# Buckling and Vibration Analysis of Cracked Composite Beam

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*Abstract*— Cracks in structural members lead to nearby changes in their stiffness and thus their static and dynamic conduct is modified. The impact of cracks on dynamic attributes like natural frequencies, methods of vibration of structures has been the subject of numerous examinations. However concentrates on identified with conduct of composite broke structures subject to in-plane loads are rare in writing. Present work manages the vibration and buckling investigation of a cantilever beam produced using graphite fiber fortified polyimide with a transverse one-edge non-spreading open break utilizing the finite element strategy. The undamaged parts of the beam are demonstrated by beam finite elements with three hubs and three degrees of opportunity at the hub. A „overall extra flexibility matrix‟ is added to the flexibility matrix of the relating non-broke composite beam element to get the aggregate flexibility matrix, and in this way the stiffness matrix in accordance with past studies. The vibration of split composite beam is processed utilizing the present definition and is contrasted and the past results. The impacts of different parameters like break area, split profundity, volume division of strands and filaments introductions upon the progressions of the natural frequencies of the beam are contemplated. It is found that, nearness of break in beam abatements the natural recurrence which is more purported when the split is close to the settled backing and the break profundity is more. The natural recurrence of the broke beam is observed to be most extreme at around 45% of volume portion of filaments and the recurrence for any profundity of break increments with the expansion of edge of strands. The static buckling load of a split composite beam is observed to diminish with the nearness of a break and the lessening is more serious with expansion in split profundity for any area of the split. Besides, the buckling load of the beam diminished with expansion in edge of the filaments and is greatest at 0 degree introduction.

***Key words:*** Buckling and Vibration Analysis, Cracked Composite Beam

## I. INTRODUCTION

Composites as structural material are being used in aerospace, military and civilian applications because of their tailor made properties. The ability of these materials to be designed to suit the specific needs for different structures makes them highly desirable. Improvement in design, materials and manufacturing technology enhance the application of composite structures. The suitability of a particular composite material depends on the nature of applications and needs. The technology has been explored extensively for aerospace and civil engineering applications, which require high strength and stiffness to weight ratio materials.

Cracks or other defects in a structural element influence its dynamical behaviour and change its stiffness and damping properties. Consequently, the natural frequencies and mode shapes of the structure contain information about the location and dimensions of the damage. Vibration analysis can be used to detect structural defects such as cracks, of any structure offer an effective, inexpensive and fast means of nondestructive testing. What types of changes occur in the vibration characteristics, how these changes can be detected and how the condition of the structure is interpreted has been the topic of several research studies in the past. The use of composite materials in various construction elements has increased substantially over the past few years.

## II. THEORY AND FORMULATIONS

Structures are debilitated by cracks. At the point when the break size increments in course of time, the structure gets to be weaker than its past condition. At last, the structure may breakdown because of a moment split. The fundamental design of the issue examined here is a composite beam of any limit condition with a transverse one-edge non-engendering open split. Be that as it may, a run of the mill broke cantilever composite beam, which has enormous applications in aviation structures and fast turbine hardware, is considered.

### A. *Methodology*

The governing equations for the vibration analysis of the composite beam with an open one-edge transverse crack are developed

The assumptions made in the analysis are:

1) The analysis is linear. This implies constitutive relations in generalized Hook's law for the materials are linear.
2) The Euler–Bernoulli beam model is assumed.
3) The damping has not been considered in this study.
4) The crack is assumed to be an open crack and have uniform depth „a‟.

### B. *Governing Equation*

The differential equation of the bending of a beam with a mid-plane symmetry ($B_{ij}$ = 0) so that there is no bending-stretching coupling and no transverse shear deformation ( $\varepsilon$ = 0) is given by;

$$IS_{11}\frac{d^4\omega}{dx^4} = q\ \ x$$

For a composite beam in which different lamina have differing mass densities, then in the above equation use, for a beam of rectangular cross-section,

$$\rho A = \rho bh = {}_{k=1}^{N} \rho b\ \ h_k - h_{k-1}$$

Thus, the natural vibration equation of a mid-plane symmetrical composite beam is given by;

$$IS_{11}\frac{d^4\omega(x,t)}{dx^4} + \rho A\frac{\partial^2\omega(x,t)}{\partial x^2} = 0$$

The natural frequency in radians/unit time are given as

$$\omega_n = \alpha^2\left(\frac{IS_{11}}{\rho AL^4}\right)^{1\ \ 2}$$

 

### *C. Mathematical Model*

The model chosen is a cantilever composite beam of uniform cross-section A, having an open-edge transverse crack of depth „a‟ at position „**$l_1$**'. The width, length and height of the beam are B, L and H, respectively in Figure. The angle between the fibers and the axis of the beam is α‟.

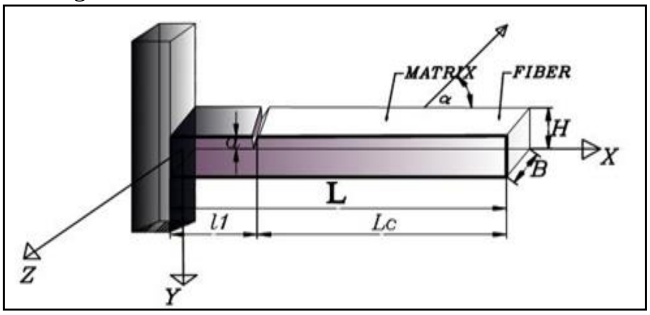


Fig. 1: Mathematical Model

### *D. Buckling Analysis Studies*

The dynamic response of a beam for a conservative system can be formulated by means of Lagrange‟s equation of motion in which the external forces are expressed in terms of time-dependent potentials

$$M \quad q \; + \quad K_e \; - P(t) \; K_g \qquad q \; = 0$$

### *E. Derivation of Element Matrices*

In the present analysis three nodes composite beam element with three degree of freedom (the axial displacement, transverse displacement and the independent rotation) per node is considered.

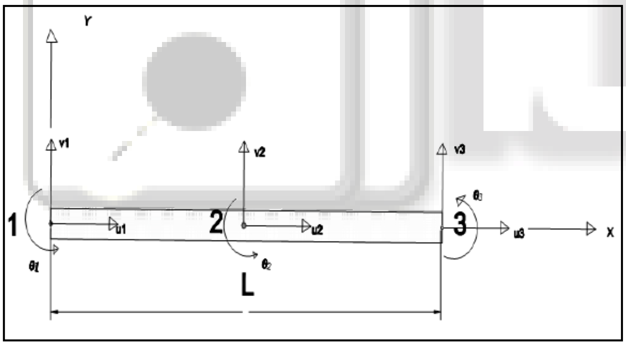


Fig. 2: Element Matrices

The linear strain can be described in terms of displacements as

$$\varepsilon \;=\; B \quad \delta$$

### *F. Stress-Strain Matrix*

$$D = \begin{matrix} S_{11} & S_{13} \\ S_{13} & S_{33} \end{matrix}$$

### *G. Element Stiffness Matrix*

Element stiffness matrix for a three-nodes composite beam element with three degrees of freedom δ = (u, v, θ) at each node, for the case of bending in the x, y plan, are given

$$K_e \;= IS_{11} \;\; _v[B]^T D \;\; B \;\; dv$$

### *H. Generalized Element Mass Matrix*

Element mass matrix of the non-cracked composite beam element is given

$$M_e \;= \rho \;\; _v[N]^T \;\; N \;\; dv$$

Where

$$M_e = \rho \mathrm{BH}L \quad \begin{matrix} \frac{2}{15} & 0 & 0 & \frac{2}{15} & 0 & 0 & \frac{-1}{30} & 0 & 0 \\ 0 & \frac{2}{15} & \frac{L}{180} & 0 & \frac{1}{15} & \frac{-L}{90} & 0 & \frac{-1}{30} & \frac{L}{180} \\ 0 & \frac{L}{180} & \frac{L^2}{1890}+\frac{H^2}{90} & 0 & 0 & \frac{-L^2}{945}+\frac{H^2}{180} & 0 & \frac{-L}{180} & \frac{L^2}{1890}-\frac{H^2}{90} \\ \frac{2}{15} & 0 & 0 & \frac{8}{15} & 0 & 0 & \frac{1}{15} & 0 & 0 \\ 0 & \frac{1}{15} & 0 & 0 & \frac{8}{15} & 0 & 0 & \frac{1}{15} & 0 \\ 0 & \frac{-L}{90} & \frac{-L^2}{945}+\frac{H^2}{180} & 0 & 0 & \frac{2L^2}{1890}+\frac{2H^2}{90} & 0 & \frac{L}{90} & \frac{-L^2}{945}+\frac{H^2}{180} \\ \frac{-1}{30} & 0 & 0 & \frac{1}{15} & 0 & 0 & \frac{2}{15} & 0 & 0 \\ 0 & \frac{-1}{30} & \frac{-L}{180} & 0 & \frac{1}{15} & \frac{-L}{90} & 0 & \frac{2}{15} & \frac{-L}{180} \\ 0 & \frac{L}{180} & \frac{L^2}{1890}-\frac{H^2}{90} & 0 & 0 & \frac{-L^2}{945}+\frac{H^2}{180} & 0 & \frac{-L}{180} & \frac{L^2}{1890}+\frac{H^2}{90} \end{matrix}$$

### *I. Geometrical Stiffness Matrix*

Geometrical stiffness matrix of the composite beam element is given as

$$K_g \;= T \;\; _0^L \; N'^{\,T} \; N' \;\; dx$$

#### *1) Computational procedure for a cracked composite beam*

A computer program is developed to perform all the necessary computations in MATLAB environment. In the initialization phase, geometry and material parameters are specified. For example for a Euler–Bernoulli composite beam model with localized crack, material parameters like modulus of elasticity, the modulus of rigidity, the Poisson ratio and the mass density of the composite beam material and geometric parameters like dimensions of the composite beam, also the specifications of the damage like size of the crack, location of the crack and extent of crack are supplied as input data to the computer program. The beam is divided into n number of elements and n+1 number of nodes. The elements of the mass matrix, elastic stiffness matrix and geometric stiffness matrix are formulated according to above expression and are obtained the non-dimensional natural frequencies and buckling load for non-cracked and cracked composite beam element. The program uses the MATLAB function, "Gauss Quadrature" to carry out the integration part. Element matrices are assembled to obtain the global matrices. Boundary conditions are imposed by elimination method. For Euler–Bernoulli composite beam with fixed-free end conditions the first three rows and columns of the global matrices are eliminated to obtain the reduced matrices.

#### *2) Flow Chart of the Program*

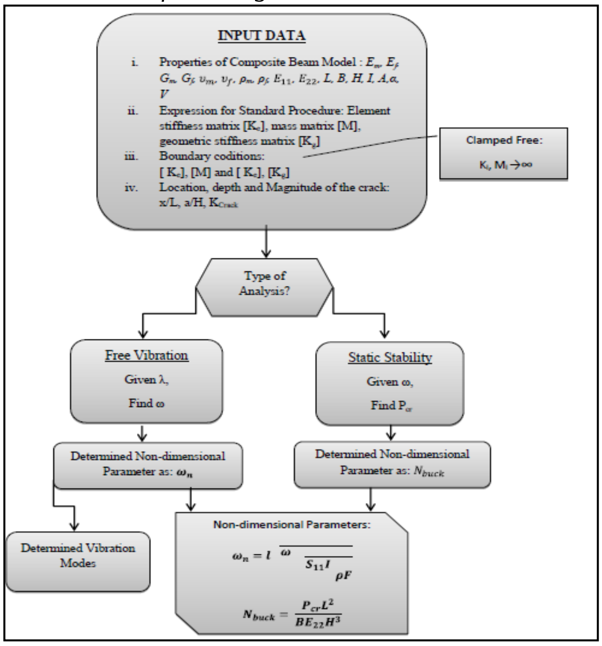


Fig. 1: Flow Chart of the Program

## III. RESULTS AND DISCUSSIONS

### *A. Convergence Study*

The convergence study is carried out for the free vibration of cracked composite beam and omitted here for sake of brevity. Based on this study, a mesh of 12 elements shows good convergence of numerical solutions for free vibration of cracked composite beam

Convergence of non-dimensional free vibration frequencies of cracked composite beam for different angle of fibers V = 0.1, a/H = 0.2, Em = 2.756; Ef = 275.6; v =.2; G =1.036; G =114.8; $\rho$ =1600; $\rho$ = 900;

| Mesh Division | Non dimensional frequencies for different angle of fibers”α”(degrees) | | |
|---|---|---|---|
| | α = 0 | α = 15 | α = 30 |
| 2 elements | 1.5982 | 1.6703 | 1.7125 |
| 4 elements | 1.6815 | 1.7255 | 1.7732 |
| 8 elements | 1.6995 | 1.7257 | 1.7748 |
| 12 elements | 1.7055 | 1.7245 | 1.7743 |
| Krawczuk & Ostachowicz | 1.7055 | 1.7245 | 1.7743 |

Table 1: Mesh Division

### *B. Comparison with Previous Studies*

#### *1) Vibration Analysis Studies*

| Angle of Fibers (degrees) | Volume of Fraction V | Present Analysis | | | Krawczuk & Ostachowicz (1995) | | |
|---|---|---|---|---|---|---|---|
| | | 1st Non-Dimensional Nat. freq | 2nd Non-Dimensional Nat. freq | 3rd Non-Dimensional Nat. freq | 1st Non-dimensional Nat. freq | 2nd Non-dimensional Nat. freq | 3rd Non-dimensional Nat. freq |
| 0 | 0.10 | 1.8798 | 4.6566 | 7.6681 | 1.85145 | 4.52827 | 7.71888 |
| 15 | | 1.8243 | 4.5300 | 7.4841 | 1.81768 | 4.51477 | 7.51418 |
| 30 | | 1.6655 | 4.1530 | 6.9033 | 1.65453 | 4.12945 | 6.89687 |
| 45 | | 1.4342 | 3.5854 | 5.9833 | 1.38995 | 3.53323 | 5.97735 |
| 60 | | 1.2083 | 3.0230 | 5.0513 | 1.15370 | 3.01580 | 5.01780 |
| 75 | | 1.0998 | 2.7514 | 4.5973 | 1.08133 | 2.74520 | 4.57040 |
| 90 | | 1.0881 | 2.7205 | 4.5410 | 1.08007 | 2.71020 | 4.51710 |
| 0 | 0.30 | 1.8771 | 4.6113 | 7.5073 | 1.85145 | 4.52827 | 7.64894 |
| 15 | | 1.8188 | 4.4873 | 7.3447 | 1.81768 | 4.44477 | 7.37372 |
| 30 | | 1.6484 | 4.0982 | 6.7804 | 1.65453 | 4.02945 | 6.92680 |
| 45 | | 1.3886 | 3.4682 | 5.7818 | 1.38995 | 3.43323 | 5.85710 |
| 60 | | 1.1068 | 2.7684 | 4.6260 | 1.15370 | 2.71580 | 4.76640 |
| 75 | | 0.948 | 2.3713 | 3.9632 | 1.08133 | 2.27052 | 4.04030 |
| 90 | | 0.9307 | 2.3263 | 3.8831 | 1.08007 | 2.21720 | 3.97620 |

Table 2: Comparison of First three Non-dimensional natural frequencies of the non-cracked composite beam as a function of the angle of fibers α, where Value of V=0.10 and 0.30.

| Angle of Fibers (degrees) | Relative Cracked Depth (a/H) | Present Analysis | | Krawczuk & Ostachowicz (1995) | |
|---|---|---|---|---|---|
| | | 1st Non-dimensional Nat. freq | 2nd Non-dimensional Nat. freq | 1st Non-dimensional Nat. freq | 2nd Non-dimensional Nat. freq |
| 0 | 0.2 | 1.7070 | 4.5477 | 1.7100 | 4.5400 |
| 15 | | 1.7260 | 4.5656 | 1.7260 | 4.5600 |
| 30 | | 1.7755 | 4.6064 | 1.7705 | 4.6000 |
| 45 | | 1.8337 | 4.6489 | 1.8297 | 4.6400 |
| 60 | | 1.8738 | 4.6771 | 1.8728 | 4.6700 |
| 75 | | 1.8875 | 4.6858 | 1.8805 | 4.6808 |
| 90 | | 1.8886 | 4.6898 | 1.8806 | 4.6820 |
| 0 | 0.4 | 1.4110 | 4.4495 | 1.4150 | 4.4315 |
| 15 | | 1.4428 | 4.6590 | 1.4458 | 4.4500 |
| 30 | | 1.5359 | 4.5075 | 1.5452 | 4.5000 |
| 45 | | 1.6723 | 4.5654 | 1.6753 | 4.5700 |
| 60 | | 1.7939 | 4.6232 | 1.7940 | 4.6232 |
| 75 | | 1.8432 | 4.6492 | 1.8332 | 4.6492 |
| 90 | | 1.8479 | 4.6497 | 1.8409 | 4.6497 |
| 0 | 0.6 | 1.2216 | 4.2150 | 1.1316 | 4.2210 |
| 15 | | 1.2530 | 4.2414 | 1.1930 | 4.2400 |
| 30 | | 1.3484 | 4.3243 | 1.3184 | 4.3300 |
| 45 | | 1.4996 | 4.4474 | 1.5004 | 4.4500 |
| 60 | | 1.6511 | 4.5732 | 1.6523 | 4.5600 |
| 75 | | 1.7189 | 4.6233 | 1.7189 | 4.6200 |
| 90 | | 1.7256 | 4.6272 | 1.7356 | 4.6272 |

Table 3: Comparison of Non-dimensional natural frequencies of the cracked composite beam as a function of the angle of fibers (α) for several values of the crack depth a/H =0.2, 0.4, 0.6 (value fraction of fibers V = 10%, crack location *x*/L = 0.1)

 

*2) Buckling Analysis Studies*

In this buckling analysis study, the results of non-cracked composite beam obtained with the present element are compared with the analytical results of Reddy (1997) and Ozturk & Sabuncu (2005).

| Angle of fibers(degree) | Present FEM | Ozturk & Sabuncu(2005) | Reddy (1997) |
|---|---|---|---|
| 0 | 4.9984 | 5.1404 | 5.14 |
| 30 | 1.6632 | - | - |
| 60 | 0.3891 | - | - |
| 90 | 0.2006 | 0.2056 | 0.205 |

Table 4: Buckling Analysis Study

## IV. CONCLUSION

The following conclusions can be made from the present investigations of the composite beam finite element having transverse non-propagating one-edge open crack. This element is versatile and can be used for static and dynamic analysis of a composite or isotropic beam.

1) From the present investigations it can be concluded that the natural frequencies of vibration of a cracked composite beam is not only the functions of the crack locations and crack depths but also the functions of the angle of fibers and the volume fraction of the fibers. The presence of a transverse crack reduces the natural frequencies of the composite beam.
2) The rate of decrease in the natural frequency of the cracked composite beam increases as the crack position approaches the fixed end.
3) The intensity of the reduction in the frequency increases with the increase in the crack depth ratio. This reduction in natural frequency along with the mode shapes of vibrations can be used to detect the crack location and its depth.

## REFERENCES

[1] Adams R. D., Cawley, P. C., Pye J. and Stone J. (1978). A vibration testing for non-destructively assessing the integrity of the structures. Journal of Mechanical Engineering Sciences 20, 93-100.
[2] Banerjee J. R. (2001). Frequency equation and mode shape formulae for composite Timoshenko beams. Composite Structures 51 (2001) 381-388.
[3] Binici B. (2005) Vibration of beams with multiple open cracks subjected to axial force. J Sound Vib; 287(1-2):277–95.