i-4, & i \text{ is odd},\ 1 \le i \le n-1 \\ 9i-3, & i \text{ is even},\ 2 \le i \le n \end{cases}$$

$$f^*(e_i^{IV}) = \begin{cases} 9i-7, & i \text{ is odd},\ 1 \le i \le n-1 \\ 9i-2, & i \text{ is even},\ 2 \le i \le n \end{cases}$$

Super mean labeling ($P_4 : C_3$) is given in figure 2.10.

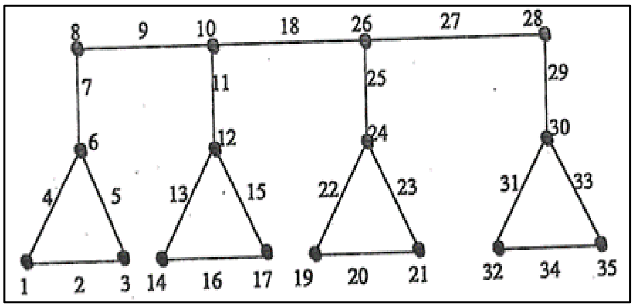


Fig. 3.7: Super mean labeling of ($P_4 : C_3$)

3) *Example: 3.7*

Super mean labeling ($P_5 : C_3$)is given in figure 3.8

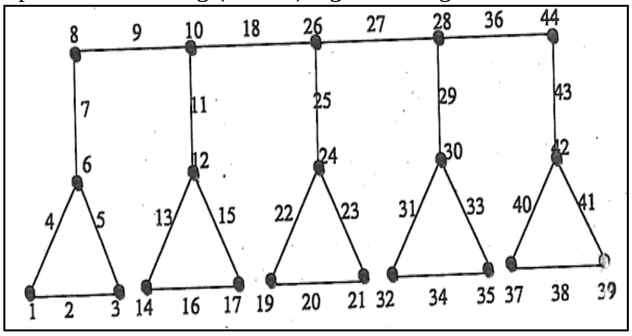


Fig. 3.8: Super mean labeling of ($P_5 : C_3$)

4) *Example: 3.8*

Super mean labeling ($P_6 : C_3$) is given in figure 3.10

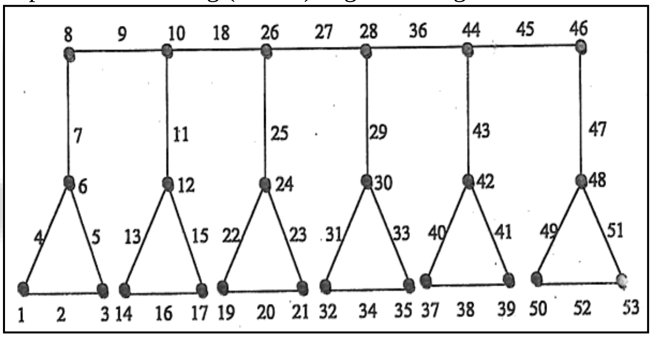


Fig. 3.9: Super mean labeling of ($P_6 : C_3$)

E. *Definition: 3.5*

Let $U_1 U_2 \ldots\ldots U_m U_1$ be the cycle C_m and let V_i be the vertex which is joined to the vertex U_i, $1 \le i \le m$, of the cycle C_m. The resultant graph is $C_m \square K_1$. Let $S_1, S_2, \ldots\ldots\ldots S_n$be the path P_n and let t_i be the vertex which is joined to the vertex S_i, $1 \le i \le n$, of the path P_n. Graph is $P_n \square K_1$. The resultant graph is G= ($C_m \square K_1$) U ($P_n \square K_1$).

1) *Theorem: 3.3*

($C_3 \square K_1$) U ($P_n \square K_1$) is a super mean graph for all $n \ge 2$.

a) Proof:

($C_3 \square K_1$) U ($P_n \square K_1$) is a super mean graph with the vertex set V(($C_3 \square K_1$) U ($P_n \square K_1$)) = {u_i ; $\le i \le 6$ } U { v_i ; $1 \le i \le n$}; U {v_i' ; $1 \le i \le n$} the edge setE(($C_3 \square K_1$) U ($P_n \square K_1$)) = {e_i ; $\le i \le 6$ } U { v_i ; $1 \le i \le n$}; U {e_i' ; $1 \le i \le n$}

The ordinary labeling($C_3 \square K_1$) U ($P_n \square K_1$) is given in figure 3.10

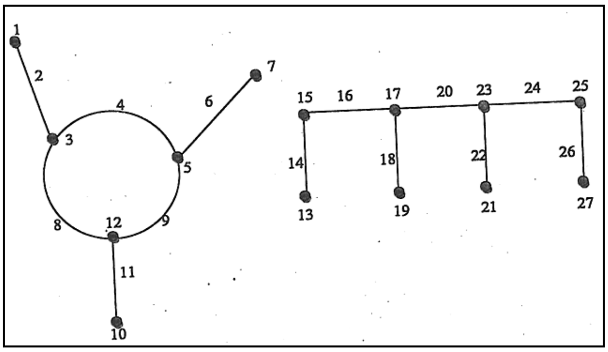


Fig. 3.10: ordinary labeling of (C_3□K_1) U (P_n□K_1)

We label the vertices of (C_3 □ K_1) U (P_n□K_1)$f(u_1)$ = 1; $f(u_2)$ = 3; $f(u_3)$ = 5; $f(u_4)$ = 7; $f(u_5)$ = 10; $f(u_6)$ = 12;

$$f(v_i) = \begin{cases} 4i + 11, \ i \text{ is odd}, \ 1 \le i \le n-1 \\ 4i + 9, \ i \text{ is even}, \ 2 \le i \le n \end{cases}$$

$$f(v_i) = \begin{cases} 9i + 9, \ i \text{ is odd}, \ 1 \le i \le n-1 \\ 4i + 11, \ i \text{ is even}, \ 2 \le i \le n \end{cases}$$

Now the induced edge tables are as follows

$f^*(e_1) = 2$; $f^*(e_2) = 4$; $f^*(e_3) = 6$; $f^*(e_4) = 9$; $f^*(e_5) = 11$; $f^*(e_6) = 8$

$f^*(e_1^{'}) = 4i + 12$, i = 1 to n ; $f^*(e_1^{'''}) = 4i + 10$, i = 1 to n

Clearly, $f(v)$ U { $f^*(e)$: e € E(G)} = { 1, 2, 3, ……., p + q }. So, f is a super mean labeling. Hence (C_3 □ K_1) U (P_n□K_1) is a super mean graph for all $n \ge 2$.

2) Example: 2.9

Super mean labeling of (C_3 □ K_1) U (P_n□K_1) is given in figure 3.12.

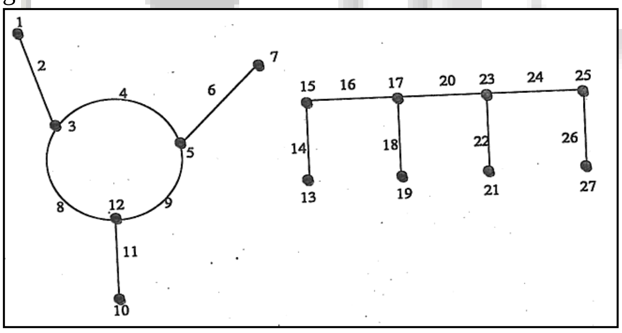


Fig. 3.11: super mean labeling (C_3 □ K_1) U (P_n□K_1)

3) Example: 2.10

Super mean labeling of (C_3 □ K_1) U (P_n□K_1) is given in figure 3.13

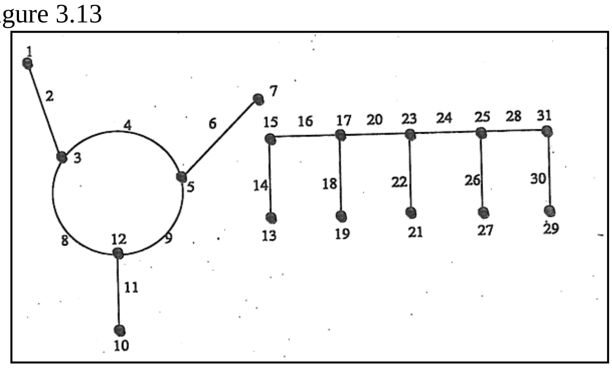


Fig. 3.12: Super mean labeling of (C_3 □ K_1) U (P_n□K_1)

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