A Mathematical Model on Visco-Elastic Elliptical Plate to Study First Two Modes of Transverse Vibration

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Abstract— Transverse vibration of elliptical plates has been a subject of study for a long time. The problems of vibration of elliptical, rectangular and circular plates etc have been extensively investigated by many researchers and are well documented. All through understanding of their vibration characteristics along with different thermal conditions is of great importance to engineers and designers making sure reliability in design procedure. In this paper, the elliptical plate material is assumed to be homogeneous, also temperature and thickness varies in x and y directions. To find the result as frequency parameter, Rayleigh Ritz method is applied. Frequency is calculated for two modes different set of value of thermal gradient and taper constant.

Key words: Vibration, Visco-Elastic, Elliptical, Plate, Taper Constant

I. INTRODUCTION

With the encroachment of equipment, plates of variable thickness are being widely used in civil, electronic, mechanical, aerospace and marine engineering purpose. Now days, it becomes very essential to study the vibration deeds of plates to a void resonance excited by internal or external forces. Current engineering formations are based on dissimilar types of design, which involve a variety of types so fan isotropic and non-homogeneous materials in the form of their formation components. Depending upon the prerequisite, stability and dependability materials are being residential so that they can be used to give better potency and competence. In the recent past, there has been a phenomenal increase in the enlargement to elastic materials due to high demand for light weight, high strength, corrosion resistance and high-temperature recital requirements in current equipment. Plates of composite materials are far and wide used in many engineering formation and machines. A number of researchers have worked on free vibration analysis of plates of dissimilar shapes and variable thickness.

Elliptical plates of unstable thickness with thermal gradient find different purpose in the manufacture of modern high speed air craft. The vibration uniqueness of such plates are of notice to the designer. Analysis of plate's vibration with complex shape and structure has been studied in large scale for many years. Vibrant behavior of complex designed structure is very much dependent on shape, size and material's property at the end on boundary conditions.

There are a huge number of applications where vibration effect is required e.g. in string and percussion instruments, in the design of loudspeakers, space shuttles, satellites where discrepancies in the temperature also affects the vibration effect. Illicit vibration effects are also necessary in health industry, paper industry, blueprint of structures, building construction, reducing soil bond and many more areas hold your attention vibration upshot.

Recently, Leissa [1, 2] gave the solutions for variable varying thickness in rectangular plate. An orthotropic rectangular plate with non-homogeneity which is parabolically varying in both directions with thermal effect is studied by Gupta, Johri and Vats [3]. Gupta and Khanna [4] discussed free vibration on visco-elastic rectangular plate having linearly varying thickness in x and y directions. Transverse vibration of rectangular plates is studied by Singh and Saxena [5] having bi-directional thickness variation. Sobotka [6, 7] has investigated the vibrational analysis of orthotropic rectangular visco-elastic plates. A. K. Gupta, Amit Kumar and Dhram Veer Gupta [8] found two mode of vibration for orthotropic parallelogram plate with parabolically varying thickness. Thermal deflection effect is discussed by Warade and Deshmukh [9] in the thin clamped circular plate where heat is partially distributive. Gupta and Kumar [10] analyzed vibrational analysis with non-homogeneity in visco-elastic rectangular plates with linearly thickness. The consequence of varying poisson ratio is analysed by Khanna A., Kaur N. and Sharma A. K. [11] on non-homogeneous rectangular plate having temperature variation. Sharma S. K., Sharma A. K. [12] has discussed the mechanical vibration with 2 dimensional thickness variation and temperature effect of rectangular plate. Square plate with variable varying thickness, temperature effect is solved by Khanna A. and Sharma A. K. [13]. Sinusoidal thickness variation has discussed by Kumar Sharma A., Sharma, S. K. [14] on visco-elastic plate with two dimensional thermal effect. Computational investigation on vibration is given by Khanna A., Kumar A., Bhatia M. [15] on 2-dimensional thermal variation with variable thickness in visco-elastic square plate. Khanna A., Sharma A. K. [16] calculated regular vibration of visco-elastic plate with temperature effect and variable varying thickness. Kumar Sharma A., Sharma S. K. [17] discussed free vibrational analysis of orthotropic visco-elastic rectangular plate with parabolic thermal effect and linear thickness variation in x and y directions. Bi-parabolic thermal and thickness variation is studied by Sharma S. K., Sharma A. K. [18] on vibration of orthotropic visco-elastic rectangular plate. Poisson ratio variation with thermal effect of non-homogeneous rectangular plate is solved by Khanna A., Kaur N., Sharma A. K. [19].Analysis of free vibration using Rayleigh-Ritz method on 2D varying thickness and thermal effect on rectangular orthotropic plate is studied by Sharma Subodh Kumar, Sharma Ashish Kumar [20]. Also Sharma Subodh Kumar, Sharma Ashish Kumar [21] solved vibration problems for visco-elastic parallelogram plate.

Hence our day after day life is precious by vibrations. As a result for improved plan of engineering formation, it is unavoidable to optimize redundant vibrations. In present work a full-fleshed attempt to assist the designer and industry people for better construction of engineering structure. The aim is to study 2D parabolic thermal effects on visco-elastic homogenous elliptical plate with linear varying thickness. All results are shown in graphical form.

II. Modeling

It is assumed that the elliptic plate has a steady two dimensional parabolic temperature distribution given by:

$$\psi = \psi_0\left(1-\frac{x^2}{a^2}-\frac{y^2}{b^2}\right) \quad (1)$$

Where ψ signifies the excess of temperature at any point on plate higher than the reference temperature and ψ_0 denotes the temperature at any point on the boundary of plate .For most of engineering materials, the temperature dependence of the modulus of elasticity is given as

$$E = E_0(1-\gamma\psi) \quad (2)$$

Where, $E_0$is the value of the Young's modulus at reference temperature i.e. $\psi = 0$ and γ is the slope of the variation of E with ψ. The modulus variation becomes:

$$E = E_0\left[`1-\alpha\left(1-\frac{x^2}{a^2}-\frac{y^2}{b^2}\right)\right] \quad (3)$$

Where $\alpha = \gamma\,\psi_0 (0 \le \alpha < 1)$ and α is length of a side of elliptical plate.

It is assumed that thickness χ varies linearly in two directions as shown below:

$$\chi = \chi_0\left(1-\beta\left(1-\frac{x}{a}-\frac{y}{b}\right)\right) \quad (4)$$

Where α is length of a side of elliptical plate and β is taper parameters in *x*- & *y*- directions respectively and $\chi = \chi_0$ at *x*=*y*=0.

III. Solution of Plate Modeling

Deflection function $W(x,y)$ of plate is assumed to be finite sum of characteristics functions is taken as:

$$W(x,y) = \phi_1 + \phi_2 \quad (5)$$

Where

$$\phi_1 = \left[A_2 * \left(1-\left(1-\frac{x}{b}\right)-\left(1-\frac{y}{b}\right)\right)^2\right]$$

And

$$\phi_2 = \left[A_2 * \left(1-\left(1-\frac{x}{a}\right)-\left(1-\frac{y}{b}\right)\right)^3\right]$$

And W, satisfying relevant geometrical boundary conditions.

Since, the plate is assumed as clamped at all the four edges, so the boundary conditions are:

$$W = W_{,x} = 0, \quad x = 0, a$$
$$W = W_{,y} = 0, \quad y = 0, a \quad (6)$$

Now assuming the non-dimensional variables as:

$$X = \frac{x}{a}, Y = \frac{y}{a}, \bar{W} = \frac{W}{a}, \bar{h} = \frac{h}{a}$$

According to Rayleigh-Ritz method, maximum strain energy and maximum kinetic energy are equal as:

$$\delta(S^* - K^*) = 0 \quad (7)$$

The kinetic energy K^* and strain energy $S^*$are:

$$K^* = \int_0^1\int_0^{\sqrt{1-\xi^2}} \chi\,\bar{W}\,dY\,dX \quad (8)$$

and

$$S^* = \int_0^1\int_0^{\sqrt{1-\xi^2}} D_1\left\{(\bar{W}_{,xx})^2 + (\bar{W}_{,yy})^2 + 2\boldsymbol{\nu}\bar{W}_{,xx}\bar{W}_{,yy} + 2(1-\boldsymbol{\nu})(\bar{W}_{,xy})^2\right\}dY\,dX \quad (9)$$

Where $\xi = x/a$, D_1 is flexural rigidity and $D_1 = \frac{E\chi^3}{12(1-\nu^2)}$.

By using the value of E and χ , the value of D_1 becomes:

$$D_1 = \left[E_0\left\{`1-\alpha\left(1-X^2-Y^2*\left(\frac{a}{b}\right)^2\right)\right\}*{\chi_0}^3\left(1-\beta\left(1-X-Y*\frac{a}{b}\right)\right)^3\right]/12(1-\nu^2) \quad (10)$$

Using equations (8) & (9) in equation (7) after putting the values of χ and D_1, one get:

$$(S^* - \lambda^2 p^2 K^*) = 0 \quad (11)$$

Where

$$S^{**} = \int_0^1\int_0^{\sqrt{1-\xi^2}}\left[1-\alpha\left(1-X^2-Y^2*\left(\frac{a}{b}\right)^2\right)\right]\left(1-\beta\left(1-X-Y*\frac{a}{b}\right)\right)^3\left\{(\bar{W}_{,xx})^2+(\bar{W}_{,yy})^2 + 2\boldsymbol{\nu}\bar{W}_{,xx}\bar{W}_{,yy} + 2(1-\boldsymbol{\nu})(\bar{W}_{,xy})^2\right\}dY\,dX \quad (12)$$

and

$$K^{**} = \int_0^1\int_0^{\sqrt{1-\xi^2}}\left[\left(1-\beta\left(1-X^2-Y^2*\left(\frac{a}{b}\right)^2\right)\right)*\bar{W}^2\right]dY\,dX \quad (13)$$

Parameter of frequency is given as

$$\lambda^2 = \frac{12\rho a^4(1-\nu^2)}{E_0{\chi_0}^2}.$$

Now, on substituting the value of W, equation consist of two unknown constants i. e. A_1&A_2 which is evaluate as follow:

$$\frac{\partial(S^{**}-\lambda^2K^{**})}{\partial A_n} = 0, \quad n = 1,2 \quad (14)$$

On simplifying, we get

$$Qn_1A_1 + Qn_2A_2 = 0, \quad n = 1,2 \quad (15)$$

Where, Qn_1, Qn_2, $(n = 1,2)$ include parametric constant and the frequency parameter. For having solution to be non-trivial, the determinant of the coefficient matrix should be zero. So we get equation of frequency as

$$\begin{vmatrix} Q_{11} & Q_{12} \\ Q_{21} & Q_{22} \end{vmatrix} = 0 \quad (16)$$

On solving equation (16), we obtained quadratic equation in λ^2. The solution value of λ^2 represent the frequency vibration of two modes i.e. λ_1 (I st Mode) & λ_2 (II nd Mode) for clamped plate with different values of thermal gradient and taper constant.

IV. Result and Discussion

Frequency equation (16) is quadratic in λ^2, so it will give two roots. The frequency is derived for the first two modes of vibration for homogenous elliptical plate having linearly varying thickness in both the directions, for various values of taper constant and thermal gradient. The value of Poisson ratio ν has been taken 0.345. These results are presented in figures (1-4) for first two modes of vibration for elliptical plate.

A. In Figure 1

It is clearly seen that as thermal gradient increases from 0 to 1 results frequency decreases. Figure1 has shown the results for the following three cases:

β= ξ = 0.0, a/b=1.5
β= ξ = 0.3, a/b=1.5
β= ξ = 0.6, a/b=1.5

B. In Figure 2

Also, it is observed that for both modes of vibration, the frequency decreases with increases in thermal gradient β from 0 to 1. Figure 2 has shown the results for the following three cases:

α = ξ=0, a/b=1.5
α= ξ =0, a/b=1.5
α= ξ =0, a/b=1.5

C. In Figure 3

It is observed that for both modes of vibration, frequency parameter increases with increases in aspect ratio a/b from 0.5 to 3. Figure 3 has shown the results for the following three cases:

α =0.0, β= 0.0
α =0.3, β= 0.3
α =0.6, β= 0.6

D. In Figure 4

It is seen that with increases in ξ from 0 to 1 then frequency increases for both modes of vibrations. Figure 4 has shown the results for the following three cases:

α = β=0, a/b=1.5
α= β =0, a/b=1.5
α= β =0, a/b=1.5

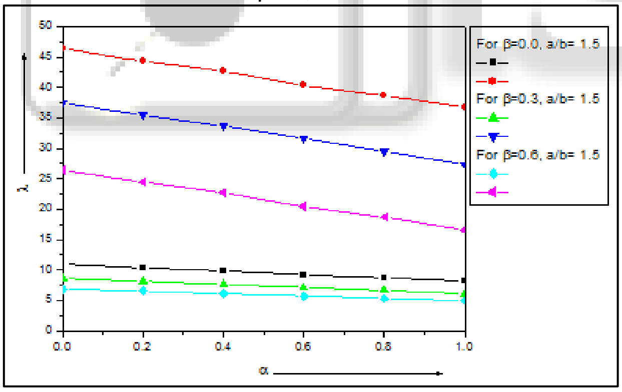

Fig. 1: Frequency of elliptic plate for different values of thermal gradient (α)

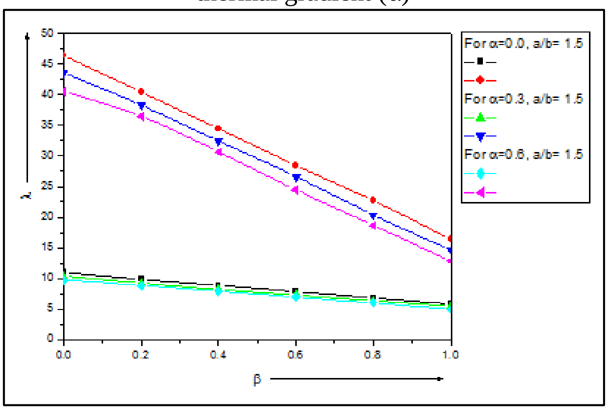

Fig. 2: Frequency of elliptic plate for different values of taper constant (β)

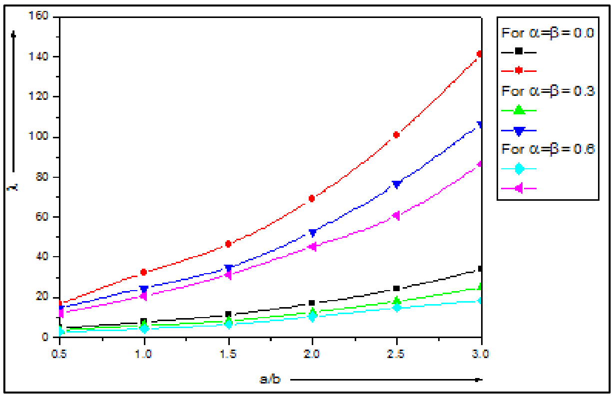

Fig. 3: Frequency of elliptic plate for different values of aspect ratio (a/b)

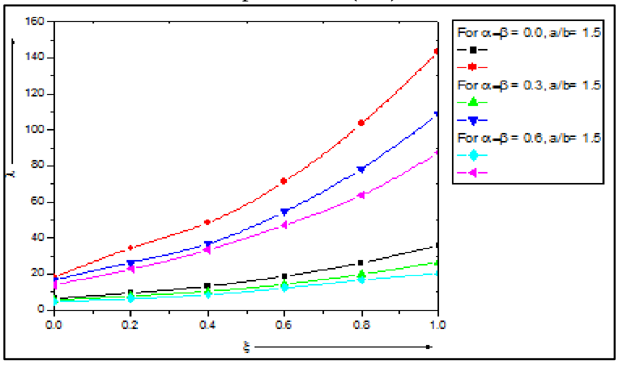

Fig. 4: Frequency of elliptic plate for different values of ξ

V. CONCLUSIONS

It can be concluded from the results that frequency parameter decreases with an increase in taper constant and thermal gradient. Also, frequency increases with increase in the value of aspect ratio. Our main aim is to provide such kind of a mathematical design so that scientist can perceive their potential in mechanical engineering field and increase strength, durability and efficiency of mechanical design and structuring with a practical approach .Actually this is the need of the hour to develop more but authentic mathematical model for the help of mechanical engineers. Therefore mechanical engineers and technocrats are advised to study and get the practical importance of the present paper and to provide much better structure and machines with more safety and economy.

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