Parametric Instability of a Tapered Beam Resting on Pasternak Foundation under various Boundary Conditions

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Abstract— The parametric instability of a tapered beam resting on Pasternak foundation subjected to axial periodic load is studied in present work. The equation of motion and associated boundary conditions are obtained using Hamilton's principle. Then, these equations of motion and associated boundary conditions are non-dimensional zed. A set of Mathieu-Hill's equations is obtained from the non-dimensional equations of motion by the application of the general Galerkin's method and these Mathieu-Hill's equations are used to obtain the zones of instability. The effect of taper& shear layer thickness on the zones of parametric instability for various boundary conditions have been investigated.

Key words: Axial Periodic Load, Parametric Instability of a Tapered Beam

I. Introduction

The problem of a beam on an elastic foundation is important in both the civil and mechanical engineering fields, since it constitutes a practical idealization for many problems (e.g. the footing foundation supporting group of columns (as shown in Figure 1) etc.

Fig. 1: Grillage Foundation

The concept of beams and slabs on elastic foundations has been extensively used by geotechnical, pavement and railroad engineers for foundation design and analysis. The analysis of structures resting on elastic foundations is usually based on a relatively simple model of the foundation's response to applied loads. A simple representation of elastic foundation was introduced by Winkler (as shown in Figure 2) in 1867. The Winkler model (one parameter model), which has been originally developed for the analysis of railroad tracks, is very simple but does not accurately represents the characteristics of many practical foundations.

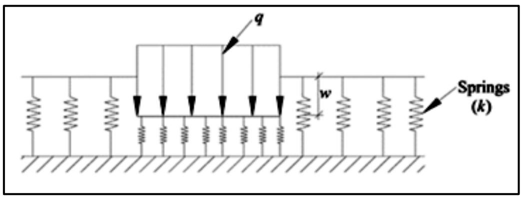

Fig. 2: Deflections of Winkler Foundation under Uniform Pressure Q

In order to eliminate the deficiency of Winkler model, improved theories have been introduced on refinement of Winkler's model, by visualizing various types of interconnections such as shear layers and beams along the Winkler springs.

These theories have been attempted to find an applicable and simple model of representation of foundation medium. To overcome the Winkler model shortcomings improved versions have been developed. A shear layer is introduced in the Winkler foundation and the spring constants above and below this layer is assumed to be different as per this formulation. The following figure shows the physical representation of the Winkler-Pasternak model.

The vibrations of continuously-supported finite and infinite beams on elastic foundation has wide applications in the design of aircraft structures, base frames for rotating machinery, railroad tracks, etc. Quite a good amount of literature exists on this topic, and valuable practical methods for the analysis of beams on elastic foundation have been suggested

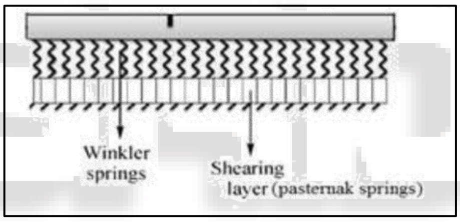

Fig. 3: Winkler-Pasternak Model

Kameswara Rao et al studied the problem of torsional vibration of long, thin-walled beams of open sections resting on Winkler-type elastic foundations using exact, finite element and approximate expressions for torsional frequency of a thin-walled beam and subjected to a time-invariant axial compressive force. It is well known that a dynamic stiffness matrix is mostly formed by frequency-dependent shape functions which are exact solutions of the governing differential equations. It overcomes the discretization errors and is capable of predicting an infinite number of natural modes by means of a finite number of degrees of freedom. This method has been applied successfully for many dynamic problems including natural vibration.

A general dynamic-stiffness matrix of a Timoshenko beam for transverse vibrations was derived including the effects of rotary inertia of the mass, shear distortion, structural damping, axial force, elastic spring and dashpot foundation. Analytical expressions were derived for the coupled bending-torsional dynamic stiffness matrix terms of an axially loaded uniform Timoshenko beam element and also a dynamic stiffness matrix is derived based on Bernoulli–Euler beam theory for determining natural frequencies and mode shapes of the coupled bending-torsion vibration of axially loaded thin-walled beams with mono-symmetrical cross sections, by using a general solution of the governing differential equations of motion including the effect of warping stiffness and axial force and Using the technical computing program Mathematica, a new dynamic stiffness matrix was derived based on the power series method for the spatially coupled free vibration analysis of thin-walled curved beam with non-symmetric cross-section on Winkler and also Pasternak types of elastic foundation and The free vibration frequencies of a beam were also derived with flexible ends resting on Pasternak soil, in the presence of a concentrated mass at an arbitrary intermediate abscissa. The static and dynamic behaviors of tapered beams were studied using the differential quadrature method (DQM) and also a finite element procedure was developed for analyzing the flexural vibrations of a uniform Timoshenko beam-column on a two-parameter elastic foundation.

Though many interesting studies are reported in the literature, the case of doubly-symmetric thin-walled open section beams resting on Winkler–Pasternak foundation is not dealt sufficiently in the available literature to the best of the author's knowledge.

In view of the above, the present paper deals with the free torsional vibrations of doubly symmetric thin-walled beams of open section and resting on Winkler-Pasternak continuous foundation. A general dynamic stiffness matrix is developed in this paper which includes the effects of warping and Winkler-Pasternak foundation on the torsional natural frequencies. The resulting highly transcendental frequency equations for a simply supported and guided-end condition are solved for varying values of warping Winkler and Pasternak foundation parameters on its frequencies of vibration. A new MATLAB code was developed based on Euler Bernoulli Beam method to solve the highly transcendental frequency equations and to accurately determine the torsional natural frequencies for various boundary conditions. Numerical results for natural frequencies for various values of warping and Winkler and Pasternak foundation parameters are obtained and presented in graphical form showing their parametric influence clearly.

II. BASICS OF EULER-BERNOULLI BEAM THEORY

A. *Euler-Bernoulli Beam Theory*

This theory is the most basic theory for beams. To derive the equation of motion for a beam that is slender, a small piece of the beam will be analyzed. The rotation of cross sections of the beam is neglected compared to the translation. In addition, the distortion due to shear is considered negligible compared to the bending deformation.

1) *Euler-Bernoulli Beam Assumptions*

- The beam is long and slender.
- The beam is loaded in its plane of symmetry.
- Torsion = 0.
- No buckling.
- No plasticity.
- Material is isotropic.
- Length >> width and length >> depth, Therefore tensile/compressive stresses perpendicular to the beam are much smaller than that of tensile/compressive stresses parallel to the beam.

The transverse displacement of the centreline of the beam is given by w, the displacement components of any points in the cross section, when plane sections remain plane and normal to the centreline, are given by

$$u = -\left(\frac{\partial w(x,t)}{\partial x}\right); v = 0; w = w(x,t)$$

This can be seen from the following figure (4):

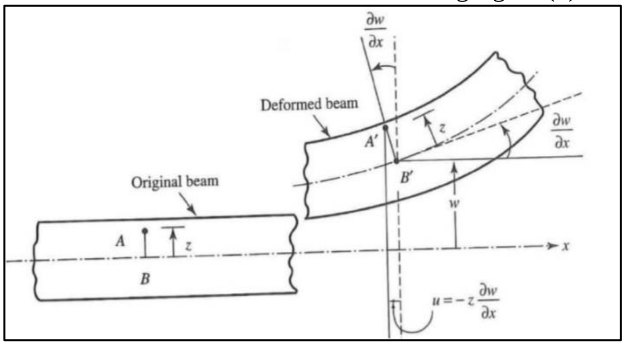

Fig. 4: Deformed Euler Bernoulli beam

Here it is assumed that the displacements are small, such that $\tan\theta = \theta$.

III. PROPOSED METHOD

A. *Formulation of Governing Differential Equation of Tapered Beam*

A general Euler's Bernoulli beam is considered which is tapered linearly in both horizontal as well as in vertical planes. Fig.3.1 shows the variation of width and depth in top and front view.

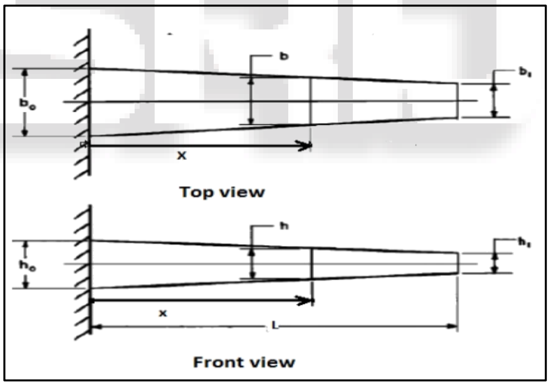

Fig. 5: Top and front view of cantilever tapered beam with linearly varying width and depth.

The width and depth are varying linearly given by

$$\text{Depth, } h = h_o - \left(\frac{x}{l}\right)(h_0 - h_1);$$

$$\text{Width, } b = b_o - \left(\frac{x}{l}\right)(b_0 - b_1);$$

Similarly area and moment of inertia will be varying accordingly

$$A = b \times h$$

$$A(x) = \left[h_o - \left(\frac{x}{l}\right)(h_0 - h_1)\right]\left[b_o - \left(\frac{x}{l}\right)(b_0 - b_1)\right];$$

$$I = \frac{1}{12}bh^3$$

$$I(x) = \frac{1}{12}\left[h_o - \left(\frac{x}{l}\right)(h_0 - h_1)\right]^3\left[b_o - \left(\frac{x}{l}\right)(b_0 - b_1)\right];$$

If we write in non-dimension form then,

Depth, $h = h_o - (\bar{x})(h_0 - h_1)$;

Width, $b = b_o - (\bar{x})(b_0 - b_1)$;

Similarly write area and area moment of inertia also in non-dimension form then,

$$A(\bar{x}) = [h_o - (\bar{x})(h_0 - h_1)][b_o - (\bar{x})(b_0 - b_1)];$$
$$I(\bar{x}) = \frac{1}{12}[h_o - (\bar{x})(h_0 - h_1)]^3[b_o - (\bar{x})(b_0 - b_1)];$$

B. Calculation of Shape Function

We are considering cubic polynomial function $W_i(\bar{x})$for various boundary conditions,

$$W_i(\bar{x}) = a\bar{x}^i + b\bar{x}^{i+1} + c\bar{x}^{i+2} + d\bar{x}^{i+3} \qquad (4.1)$$

Now calculate shape function for clamped-free, clamped-clamped, clamped-pined, pin-pined.

1) Clamped-Free Beam

The shape function $W_i(\bar{x})$ for clamped-free beam,

$$W_i(\bar{x}) = a\bar{x}^i + b\bar{x}^{i+1} + c\bar{x}^{i+2} + d\bar{x}^{i+3}$$

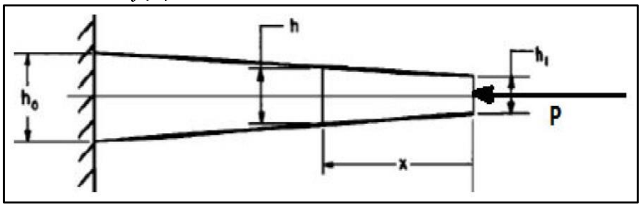

Fig. 6: clamped- free beam

Now apply the boundary conditions for clamped-free beam, and find out the constants in above equation,

$$W_i(\bar{x}) = 0\,;\ \frac{dW_i(\bar{x})}{d\bar{x}} = 0;\ at\ \bar{x} = 0$$
$$\frac{d{W_i}^2(\bar{x})}{d\bar{x}^2} = 0\,;\ \frac{d{W_i}^3}{d\bar{x}^3} = 0;\ at\ \bar{x} = 1$$

Then we will get,

$$W_i(\bar{x}) = (i+2)(i+3)\bar{x}^{i+1} - i(i+1)(i+3)\bar{x}^{i+2} + i^2(i+1)\bar{x}^{i+3} \qquad (4.2)$$

2) Clamped-Clamped Beam

The shape function $W_i(\bar{x})$ for clamped-clamped beam,

$$W_i(\bar{x}) = a\bar{x}^i + b\bar{x}^{i+1} + c\bar{x}^{i+2} + d\bar{x}^{i+3}$$

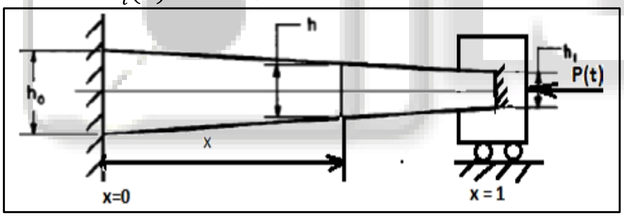

Fig. 7: clamped-clamped beam

Now apply the boundary conditions for clamped-clamped beam, and find out the constants in above equation,

$$W_i(\bar{x}) = 0\,;\ \frac{dW_i(\bar{x})}{d\bar{x}} = 0;\ at\ \bar{x} = 0\ \&\ \bar{x} = 1$$

Then we will get,

$$W_i(\bar{x}) = \bar{x}^{i+1} - 2\bar{x}^{i+2} + \bar{x}^{i+3} \qquad (4.3)$$

3) Clamped-Pinned Beam

The shape function $W_i(\bar{x})$ for clamped-pinned beam,

$$W_i(\bar{x}) = a\bar{x}^i + b\bar{x}^{i+1} + c\bar{x}^{i+2} + d\bar{x}^{i+3}$$

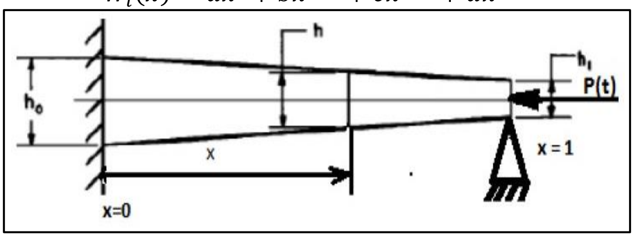

Fig. 8: Clamped-Pined Beam

Now apply the boundary conditions for clamped-pinned beam, and find out the constants in above equation,

$$W_i(\bar{x}) = 0\,;\ \frac{dW_i(\bar{x})}{d\bar{x}} = 0;\ at\ \bar{x} = 0$$
$$W_i(\bar{x}) = 0\,;\ \frac{d{W_i}^2(\bar{x})}{d\bar{x}^2} = 0;\ at\ \bar{x} = 1$$

Then we will get,

$$W_i(\bar{x}) = (i+1)\bar{x}^{i+1} - 0.5(i+1)^2(i+3) - i(i+1)(i+2)\bar{x}^{i+2} + (i+1)\bar{x}^{i+3} \qquad (4.4)$$

4) Pinned-Pinned Beam

The shape function $W_i(\bar{x})$ for pinned-pinned beam,

$$W_i(\bar{x}) = \sin(\pi i\bar{x}) \qquad (4.5)$$

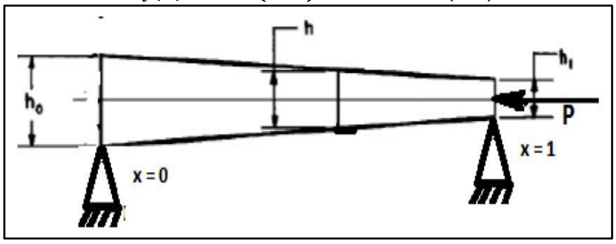

Fig. 9: pined-pined beam

Now apply the boundary conditions for pinned-pinned beam, and find out the constants in above equation

$$W_i(\bar{x}) = 0\,;\ \frac{d{W_i}^2(\bar{x})}{d^2\bar{x}} = 0;\ at\ \bar{x} = 0\ \&\ \bar{x} = 1$$

C. Modeling of Euler Bernoulli Beam on Pasternak foundation

An Euler-Bernoulli beam of length 'l' resting on a Pasternak foundation with a periodic force $P(t) = P_0 + P_1\cos\omega t$ acting axially on it is shown in fig:1, ω being the frequency of the applied load, t being the time and P_0 and P_1 are the static and dynamic load amplitudes respectively. The beam may be subjected to any of the boundary conditions as clamped-free, clamped-pined, pined-pined, and clamped-clamped. The beam having thickness h, width b, moment of inertia I, Young's modulus E, shear modulus Gs, and spring constant K.

$$P(t) = P_0 + P_1\cos\omega t$$

ω = Frequency of the load
t = time
P_0 = Static amplitude
P_1 = dynamic amplitude

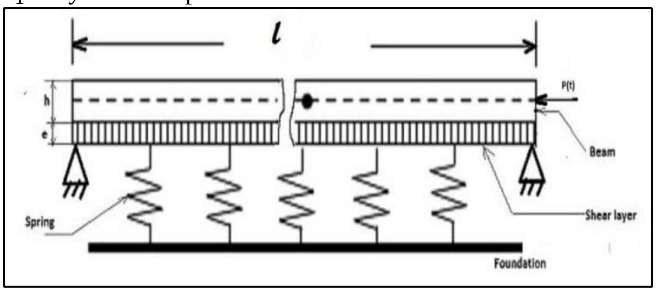

Fig. 10: Beam resting on Pasternak foundation subjected to axial periodic load

The expressions for kinetic energy, potential energy & work done are as follows

Kinetic energy:

$$T = \frac{1}{2}\int_0^l m\left(\frac{\partial w}{\partial t}\right)^2 dx \qquad (4.6)$$

Potential energy:

$$U = \frac{1}{2}\int_0^l EI\left(\frac{\partial^2 w}{\partial t^2}\right)^2 dx \qquad (4.7)$$

Strain energy due to spring:

$$U_{spring} = \frac{1}{2}K\int_0^l (W)^2\, dx \qquad (4.8)$$

Strain energy due to shear layer:

$$U_{Shear\ layer} = \frac{1}{2} GA \int_0^l \left(\frac{\partial w}{\partial x}\right)^2 dx \qquad (4.9)$$

Work potential:

$$W_p = \frac{1}{2}\int_0^l P \left(\frac{\partial w}{\partial x}\right)^2 dx \qquad (4.10)$$

Now non-dimension the above equations,

$$x = l.\bar{x} \implies dx = l.d\bar{x}$$

Where $x = 0 \longrightarrow \bar{x} = 0$

$$x = l \longrightarrow \bar{x} = 1$$

$$w = l.\bar{w} \longrightarrow dw = l.d\bar{w}$$

$$\frac{\partial w}{\partial t} = \frac{1}{t_0}\left(\frac{\partial \bar{w}}{\partial t}\right);\ \because \frac{\partial \bar{t}}{\partial t} = \frac{1}{t_0}$$

$$\frac{\partial^2 w}{\partial x^2} = \frac{1}{l}\frac{\partial^2 \bar{w}}{\partial \bar{x}^2}$$

$$a = h_0(1 - \bar{x} \times \alpha_1);\ \&\ b = b_0(1 - \bar{x} \times \alpha_2)$$

$$\alpha_1 = \frac{h_0 - h_1}{h_0}$$

$$\alpha_2 = \frac{b_0 - b_1}{b_0}$$

After non-dimensionalized of the Eqs. 3.6 to 3.10 the following equations are obtained.

$$T = \frac{El^3}{24}\int_0^l ab \left(\frac{\partial \bar{w}}{\partial \bar{t}}\right)^2 d\bar{x} \qquad (4.11)$$

$$U = \frac{El^3}{24}\int_0^l a^3 b \left(\frac{\partial^2 \bar{w}}{\partial \bar{x}^2}\right)^2 d\bar{x} \qquad (4.12)$$

$$U_{spring} = \frac{1}{2} Kl \int_0^l (\bar{W})^2 d\bar{x} \qquad (4.13)$$

$$U_{Shear\ layer} = \frac{1}{2} G_s \delta b \int_0^l l \left(\frac{\partial \bar{w}}{\partial \bar{x}}\right)^2 d\bar{x} \qquad (4.14)$$

$$W_p = \frac{\bar{P}El^3}{24}\int_0^l a^3 b \left(\frac{\partial \bar{w}}{\partial \bar{x}}\right)^2 d\bar{x} \qquad (4.15)$$

Now apply above Eqs: 4.11 to 4.12 in Anderson's equation

$$\frac{d}{d\bar{t}}\left[T + U - W_p\right] = 0 \qquad (4.16)$$

Split the above equation

$$\frac{dT}{d\bar{t}} = 0$$

$$\frac{dT}{d\bar{t}} = \frac{d}{d\bar{t}}\left[\int_0^l ab \left(\frac{\partial \bar{w}}{\partial \bar{t}}\right)^2 d\bar{x}\right]$$

$$\frac{dT}{d\bar{t}} = \frac{d}{d\bar{t}}\left[\int_0^l ab\left(\sum_i \sum_j W_i W_j \dot{f}_i \dot{f}_j\right) d\bar{x}\right]$$

Here

$$\frac{\partial \bar{W}}{\partial \bar{t}} = \frac{\partial}{\partial \bar{t}}\left(\sum w_i f_i\right) = \sum w_i \dot{f}_i$$

and $\frac{\partial}{\partial \bar{t}}\left[\sum_i \sum_j W_i W_j \dot{f}_i \dot{f}_j\right] = 2\sum_i \sum_j W_i W_j \ddot{f}_i \dot{f}_j$

$$\frac{dT}{d\bar{t}} = \sum_i \sum_j [M_{ij}] \ddot{f}_i \dot{f}_j$$

Where

$$\frac{dT}{d\bar{t}} = \int_0^l 2\sum_i \sum_j W_i W_j \ddot{f}_i \dot{f}_j\ d\bar{x}$$

Mass Matrix $[M_{ij}] = \int_0^l 2abW_iW_j\ d\bar{x}$

$$\frac{dU}{d\bar{t}} = \frac{d}{d\bar{t}}\left[\int_0^l a^3 b \left(\frac{\partial^2 \bar{w}}{\partial \bar{x}^2}\right)^2 d\bar{x}\right]$$

$$\frac{dU}{d\bar{t}} = \sum_i \sum_j [K_{ij}] f_i \dot{f}_j$$

Stiffness matrix $[K_{ij}] = \int_0^l 2a^3 b \ddot{W}_i \ddot{W}_j\ d\bar{x}$

$$\frac{dU_{spring}}{d\bar{t}} = \frac{d}{d\bar{t}}\left[\frac{12K}{El^2}\int_0^l (\bar{W})^2 d\bar{x}\right]$$

$$\frac{dU_{spring}}{d\bar{t}} = \sum_i \sum_j [K_{ij}]_{spring} f_i \dot{f}_j$$

Stiffness matrix due to spring

$$[K_{ij}]_{spring} = \frac{24K}{El^2}\int_0^l \bar{W}_i \bar{W}_j d\bar{x}$$

$$\frac{dU_{Shear\ layer}}{d\bar{t}} = \frac{d}{d\bar{t}}\left[\frac{12G_s\delta b}{El^2}\int_0^l \left(\frac{\partial \bar{w}}{\partial \bar{x}}\right)^2 d\bar{x}\right]$$

$$\frac{dU_{Shear\ layer}}{d\bar{t}} = \sum_i \sum_j [K_{ij}]_{shear-layer} f_i \dot{f}_j$$

Stiffness matrix due to shear-layer

$$[K_{ij}]_{shear-layer} = \frac{12G_s\delta b}{El^2}\int_0^l \dot{\bar{W}}_i \dot{\bar{W}}_j d\bar{x}$$

Similarly work potential

$$\frac{dW_p}{d\bar{t}} = \frac{d}{d\bar{t}}\left[\int_0^l \bar{p} a^3 b \left(\frac{\partial \bar{w}}{\partial \bar{x}}\right)^2 d\bar{x}\right]$$

$$\frac{dW_p}{d\bar{t}} = \sum_i \sum_j [h_{ij}] f_i \dot{f}_j$$

$$[h_{ij}] = 2\int_0^l \bar{p} a^3 b \dot{W}_i \dot{W}_j . d\bar{x}$$

Now re-write the Anderson's equation,

$$\frac{dT}{d\bar{t}} + \frac{dU}{d\bar{t}} + \frac{dU_{spring}}{d\bar{t}} + \frac{dU_{shear-layer}}{d\bar{t}} - \frac{dW_p}{d\bar{t}} = 0$$

Now, let the transverse displacement as

$$\bar{W}(\bar{x}, \bar{t}) = \sum_{t=1}^{N} W_i(\bar{x}).f_i(\bar{t}) \qquad (4.17)$$

Substitute above equations in Anderson's equation, then we will get

$$\sum_i \sum_j [M_{ij}] \ddot{f}_i \dot{f}_j + \sum_i \sum_j [K_{ij}] f_i \dot{f}_j + \sum_i \sum_j [K_{ij}]_{spring} f_i \dot{f}_j + \sum_i \sum_j [K_{ij}]_{shear-layer} f_i \dot{f}_j - \bar{p}\sum_i \sum_j [h_{ij}] f_i \dot{f}_j = 0$$

$$[M]\{\ddot{f}\} + [K]\{f\} + [K]_{spring}\{f\} + [K]_{shear-layer}\{f\} - \bar{p}[h]\{f\} = \{\varphi\}$$

$$[M]\{\ddot{f}\} + [K]\{f\} + [K]_{spring}\{f\} + [K]_{shear-layer}\{f\} - \bar{p}_0[H]\{f\} - \bar{p}_1\cos(\bar{\omega}\bar{t})[H]\{f\} = \{0\}$$

$$[M]\{\ddot{f}\} + [K_1]\{f\} - \bar{p}_1\cos(\bar{\omega}\bar{t})[H]\{f\} = \{0\}$$

Where

$$[K_1] = [K] + [K]_{spring} + [K]_{shear-layer} - \bar{p}_0[H]$$

Using the model matrix $[Q]$ as a coordinate transformation matrix, we can uncouple the EOM, so if we use the transformation $\{f\} = [Q]\{p\}$, where $\{p\}$ is set of principal coordinates or say normal coordinates

$$[Q]^T[M][Q]\{\ddot{p}\} + [Q]^T[K_1][Q]\{p\} - \bar{p}_1\cos(\bar{\omega}\bar{t})[Q]^T[H][Q]\{p\} = \{0\}$$

So

$$[M]\{\ddot{p}\}[K_{11}]\{p\} - \bar{p}_1\cos(\bar{\omega}\bar{t})[Q]^T[H][Q]\{p\} = \{0\}$$

Here

$$[M_{11}] = [Q]^T[M][Q]$$

$$[K_{11}] = [Q]^T[K][Q]$$

Now multiplying the equation (3.17) by inverse of $[M_{11}]$, we get

$$\{\ddot{p}\} + {\omega_i}^2\{p\} - \bar{p}_1\cos(\bar{\omega}\bar{t})[M_{11}]^{-1}[Q]^T[H][Q]\{p\} = 0 \qquad (4.18)$$

Here,

$${\omega_i}^2 = [M_{11}]^{-1}[K_{11}]$$

1) Regions of Instability

The regions of instability are found out using the result of Hsu's equation (*3.19*) and result of Mathieu equation (*3.20*). Hsu's equation is used for finding the instability regions for the combination resonance and whereas Mathieu equation for finding the instability regions for the simple resonance.

$$\omega = \omega_i + \omega_j \pm \left[\varepsilon \left(\frac{b_{ij}\times b_{ji}}{\overline{\omega_i}\times\overline{\omega_j}}\right)^{\frac{1}{2}}\right] + ()(\varepsilon^2) \qquad (4.19)$$

$$\omega = 2\omega_i \pm \varepsilon \frac{b_{ij}}{\overline{\omega_i}} + ()(\varepsilon^2) \qquad (4.20)$$

Here b_{ij} are the elements of

$$[B] = -[Q]^{-1}[M]^{-1}[H][Q] \ \& \ \epsilon = \bar{p}\left(\frac{1}{2}\right)$$

Also $(0 < \epsilon < 1)$

IV. NUMERICAL RESULTS AND DISCUSSION

The natural frequencies were obtained for relevant values of system parameters and these were compared with those given by W.T. Thomson [3] and good agreement was observed. Also, the first three natural frequencies were obtained for the reference Euler beam corresponding to each boundary condition and the results obtained were found to be agree.

A. Basic Information

The basic configuration of problem considered here, a beam of length l which is subjected to an axial load(t). Taking, $\bar{p}_0$=0.5, l=50 cm, h_0=2cm, h_1=1cm, b_1=2cm, b_1=1cm.

Natural frequencies	Clamped-free beam	Clamped-pined beam	Clamped-Clamped beam	Pined-pined beam
1	2.7452	37.7823	65.4821	26.8844
2	51.1721	140.1434	180.1472	115.7377
3	170.1085	298.6804	353.0172	260.4956

Table 1: Non-foundation with various boundary conditions dimensionalized natural frequencies of tapered beam

The Euler Bernoulli beam resting on a Pasternak foundation with various boundary conditions for tapered beam with l= 50 cm, h_0=2 cm, h_1=1 cm, b_0=2 cm, $b_1$1 cm, shear thickness e=0.5 cm, K (spring constant) = 2000 $N/_{cm^2}$, Gs(Shear modulus) = 80e5$N/_{cm^2}$, E (Young's modulus) = 210e5$N/_{cm^2}$.

Natural frequencies	Clamped-free beam	Clamped-pined beam	Clamped-clamped beam	Pined-pined beam
1	3.49044	37.7988	65.6323	27.1581
2	51.5501	140.3677	180.3532	116.0133
3	170.4054	298.922	353.2447	260.7762

Table 2: Non-dimension analyzed natural frequencies of tapered beam resting on Pasternak foundation

Figure 11 below shows the parametric instability of tapered beam subjected to axial periodic load with various boundary conditions such as, a. clamped-free beam, b. clamped-pined beam, c. clamped-clamped beam, and d. pined-pined beam.

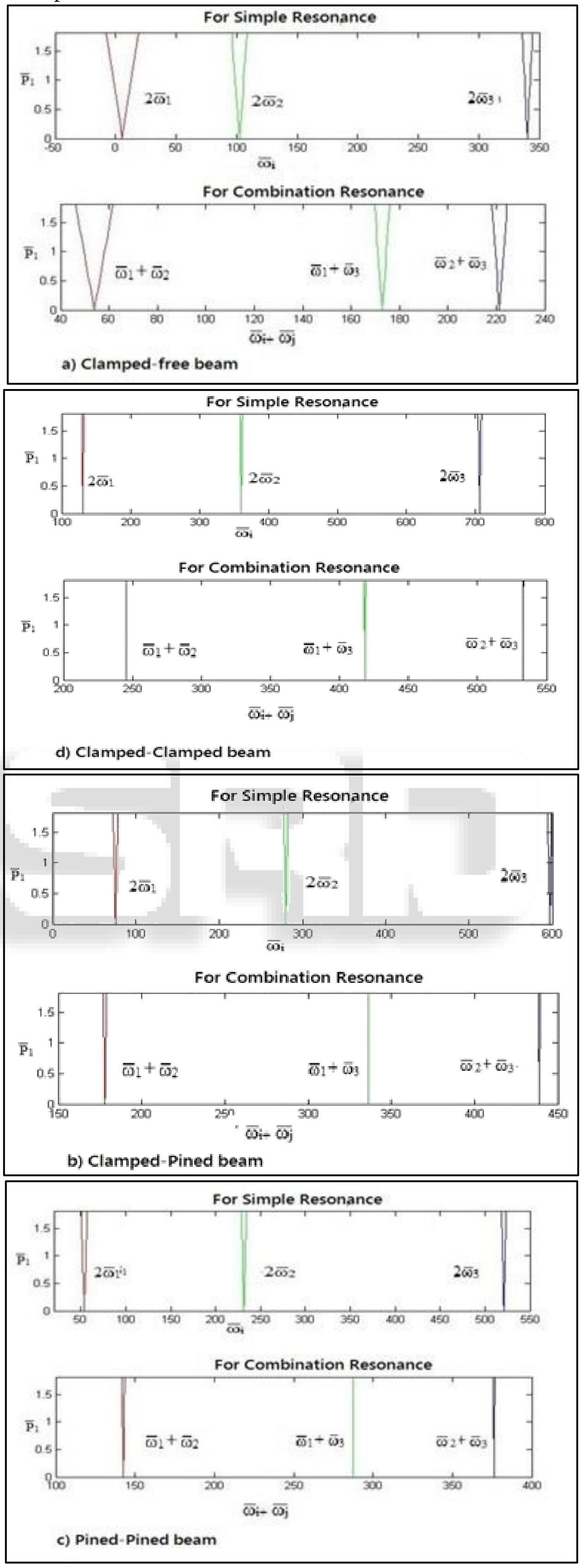

Fig. 11: Zones of instability for a tapered beam without Pasternak foundation

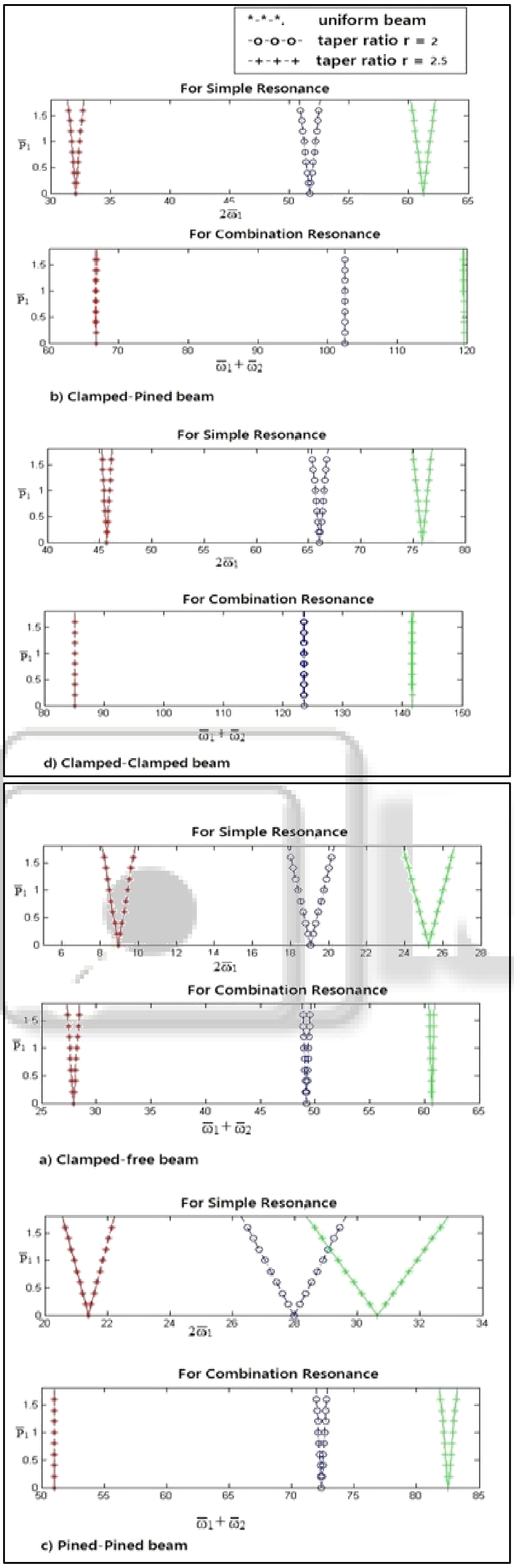

Fig. 12: Zones of instability for convergent tapered beam resting on Pasternak foundation (taper ratio = h_0/h_1 & b_0/b_1)

Fig: 12 addresses the parametric instability of tapered beam resting on Pasternak foundation subjected to axial periodic load with various boundary conditions. From the above figure it can be seen that parametric instability zones are widest in Clamped-Free beam when compared to the other boundary conditions, so we can say Clamped-Free beam is more unstable of the four configurations.

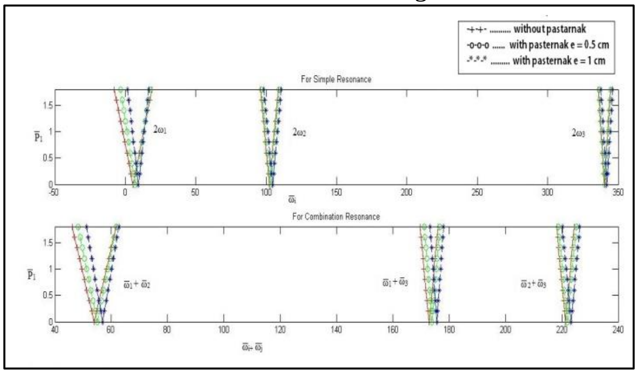

Fig. 13: Zones of instability for a fixed-free tapered beam resting on Pasternak foundation with varying shear layer thickness.

Fig: 13 shows the comparison of zones of instability for fixed-free taper beam resting on Pasternak foundation to explain the effect of shear layer thickness. An increase in the shear layer thickness stabilizes dynamically has been noticed.

V. CONCLUSIONS AND SCOPE FOR FUTURE WORK

A. *Conclusion*

The parametric instability of a tapered beam resting on Pasternak foundation subjected to axial periodic load is studied by analytical method. The programming has been developed using MATLAB. The following conclusions are drawn from the present work.

1) Instability regions for simple resonance are wider than the instability regions for combination resonance for taper beam.
2) An increase in the non-dimensional dynamic load in uniform and non-uniform Euler Bernoulli beams, make the instability regions become wider.
3) An increase in the divergent taper ratio in Euler Bernoulli beams causes the instability regions to become wider.
4) An increase in the thickness of the shear layer stabilizes the beam dynamically, an increase in the stiffness of the foundation is seen to have similar effect.
5) For clamped-free and clamped-pined cases, divergent taper is dynamically more unstable compared to convergent taper.

B. *Scope for Future Work*

1) FEM analysis of the same problem.
2) Consideration of other boundary conditions.
3) Consideration of other foundations such as the Winkler foundation.

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