Near Mean Cordial - Path Related Graphs

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Abstract— Let G = (V,E) be a simple graph. A Near Mean Cordial Labeling of G is a function f : V(G) → {1,2,3,…,p−1,p+1} such that for each edge uv the induced map f* defined by f*(uv) $=\begin{cases} 1 & if(f(u)+f(v)) \equiv 0\ (mod 2) \\ 0 & else \end{cases}$, and it satisfies the condition $|e_f(0) - e_f(1)| \leq 1$, where $e_f(0)$ and $e_f(1)$ represent the number of edges labeled with 0 and 1 respectively. A graph is called a Near Mean Cordial Graph if it admits a Near Mean Cordial Labeling. In this paper, It is proved that the graphs P_n, SP (P_n,$K_{1,m}$) and $B_{m,n}$ are Near Mean Cordial Graphs. AMS Mathematics subject classification 2010: 05C78.

Key words: Cordial Labeling, Mean Cordial Labeling, Near Mean Cordial Labeling and Near Mean Cordial Graphs

I. INTRODUCTION

By a graph, it means a finite undirected graph without loops or multiple edges. For graph theoretic terminology, we referred Harary[4]. For labeling of graphs, we referred Gallian[1].

A vertex labeling of a graph G is an assignment of labels to the vertices of G that induces for each edge *uv* a label depending on the vertex labels of *u* and *v*.

A graph *G* is said to be labeled if the *n* vertices are distinguished from one another by symbols such as $v_1, v_2, \ldots, v_n$. In a labeling of a particular type, the vertices are assigned distinct values from a given set, which induces distinguish edge values satisfying certain conditions. The concept of graceful labeling was introduced by Rosa[3] in 1967 and subsequently by Golomb[2]. In this paper , It is proved that P_n, $B_{m,n}$, SP(P_n,$K_{1,m}$) are Near Mean Cordial Graphs.

II. PRELIMINARIES

A. Definition 2.1:

Let G = (V,E) be a simple graph. Let f:V(G)→{0,1} and the induced edge label, assigning $|f(u) - f(v)|$ is called a Cordial Labeling if the number of vertices labeled 0 and the the number of vertices labeled 1 differ by atmost 1 and also the number of edges labeled 0 and the the number of edges labeled 1 differ by atmost 1. A graph is called Cordial if it has a cordial labeling.

B. Definition 2.2

Let G = (V,E) be a simple graph. G is said to be a Mean Cordial Graph if f:V(G)→{0,1,2} such that for each edge uv the induced map f*defined by f*(uv) $=\left\lfloor \frac{f(u)+f(v)}{2} \right\rfloor$ where $\lfloor x \rfloor$ denote the least integer which is ≤ x and $|e_f(0) - e_f(1)| \leq 1$ where $e_f(0)$ is the number of edges with zero label. $e_f(1)$ is the number of edges with one label.

C. Definition 2.3

Let G = (V, E) be a simple graph. A Near Mean Cordial Labeling on G is a function in f: V(G) → {1,2,3,…,p−1,p+1} such that for each edge uv the induced map f* defined by f*(uv) $=\begin{cases} 1 & if(f(u)+f(v)) \equiv 0\ (mod 2) \\ 0 & else \end{cases}$ and it satisfies the condition $|e_f(0) - e_f(1)| \leq 1$. Where $e_f(0)$ and $e_f(1)$ represent the number of edges labeled with 0 and 1 respectively. A graph is called a Near Mean Cordial Graph if it admits a near mean cordial labeling.

D. Definition 2.4

If all the vertices in a walk are distinct, then it is called a path P_n and a path of length k is denoted by P_{k+1}.

E. Definition 2.5

A graph obtained from a path of length $n - 1$ by attaching root of a star $K_{1,m}$ at one end of the path. It is denoted by SP (P_n,$K_{1,m}$).

F. Definition 2.6

The Bistar $B_{m,n}$ is a graph obtained from K_2 by joining the centers of $K_{1,m}$ at one end and $K_{1,n}$ at the other end of K_2.

III. MAIN RESULTS

A. Theorem 3.1

P_n is a Near Mean Cordial Graph.

1) Proof

Let V(P_n) ={u_i: 1 ≤ i ≤ n}.
Let E (P_n) = {(u_iu_{i+1}): 1 ≤ i ≤ n − 1} .

a) Case (i): when n is odd

Define f : V(P_n) → {1,2,3,…,n−1,n+1} by

$$f(u_1) = 1, \quad f(u_{n-1}) = n+1$$
$$f(u_{2i+1}) = i+1, \quad 1 \leq i \leq \frac{n-1}{2}$$
$$f(u_{2i}) = \frac{n+1}{2} + i, \quad 1 \leq i \leq \frac{n-3}{2}$$

The induced edge labeling are

$$f^*(u_i\,u_{i+1}) = \begin{cases} 1 & \text{if } f(u_i) + f(u_{i+1}) \equiv 0 \ (\text{mod } 2) \\ 0 & \text{else} \end{cases}, \quad 1 \leq i \leq n-1$$

Here,

$$e_f(0) = e_f(1) = \frac{n-1}{2}.$$

Hence, it satisfies the condition $|e_f(0) - e_f(1)| \leq 1$.
Hence, P_n is a Near Mean Cordial Graph .

For example, the Near Mean Cordial Labeling of P_7 is shown in the Figure 1.

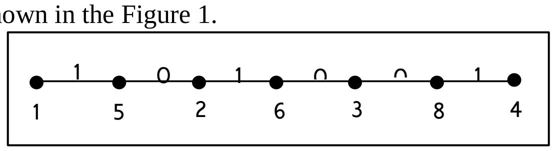

Fig. 1: Graph

b) Case (ii): when n is even

Define f : V(P_n) → {1,2,3,…,n−1,n+1} by

$$f(u_1) = 1, \quad f(u_n) = n+1$$
$$f(u_{2i+1}) = i+1, \quad 1 \leq i \leq \frac{n-2}{2}$$

$$f(u_{2i}) = \frac{n}{2} + i, \quad 1 \le i \le \frac{n-2}{2}$$

The induced edge labeling are

$$f^*(u_i\, u_{i+1}) = \begin{cases} 1 & \text{if } f(u_i) + f(u_{i+1}) \equiv 0 \pmod 2 \\ 0 & \text{else} \end{cases}, \quad 1 \le i \le n-1$$

c) Sub case(i): when $n \equiv 0 \pmod 4$

Here, $e_f(0) = \frac{n}{2}$ and $e_f(1) = \frac{n}{2} - 1$.

Hence, it satisfies the condition $|e_f(0) - e_f(1)| \le 1$.

Hence, P_n is a Near Mean Cordial Graph.

For example, the Near Mean Cordial Labeling of P_8 is shown in the Figure 2.

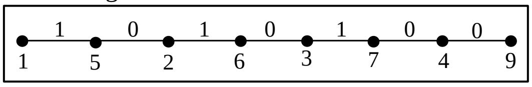

Fig. 2: Graph

d) Sub case(ii): when $n \equiv 2 \pmod 4$

Here, $e_f(0) = \frac{n}{2} - 1$ and $e_f(1) = \frac{n}{2}$.

Hence, it satisfies the condition $|e_f(0) - e_f(1)| \le 1$.

Hence, P_n is a Near Mean Cordial Graph.

For example, the Near Mean Cordial Labeling of P_6 is shown in the Figure 3.

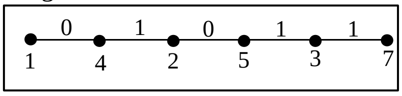

Fig. 3: Graph

B. *Theorem 3.2*

$SP(P_n,K_{1,m})$ is a Near Mean Cordial Graph.

1) *Proof:*

Let $V(SP(P_n,K_{1,m})) = \{u_i: 1 \le i \le n,\ v_i: 1 \le i \le m\}$.

Let $E\ (SP(P_n, K_{1,m})) = \{(u_i u_{i+1}) : 1 \le i \le n-1\} \cup \{(u_n v_i) : 1 \le i \le m\}$.

a) Case (i): when n is even

Define $f : V(SP(P_n, K_{1,m})) \to \{1,2,3,\ldots,m+n-1,m+n+1\}$ by

$$f(u_{2i-1}) = i, \quad 1 \le i \le \frac{n}{2}$$

$$f(u_n) = m+n+1$$

$$f(u_{2i}) = (m+1) + \frac{n}{2} + (i-1), \quad 1 \le i \le \frac{n}{2} - 1$$

$$f(v_i) = \frac{n}{2} + i, \quad 1 \le i \le m$$

b) Case (ii): when n is odd

Define $f : V(SP(P_n, K_{1,m})) \to \{1,2,3,\ldots,m+n-1,m+n+1\}$ by

$$f(u_{2i-1}) = i, \quad 1 \le i \le \frac{n+1}{2}$$

$$f(u_{n-1}) = m+n+1$$

$$f(u_{2i}) = (m+1) + \frac{n+1}{2} + (i-1), \quad 1 \le i \le \frac{n-3}{2}$$

$$f(v_i) = \frac{n+1}{2} + i, \quad 1 \le i \le m$$

The induced edge labelings are

$$f^*(u_i\, u_{i+1}) = \begin{cases} 1 & \text{if } f(u_i) + f(u_{i+1}) \equiv 0 \pmod 2 \\ 0 & \text{else} \end{cases}, \quad 1 \le i \le n$$

$$f^*(u_n\, v_i) = \begin{cases} 1 & \text{if } f(u_n) + f(v_i) \equiv 0 \pmod 2 \\ 0 & \text{else} \end{cases}, \quad 1 \le i \le m$$

When n is even and m is odd

Here, $e_f(0) = e_f(1) = \frac{m+n-1}{2}$.

Hence, it satisfies the condition $|e_f(0) - e_f(1)| \le 1$.

When n is odd and m is odd

Here, $e_f(0) = \frac{m+n}{2}$ and $e_f(1) = \frac{m+n}{2} - 1$.

Hence, it satisfies the condition $|e_f(0) - e_f(1)| \le 1$.

Hence, $SP(P_n,K_{1,m})$ is a Near Mean Cordial Graph .

For example, the Near Mean Cordial Labeling of $SP(P_6,K_{1,3})$ and $SP(P_5,K_{1,5})$ is shown in Figures 4 and 5.

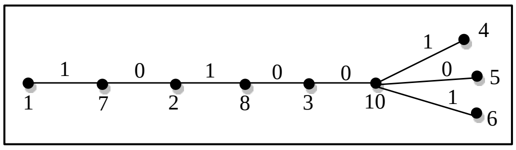

Fig. 4: Graph

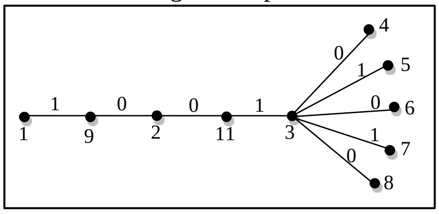

Fig. 5: Graph

When n is even and m is even

When $n \equiv 0 \pmod 4$

Here,

$$e_f(0) = \frac{m+n}{2} \text{ and } e_f(1) = \frac{m+n}{2} - 1.$$

When $n \equiv 2 \pmod 4$

Here,

$$e_f(1) = \frac{m+n}{2} \text{ and } e_f(0) = \frac{m+n}{2} - 1.$$

Hence, it satisfies the condition $|e_f(0) - e_f(1)| \le 1$.

When n is odd and m is even

Here,

$$e_f(0) = e_f(1) = \frac{m+n-1}{2}$$

Hence, it satisfies the condition $|e_f(0) - e_f(1)| \le 1$.

Hence, $SP(P_n,K_{1,m})$ is a Near Mean Cordial Graph.

For example, the Near Mean Cordial Labeling of $SP(P_4,K_{1,4})$, $SP(P_5,K_{1,6})$ and $SP(P_2,K_{1,4})$ is shown in Figures 6.

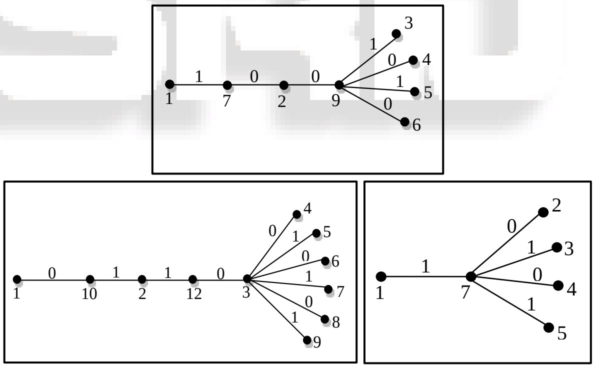

Fig. 6: Graph

C. *Theorem 3.3*

$B_{m,n}$ is a Near Mean Cordial Graph.

1) *Proof*

Let $V(B_{m,n}) = \{u, v, (u_i: 1 \le i \le m), (v_i: 1 \le i \le n)\}$.

Let $E\ (B_{m,n}) = \{\{uv\} \cup \{(uu_i) : 1 \le i \le m\} \cup \{(vv_i) : 1 \le i \le n\}\}$.

a) Case (i): when m = n

Define $f : V(B_{m,n}) \to \{1,2,3,\ldots,m+n+1,m+n+3\}$ by

$$f(u) = 2m+1, \quad f(v) = 2n+3$$

$$f(u_i) = 2i-1, \quad 1 \le i \le m$$

$$f(v_i) = 2i, \quad 1 \le i \le n$$

The induced edge labeling are

$$f^*(uv) = 1$$

$$f^*(uu_i) = 1, \quad 1 \le i \le m$$

$$f^*(vv_i) = 0, \quad 1 \le i \le n$$

Here

$$e_f(0) = \frac{m+n}{2} \text{ and } e_f(1) = \frac{m+n}{2} + 1.$$

Hence, it satisfies the condition $|e_f(0) - e_f(1)| \leq 1$.

Hence, $B_{m,n}$ is a Near Mean Cordial Graph.

For example, the Near Mean Cordial Labeling of $B_{4,4}$ is shown in the Figure 7.

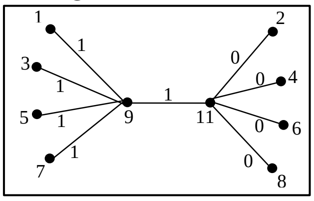

Fig. 7: Graph

b) Case (ii): when m < n

Define $f : V(B_{m,n}) \rightarrow \{1,2,3,\ldots,m+n+1,m+n+3\}$ by

$$f(u) = m+n+1, \; f(v) = m+n+3$$
$$f(u_i) = 2i-1, \quad 1 \leq i \leq m$$
$$f(v_i) = 2i, \quad 1 \leq i \leq m$$
$$f(v_i) = m+i, \quad m+1 \leq i \leq n$$

c) sub case(i): when m is odd,nis odd and when m is even ,nis even

The induced edge labelings are

$$f^*(uv) = 1$$
$$f^*(uu_i) = 1, \; 1 \leq i \leq m$$
$$f^*(vv_i) = 0, \; 1 \leq i \leq m$$
$$f^*(v \;\; v_i) = \begin{cases} 1 & \text{if } f(v) + f(v_i) \equiv 0 \pmod 2 \\ 0 & \text{else} \end{cases}, \; m+1 \leq i \leq n$$

Here,

$$e_f(0) = \frac{m+n}{2} \text{ and } e_f(1) = \frac{m+n}{2} + 1.$$

d) sub case(ii): when m is odd, nis even and when m is even ,nis odd

The induced edge labelings are

$$f^*(uv) = 1$$
$$f^*(uu_i) = 0, \; 1 \leq i \leq m$$
$$f^*(vv_i) = 1, \; 1 \leq i \leq m$$
$$f^*(v \;\; v_i) = \begin{cases} 1 & \text{if } f(v) + f(v_i) \equiv 0 \pmod 2 \\ 0 & \text{else} \end{cases}, \; m+1 \leq i \leq n$$

Here

$$e_f(0) = \frac{m+n+1}{2} = e_f(1).$$

Hence, In all the cases it satisfies the condition $|e_f(0) - e_f(1)| \leq 1$.

Hence, $B_{m,n}$ is a Near Mean Cordial Graph .

For example, the Near Mean Cordial Labeling of $B_{3,5}$, $B_{2,6}$, $B_{5,8}$ and $B_{6,7}$ are shown in Figures 8.

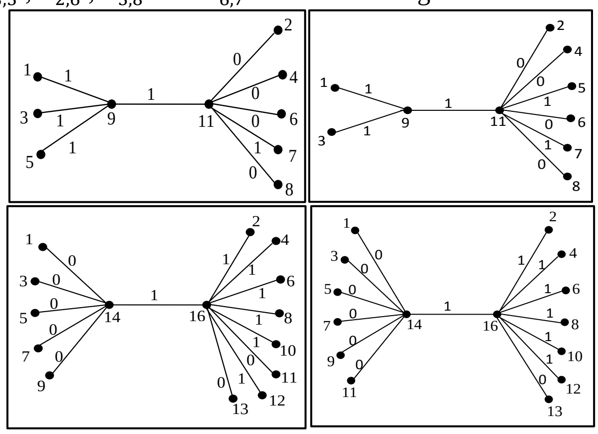

Fig. 8: Graph

REFERENCES

[1] G.J. Gallian, A Dynamic survey of graph labeling, The electronic journal of combinotorics, 16 (2009), #DS6.
[2] S.W Golomb, How to number a graph in graph Theory and computing, R.C. Read, ed., Academic Press, New York (1972), 23-37.
[3] A. Rosa, On certain valuations of the vertices of a graph, Theory of graphs (International Symposium, Rome), July (1966).
[4] R. Sridevi, S. Navaneethakrishnan, Some New Graph Labeling Problems and Related Topics(thesis) jan 2013.
[5] Frank Harary, Graph Theory, Narosa publishing house pvt. Ltd., 10th reprint 2001.
[6] A. NellaiMurugan, and G. Esther, Some Results On Mean Cordial Graphs International journal of Mathematics Trends and Technology- volume11 No.2- Jul 2014.