The Statistical Modeling of Wavelet Coefficients as a Tool for Image De-Noising

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Abstract— This paper proposes a spatially adaptive statistical model for wavelet image coefficients in order to perform image de-noising. The wavelet coefficients are modeled as zero-mean Gaussian random variables with high local correlation. This model is developed in a Bayesian framework, where a Maximum Likelihood (ML) estimator evaluates the variance of the blocks to which the wavelet subbands have been segmented. Then, applying the Minimum Mean Squared Error (MMSE) estimation procedure, the original or de-noised wavelet image coefficients are estimated. The reliable estimation of local variance is performed by making the assumption that variance is locally smooth. The validity of this assumption is boosted by segmenting the wavelet subbands into blocks of variable size. The segmentation employs quad-tree decomposition of the image and a linear transfer of the resulted tree on the wavelet subbands. This decomposition identifies object boundaries and defines more accurately the regions of smooth variance instead of dividing them in to blocks of fixed size. The extensive experimental evaluation, shows that the proposed scheme demonstrates very good performance as far as PSNR measures and visual quality are concerned with respect to others state of the art de-noising schemes.

Key words: Wavelets, De-Noising, Quad-Tree Decomposition

I. INTRODUCTION

An image is often corrupted by noise during its acquisition or transmission. Image de-noising is used to remove the additive noise while retaining as much as possible the important image features. In the recent years there has been a fair amount of research on filtering and wavelet coefficients thresholding, because wavelets provide an appropriate basis for separating noisy signal from the image signal. These wavelet-based methods mainly rely on thresholding the Discrete Wavelet Transform (DWT) coefficients, which have been affected by Additive White Gaussian Noise (AWGN).

Since the work of Donoho and Johnstone [1]-[4], there has been a lot of research on the way of defining the threshold levels and their type (i.e. hard or soft threshold). These algorithms usually perform global thresholding of wavelet coefficients by retaining only large coefficients and setting the rest to zero. Thus, they do not present spatial adaptivity and their performance in real life images is not sufficiently effective.

A wide class of image processing algorithms is based on the DWT. The transform coefficients within the subbands can be locally modeled as independent identically distributed (i.i.d) random variables with Generalized Gaussian Distribution (GGD) [5]. In that sense, the de-noised coefficients may be evaluated by an MMSE (Minimum Mean Square Error) estimator, in terms of the noised coefficients and the variances of signal and noise. The signal variance is locally estimated by an ML (Maximum Likelihood) estimator, whereas noise variance is estimated from the first level diagonal details. Therefore, the de-noised coefficients are statistically estimated in small regions for every subband instead of applying a global threshold [6]. These methods present efficient results but their spatial adaptivity is not well suited near object edges where the variance field is not smoothly varied. In [7] a similar spatially adaptive model for wavelet image coefficients was used to perform image de-noising via wavelet thresholding.

The present work employs the spatially adaptive model as in [6] and performs MMSE coefficient estimation, rather than coefficient thresholding as in [7]. However, it differs from [6] in the way that the underlying variance field is estimated. In our work, this estimation is performed in a variable block size framework in contradiction to [6] where a fixed block size framework is used. The subbands segmentation into blocks of variable size is performed by quad-tree decomposition of the noisy image and a linear transfer of the resulted tree on the wavelet subbands. This approximation is rational if the spatial dependency among the image and the wavelet subbands is considered. This decomposition identifies object boundaries and defines more accurately the regions of smooth variance instead of dividing them in to blocks of standard size. In addition, the algorithm becomes faster as fewer blocks are created.

This paper is organized as follows. Section 2 describes the proposed de-noising algorithm. The experimental results are presented in Section 3 and the conclusions are summarized in Section 4.

II. THE PROPOSED DE-NOISING ALGORITHM

A. *The Statistical Model*

The statistical model of the proposed de-noising algorithm is shown in Fig. 1. A noise contaminated image may be formulated as in the illustrated block-diagram. A “clean” image, x, is decomposed by DWT providing the wavelet coefficients X(k). These coefficients, which may be locally considered as i.i.d GGD random variables with variance) σ^2_X (k), are corrupted by additive i.i.d Gaussian noise samples, n(k), to produce the observed wavelet coefficients of the noisy image, Y(k).

Let W and W^{-1} denote the two dimensional DWT and its inverse respectively. The relationship between image and transform coefficients is:

$$X=Wx \text{ and } x=W^{-1}X \qquad (1)$$

The “clean” coefficients, *X*, may be estimated from the observed coefficients, *Y*, if noise variance, σ^2_n and image variance, σ^2_X (k) are known. Here, a robust median estimator of the highest subband diagonal coefficients (i.e. HH_1) estimates the noise variance [2]. Also, an ML estimation of image variance is performed for every transform coefficient, using the observed noisy data in its local neighborhood. In this work, the local neighborhood is

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defined by segmenting the image or every subband into variable size blocks by quad-tree decomposition. Finally, an MMSE estimator provides an estimate of the "clean" coefficients, $\hat{X}(k)$. The reconstructed de-noised image is given by:

$$\hat{x} = W^{-1}\hat{X} \quad (2)$$

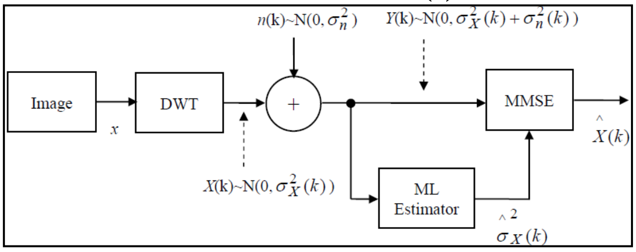

Fig. 1: Block-diagram of the proposed de-noising algorithm. The statistical model is based on an ML estimator for an estimate of the underlying variance and an MMSE estimator for the evaluation of the "clean" wavelet coefficients.

B. The Mathematical Estimation

It is known that the best estimate of a random variable x, given by the MMSE estimator, is:

$$\hat{X} = E[X] \quad (3)$$

Also, under the assumptions of independence and Gaussian distribution of the random variables, it is known that [8]:

$$f_X \sim N(0, \sigma_X^2)$$
$$f_Y \sim N(0, \sigma_X^2 + \sigma_n^2)$$
$$f_{Y|X} \sim N(x, \sigma_n^2) \quad (4)$$

The last equation describes the conditional distribution of the observed values when the "clean" values are known. The Bayesian estimation of the "clean" values, given the observed values, is:

$$f_{X|Y} = \frac{f_{Y|X} f_X}{f_Y} = \frac{N(x, \sigma_n^2) N(0, \sigma_X^2)}{N(0, \sigma_X^2 + \sigma_n^2)} \quad (5)$$

If the Gaussian distributions are replaced by their explicit forms, equation (5) results in:

$$f_{X|Y} \sim N\left(\frac{\sigma_X^2}{\sigma_X^2 + \sigma_n^2} y, \frac{\sigma_X^2 \sigma_n^2}{\sigma_X^2 + \sigma_n^2}\right) \quad (6)$$

The estimated "clean" wavelet coefficients result in considering equations (3) and (6):

$$\hat{X}(k) = \frac{\sigma_X^2}{\sigma_X^2 + \sigma_n^2} Y(k) \quad (7)$$

But in fact σ_X^2 is not known, so we employ an ML estimator in order to have an estimate, $\hat{\sigma}_x^2$, for a local neighborhood, where variance is assumed to be constant. The ML estimate is defined as: 2^Xσ

$$\hat{X}_{ML}(y) = \arg\max_x f_{Y|X}(y \mid x) \quad (8)$$

In our case, this takes the following form:

$$\hat{X}_{ML}(y) = \arg\max_{\sigma_y^2} \prod_{j \in N} f(y, \sigma_y^2) \quad (9)$$

Where N is the local neighborhood. The maximum of the above equation is found to be for:

$$\sigma_y^2 = \frac{1}{M} \sum_{k=1}^{M} Y^2(k) \quad (10)$$

Where M represents the number of wavelet coefficients residing in the local neighborhood N.

Therefore, the estimate of the "clean" coefficients variance is:

$$\hat{\sigma}_x^2 = \frac{1}{N} \sum_{k=1}^{M} Y^2(k) - \sigma_n^2 \quad (11)$$

Finally, the "clean" coefficients are estimated by combining (7) and (11):

$$\hat{X}(k) = \frac{\hat{\sigma}_x^2}{\hat{\sigma}_x^2 + \sigma_n^2} Y(k) \quad (12)$$

Where the noise variance is estimated, as it was stated in the previous subsection, by:

$$\sigma_n^2 = \left[\frac{median(|Y(k)|)}{0.6745}\right]^2 \quad (13)$$

Where Y(k) represents the coefficients of HH_1 subband.

C. The Proposed Method of De-Noising

Applying the aforementioned analysis, the noise contaminated image is subjected to DWT and the "clean" coefficients are estimated by equations (11), (12) and (13). Then, the "clean" image is attained by reconstruction employing the inverse DWT. However, the above mentioned equations are valid assuming that the underlying variance field in the subbands is varying smoothly in a local neighbourhood. The local neighbourhood may be defined by segmenting the subbands in blocks of fixed size [6] or in blocks of variable size as it is proposed in this work.

The proposed method of segmentation is based on quad-tree decomposition (QTD) of the noisy image applying an intensity difference splitting criterion. According to this criterion, a parent block splits into four children blocks if the intensity gradient within block is greater than a predefined threshold. Fig. 2 illustrates the segmentation of "Lena" image and its associated subbands with QTD. The image is transformed by DWT and the segmentation tree is linearly transferred in every subband because there is a spatial dependency among the subbands and the image. This decomposition identifies object boundaries and defines more accurately the regions of smooth variance instead of dividing them in to blocks of fixed size. Also, the computational complexity is lowered because the execution of the algorithm is performed in fewer blocks.

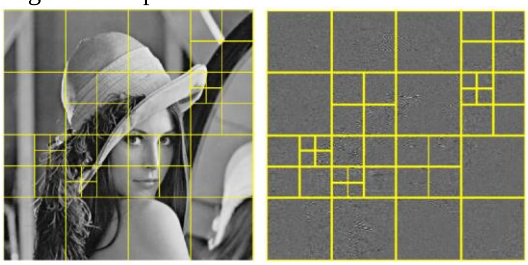

Fig. 2: QTD segmentation into blocks of variable size. (a) Noisy image; (b) Horizontal subband (HH_1) after DWT

III. Experimental Results

The experimental evaluation is performed on two gray scale images like "Lena" and "Barbara" of size 512X512 pixels at different noise levels. The wavelet transform employs

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Daubechies's least asymmetric compactly supported wavelet with eight vanishing moments [9] at four levels of decomposition. The objective quality of the reconstructed image is measured by:

$$PSNR = 10\log_{10}\frac{255^2}{mse} \qquad (14)$$

Where *mse* is the mean square error between the original (i.e. *x*) and the de-noised image (i.e. $\hat{x}$) with size *M* x *N*:

$$mse = \frac{1}{MxN}\sum_{i=1}^{M}\sum_{j=1}^{N}[x(i,j) - \hat{x}(i,j)]^2 \qquad (15)$$

To assess the performance of our proposed method, it is compared with SureShrink [3], BayesShrink [7], NormalShrink [10], Wiener [11] and LAWML [6]. The first of the above mentioned algorithms is the hard-thresholding of wavelet coefficients using a constant threshold for all subbands that is estimated from HH_1 subband. The second algorithm uses spatially adaptive wavelet thresholding. The third algorithm employs the same principle as the previous one in order to estimate subband dependent threshold. The fourth algorithm is based on Matlab's image de-noising algorithm and the last one employs the statistical modeling of wavelet coefficients in order to estimate the "clean" coefficients using the observed ones and estimating the underlying variance field in a local neighborhood of fixed size. The PSNR from the various methods are compared in Table 1 and the best ones are highlighted with bold fonts.

	SureShrink	**BayesShrink**	**NormalShrink**	**Wiener**	**LAWML**	**QTD**
			Lena			
σ=10	33.34	33.16	32.80	33.59	**33.87**	33.60
σ=15	31.22	31.18	30.89	31.12	**31.58**	31.52
σ=20	29.80	29.82	29.60	28.99	29.91	**30.17**
σ=25	28.67	28.87	28.54	27.19	28.56	**29.11**
σ=30	28.07	**28.12**	27.72	25.67	27.62	28.07
			Barbara			
σ=10	31.47	31.25	30.63	29.90	**32.49**	31.16
σ=15	28.97	28.76	28.02	28.30	**30.08**	29.34
σ=20	27.32	27.20	26.38	26.84	**28.33**	27.97
σ=25	26.13	26.09	25.13	25.51	27.11	**27.15**
σ=30	25.14	25.17	24.31	24.28	25.95	**26.02**

Table 1: PSNR comparative results for the test images "Lena" and "Barbara" and for various values of noise standard deviation.

It is apparent that our proposed method, called QTD, presents a very competitive performance compared to all other methods in almost the whole examining range. For example, in "Lena" QTD outperforms LAWML by 0.45 dB for σ=30, that is, in a heavily noise contaminated image. Similarly, in "Barbara" QTD outperforms LAWML by 0.1 dB for σ=30. Fig. 3 demonstrates the subjective quality performance among the three methods that present the best PSNR results for the test image "Lena". The tested image is heavily noise contaminated with a noise standard deviation of σ=30. It may be observed that the quality of the de-noised image of our proposed method is better than SureShrink and BayesShrink, both in low and high textured areas.

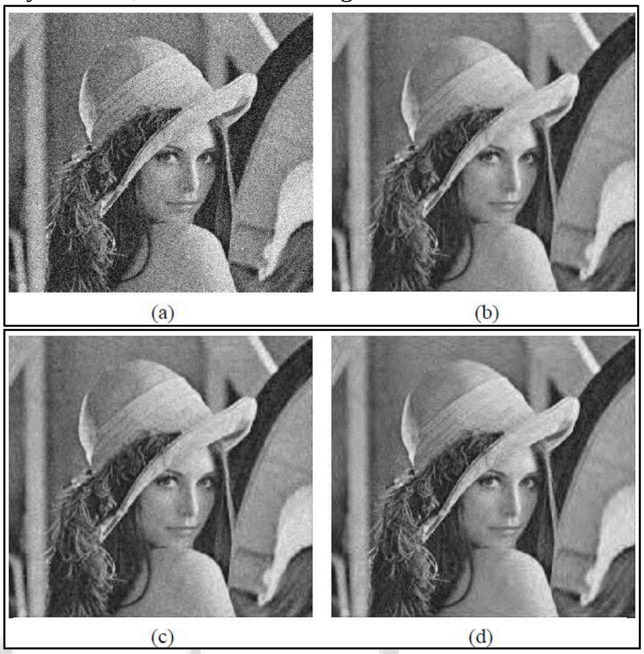
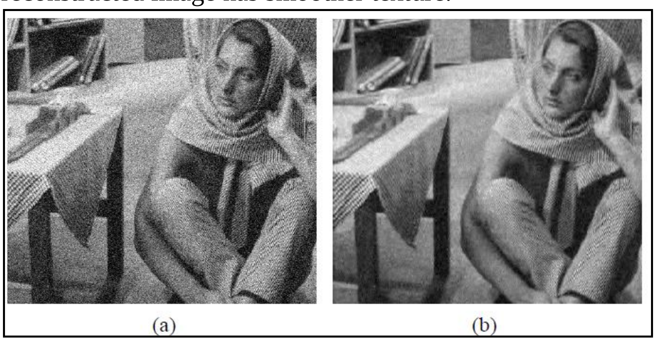


Fig. 3: Subjective quality performance comparison for σ=30. (a) Noisy image (b) SureShrink (c) BayesShrink (d) QTD.

In the same way, Fig.4 demonstrates the subjective quality performance among the three methods that present the best PSNR results for the test image "Barbara". The tested image is heavily noise contaminated with a noise standard deviation of σ=30. Finally, Fig. 5 shows the quality difference between the best two de-noising algorithms, LAWML and QTD. The reconstructed image has been magnified in order to observe differences around a region that contains low and high texture areas. It may be observed that our proposed algorithm QTD performs equally well to LAWML around the kerchief stripes of "Barbara" and outperforms LAWML around her face, where the reconstructed image has smoother texture.



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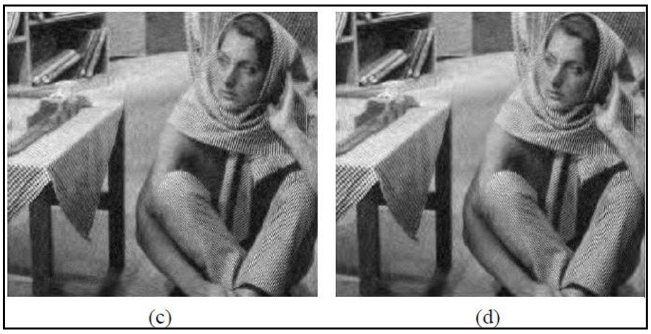

Fig. 4: Subjective quality performance comparison for σ=30. (a) Noisy image; (b) BayesShrink; (c) LAWML; (d) QTD.

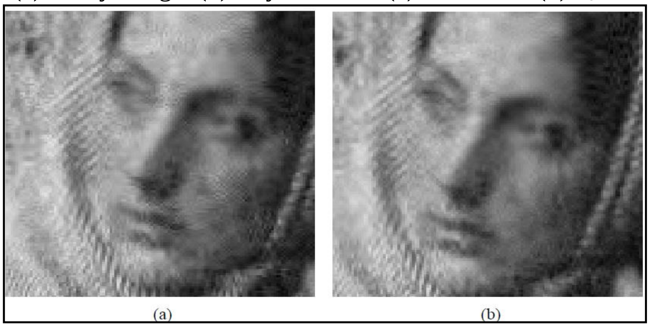

Fig. 5: Subjective quality performance comparison for σ=30. (a) LAWML (b) QTD.

IV. CONCLUSIONS

In this paper, a new method for recovering an image from noise contamination effectively is proposed. It is based on the wavelet decomposition of the image and the Generalized Gaussian Distribution modelling of the subband coefficients. The proposed method employs a spatially adaptive model and performs MMSE coefficient estimation instead of the classical threshold estimation. Specifically, it segments the subbands into blocks of variable size and estimates the variance in each block assuming that it is smoothly varying in a local neighborhood. The segmentation is performed by quad-tree decomposition employing an intensity splitting criterion and the resulting tree is transferred linearly to the subbands. This decomposition identifies object boundaries and defines more accurately the regions of smooth variance instead of dividing them in to blocks of standard size. The noise variance is estimated by the robust estimator used by SureShrink method. Finally, the "clean" coefficients are estimated by an MMSE estimator and the "clean" image is recovered by an inverse transform.

The proposed method is tested with two grey scale images for various values of noise variance and its performance is compared with other de-noising algorithms. The experimental evaluation showed that the proposed method has a very good performance, providing reconstructed images with fairly good quality.

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