Estimation of Frequency Offset in Wireless transceivers using Subspace Fitting

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Abstract— Wireless devices in future have to support many applications with low latency and more reliability. Optimizing wireless communication transmitters and receivers with different characteristics of circuit has more hardware limitations. Exchanging between transmitter and receiver wireless operations time division multiplexing (TDM) for a given source voltage load can change. The quick change in load results in a change in source voltage which effects carrier frequency offset. This offset could not be calculated using conventional methods. This paper presents calculation of transient offset frequency by considering second order control system (Under damped). The transient carrier frequency offset is compensated by parametric estimation algorithm having low complexity. This utilizes the null space of the HANKEL-like matrix obtained from angle difference of the 2 halves of the training sequence. Weighted subspace fitting algorithm is used to calculate mean square error of the estimated parameters in the presence of noise.

Key words: TDM, Carrier Frequency Offset, Hankel Matrix

I. INTRODUCTION

Communication systems are broadly classified into wire based communication and wireless based communications. Wire based communications is used to convey data from one end to another. Optical cables plays a very important role in wire based communication.. In the past few years communication has grown from low end communication to high end satellite communications.

To meet fast growing demand of wireless multimedia services, a high speed data access mechanism is required [8]. Various methods were proposed in to achieve high data [1]. Multiple-input multiple- output (MIMO) concept is the one different from smart antenna techniques developed to enhance beam forming and diversity.

II. REVIEW OF EXISTING METHODS

A. Conventional CFO Estimation

The receiver estimates the Carrier Frequency in OFDM symbol with preamble, which has identical halves and T is the time separation between the two halves [9]. The received signal with CFO and ˇn(t) is small to n(t). In conventional CFO estimation, we assume that the CFO is constant. Then, receiver computes the CFO in OFDM symbol, which has identical halves. Transmitted OFDM signal is

$$S(t)=\begin{cases} s_1(t) & 0 \le t \le T \\ s_2(t) & T \le t \le 2T \end{cases}$$

The received signal with CFO is

$$r(t)=\left(\int_{-\infty}^{+\infty} h(v)s(t-v)dv + n(t)\right)e^{\int_0^t j\Delta\omega(\tau)d\tau} + \breve{n}(t) \quad (1)$$

Where $n(t)$ and $\breve{n}(t)$are the white Gaussian noises at receiver , $h(t)$ is the impulse response of channel, $s(t)$ is the data; and $\Delta\omega(t)$ is the offset.

In conventional frequency offset computation [5], CFO is constant, and equation (1) becomes

$$r(t)=\left(\int_{-\infty}^{+\infty} h(v)s(t-v)dv + n(t)\right)e^{j\Delta\omega t} \quad 0 \le t \le 2T$$

The first half of the data received is

$$r_1(t)=\left(\int_{-\infty}^{+\infty} h(v)s(t-v)dv + n_1(t)\right)e^{j\Delta\omega t} \quad 0\le t \le T$$

and second half of the received data is

$$r_2(t)=\left(\int_{-\infty}^{+\infty} h(v)s(t-v)dv + n_2(t)\right)e^{j\Delta\omega t} \quad 0\le t \le T$$

Where $r1$ (t) and $r2$ (t) are the received data with white Gaussian noises $n1(t)$ and $n2(t)$ respectively.

Where $n1(t)$ and $n2(t)$ are the additive white noises in $r1(t)$ and $r2(t)$, respectively. The conventional offset computation has a constant phase difference. Then computation based on Maximum likelihood for CFO, denoted by $\Delta\hat{}\omega$, is

$$\Delta\hat{\omega}=\frac{\angle\left(\sum_{n=0}^{N-1} r_1^*(nT_s)r_2(nT_s)\right)}{T}$$

Where N is the length, .r1 (nTs) and r2 (nTs) are the received OFDM data and Fs=1/Ts is the Nyquist rate.

B. Model of Carrier Frequency Offset

The output voltage in a dc to dc converter, the load current is come close to a second order system with change in step. The relation between the VCO voltage and the output frequency is linear. The step response of a under-damped system second order and output is an exponentially damped sinusoid. Carrier offset, denoted by $\Delta\omega T$ (t), can be modeled as

$$\Delta\omega T(t) = \alpha e^{-\zeta}\omega_n t \, sin\left(\omega_{n\sqrt{1-\zeta^{2t+\emptyset}}}\right) \quad t > 0,0 < \zeta<1$$

Since $\zeta < 1$ second order system is under damped. Here, ζ and ωn are the damping factor and undamped natural frequency, respectively. α *and* Φ are the gain And phase of the output, respectively. The Offset estimation has two blocks: a steady state offset estimation a transient offset estimation.

Thus, received signal becomes

$$r(t) = \left(\int_{-\infty}^{+\infty} h(v)s(t-v)dv + n(t)\right)e^{\int_0^t j(\Delta\omega_T)(\tau)+\Delta\omega_T(\tau)+\Delta\omega_s)d\tau}$$

The steady state offset is determined and separated from the source signal for the determination of the carrier offset.

The received preamble OFDM symbol in two halves is

$$r_1(t) = \left(\int_{-\infty}^{+\infty} h(v)s(t-v)dv + n_1\,(t)\right)e^{\int_0^t j\Delta\omega_T(\tau)d\tau}$$

$$r_2(t) = \left(\int_{-\infty}^{+\infty} h(v)s(t-v)dv + n_2\,(t)\right)e^{\int_0^{t+T} j\Delta\omega_T(\tau)d\tau}$$

$\psi(t)$ is angle difference between two identical halves.

$$\varphi(t) = \angle\left(r_1^*(t)r_2(t)\right) = \int_t^{t+T} \Delta\omega T(\tau)d\tau + n\varphi(t)$$

Now $\psi(t)$ is approximated as an exponentially damped sinusoid.

III. ESTIMATION METHODS

A. *Estimation Based on Subspace*

Estimation of damping factor and undamped natural frequency of damped sinusoid in existing algorithms are generic and are of four types: direct fitting, covariance based linear Prediction, direct fitting with DFT based algorithms and subspace (SVD) based linear prediction. The linear estimation first create a Hankel matrix Ψ. A nonzero vector in the null space of the noise free prediction matrix constructs the coefficients of signal AR model. Then complex roots of the estimated polynomial are used to determine parameters of damped sinusoid.

$$\psi = \begin{bmatrix} \varphi(0) & \varphi(1) & \dots & \varphi(L-1) \\ \varphi(1) & \varphi(2) & \dots & \varphi(L) \\ \vdots & \vdots & \ddots & \vdots \\ \varphi(N-L-2) & \varphi(N-L-1) & \dots & \varphi(N-1) \end{bmatrix}$$

Where L is the order of prediction filter is used to maximize estimation of the performance in the problems of linear prediction subspace based like Kumaresan–Tuft (KT) method. The analysis for the KT linear prediction method is

$$Var(\omega_{s)} \approx \frac{4}{3\gamma L(N-L)^2} \quad 1 \le L \le N-L$$

Where L and N are the number of columns and signals, γ is the SNR. Let L is approximated as $L \approx N/3$ so that error estimated is minimum. The rank of noise free estimation matrix does not change by increasing L since the rank depends on order of the signal. At the same time for large values of L, the columns of estimation matrix are less correlated, so that it modified to a matrix with good condition. Now the predicted signal space and the null space become more accurate. This motivates to build the estimation matrix of type Hankel-like matrix. Time delay r is used to form the Hankel-like matrix. If there is one damped sinusoid, the time delay r *is* chosen so that the columns of the estimation matrix are less correlated. The proposed estimator provides good estimation accuracy. Select the value of r *so* that it minimizes MSE.

The proposed method of subspace based algorithm has the estimation matrix of type Hankel –like matrix.

In above equation, r is a non-negative integer time delay and $\boldsymbol{\psi_r}$ is the addition of noise and signal i.e $\boldsymbol{\psi_r} = \boldsymbol{H_r} + \boldsymbol{w_r}$, where $\boldsymbol{H_r}$ is a noise free signal matrix and $\boldsymbol{w_r}$ is a Hankel – like matrix with noise, That is,

$$\boldsymbol{w_r} = \begin{bmatrix} \boldsymbol{n_\varphi(0)} & \boldsymbol{n_\varphi(r)} & \boldsymbol{n_\varphi(2r)} \\ \boldsymbol{n_\varphi(1)} & \boldsymbol{n_\varphi(r+2)} & \boldsymbol{n_\varphi(2r+1)} \\ \vdots & \vdots & \vdots \\ \boldsymbol{n_\varphi(N-2r-1)} & \boldsymbol{n_\varphi(N-r-1)} & \boldsymbol{n_\varphi(N-1)} \end{bmatrix}$$

Consider, the singular value decomposition of $\boldsymbol{H_r}$ is as

$$\boldsymbol{H_r} = [\boldsymbol{U_s} \quad \boldsymbol{U_0}] \begin{bmatrix} \boldsymbol{\sum_s} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} \end{bmatrix} \begin{bmatrix} \boldsymbol{V_s^H} \\ \boldsymbol{v_0^H} \end{bmatrix} = \boldsymbol{U_s \sum_s V_s^H}$$

Then, the null space of *Hr* is spanned by
$v0 = [\boldsymbol{v00}, \boldsymbol{v01}, \boldsymbol{v02}]^{\boldsymbol{T}}$.
Thus, $\boldsymbol{e^{ar \pm j\omega 2r}}$ are two zeros of the prediction Polynomial

$$\boldsymbol{B(z)} = \boldsymbol{v_{00}} + \boldsymbol{v_{01}\, z^{-1}} + \boldsymbol{v_{02}\, z^{-2}}$$

The earlier methods dissolve the noise into L elements by using the truncated singular value decomposition and most of the noise elements are eliminated for large value of L.

B. *WSF Based Estimation*

The previous methods have no concept of matrix's subspace structure estimation. The proposed paper uses a weighted subspace fitting method/algorithm; it reduces mean square error by utilizing subspace structure of the noise matrix. The proposed design is compared with existing design in terms of more accuracy with less complexity.

The singular value decomposition of Ψr is shown below

$$\psi_r = [\overline{U_s} \quad \overline{U_0}] \begin{bmatrix} \sum_s & 0 \\ 0 & \sum_0 \end{bmatrix} \begin{bmatrix} \overline{V_s^H} \\ \overline{v_0^H} \end{bmatrix} = \overline{U_s \sum_s V_s^H} + \overline{U_0 \sum_0 v_0^H}$$

Then comparing, we define the estimation error

$\Delta v_0 = v_0 - \overline{v_0}$ .SNR is given by

$$\Delta v_0 \approx -V_s \textstyle\sum_s^{-1} U_s^H W_r v_0$$

It is oblivious that $E[\Delta v_0] = 0$ since $[W_r] = 0$
.Mean square error (MSE) of v_0
$E[\|\Delta v_0\|^2] = Tr(\ U_s \sum_s^{-2} U_s^H \Gamma)$
Where Γ is defind as $[W_r v_0 v_0^H W_r^H]$. Since $n_\varphi(k_1 T_s)$ and $n_\varphi(k_2 T_s)$ as independent when $k_1 \ne k_2$.
Le output voltage $v_0 = [v_{00}, v_{01}, v_{02}]^T$.

$$[\Gamma]_{i,i} = \frac{|v_{00}|^2}{\rho((i-1)T_s)} + \frac{|v_{01}|^2}{\rho((i+r-1)T_s)} + \frac{|v_{02}|^2}{\rho((i+2r-1)T_s)}$$

$$[\Gamma]_{i,i+r} = \frac{v_{00}^* v_{01}}{\rho((i+r-1)T_s)} + \frac{v_{01}^* v_{02}}{\rho((i+2r-1)T_s)}$$

$$[\Gamma]_{i,i+2r} = \frac{v_{00}^* v_{02}}{\rho((i+2r-1)T_s)}$$

To reduce the MSE of the estimated output voltage, we propose a weighted subspace fitting method. Where, the proposed model can be modeled as $\psi_{rD} = D\psi_r = DH_r + DW_r$
Where D is an invertible weighting matrix. So, the proposed algorithm has edge over computation complexity while comparing to other algorithms.

IV. RESULTS

The wireless transceiver carrier offset is estimated by several parameters like a, ωs, $\varphi\psi$, $\alpha\psi$ with some known values. The typical values are picked according to the parametric estimation results. We assume number of the observations, N is set to be 128.

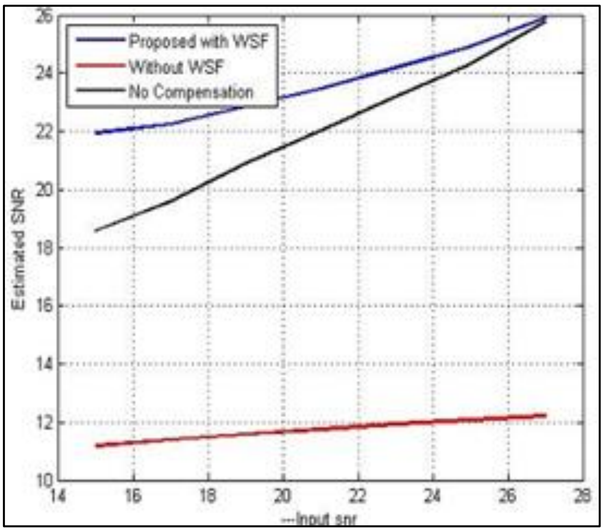

Fig. 1: Shows SNR detection of the received data symbols with different methods: without correcting the carrier offset, with correcting carrier offset by WSF

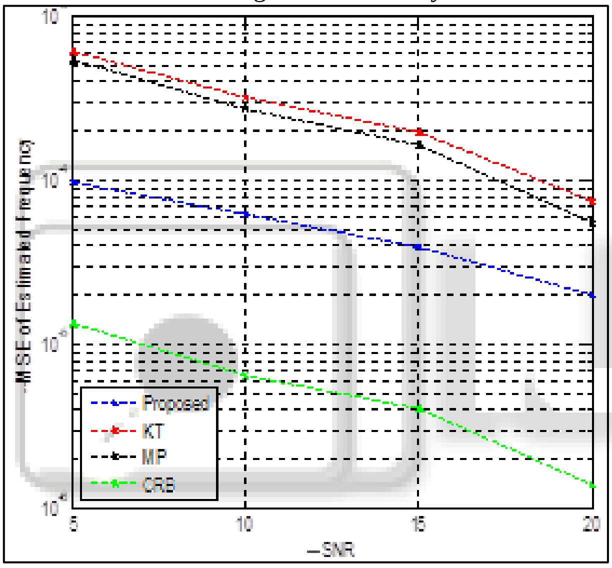

Fig. 2: Shows MSE of the Frequency estimated.

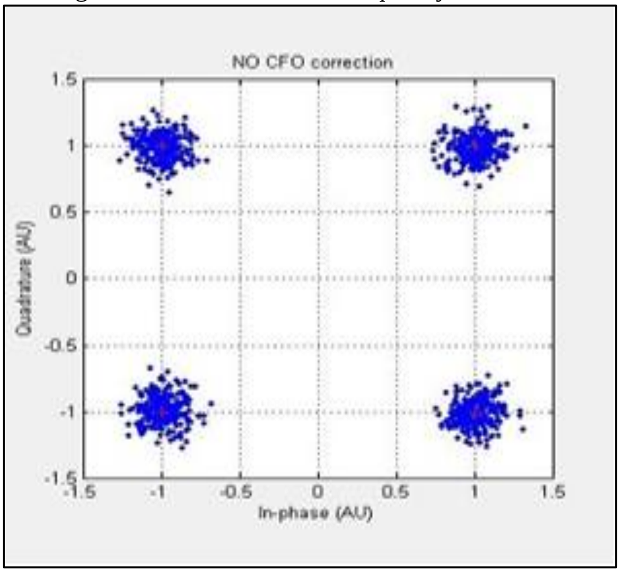

Fig. 3: shows the demodulated symbols without removing the transient CFO from the received samples.

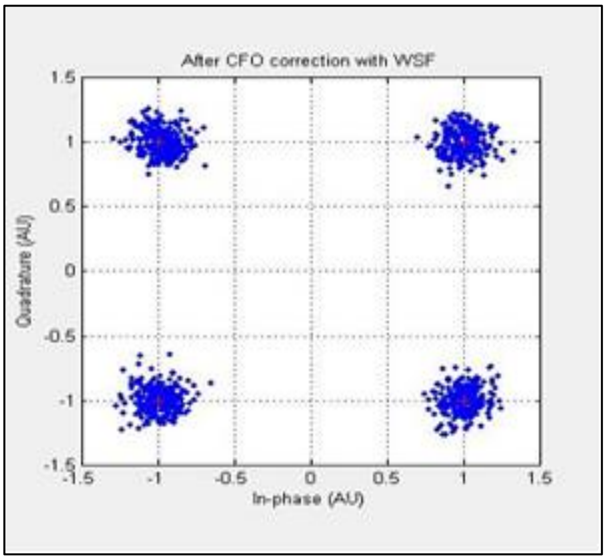

Fig. 4: Shows receiving samples after removal of carrier offset with and without WSF.

Following table shows comparison of computational time (micro -sec) for proposed and existing methods

Sl.No	Algorithm	Time required for computation (µ-Sec)
1.	Proposed algorithm (Weighted Subspace fitting)	1.98
2.	Existed Matrix pencil algorithm	35.56
3.	Existed Kumaresan-Tuft algorithm	103.52

Table 1

V. CONCLUSION

Wireless communications has grown from Daily life applications to satellite communications.

A major problem in Wireless communications is Transient carrier offset estimation which results due to change in source voltage. This error degrades the system performance. We modeled the problem as the step response of second order under-damped system. The proposed method of subspace based algorithm has the estimation matrix of type Hankel –like matrix. To improve the estimation accuracy The weighted subspace fitting algorithm is also proposed.

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