Analysis of an Asymmetric Spur Gear with Multiple Holes as Stress Reliving Feature

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Abstract— Mechanical power and torque transmission from one point to another point with minimum wastage always been a challenging topic in the field of research in the area of Gear design. Gears are not designed any more with a complete standard methodology. Rather it is now designed in customized way with different profile for different applications accordingly. The best example in this area is asymmetric gears. Asymmetric gears are mainly used in aviation vehicles and for this reason reduction of gear weight without compromising its torque transmitting capability much. This demand from practical field drives the necessity to do research in the area of asymmetric gear design with stress reliving features. Many researchers have already done on asymmetric gear design with different stress relieving features. Till now all the researches have been concentrated on any single type stress relieving features. Influence of multiple stress relieving features with different shape has not been investigated much. In the present work the research has been done in this area.

Key words: Asymmetric Gear, Stress Relieving Feature, Circular Hole, Elliptical Hole, Von-Misses Stress

I. INTRODUCTION

Designing gears for aviation vehicle always a challenging job. Here the gears should be as light as possible without lowering or reducing its torque transmission capability. Concept of porous gear evolved out of this practical field demand. But perfect or optimized design of gears with stress relieving features is a very tedious job. Investigations to understand the influence of different stress relieving features with different shapes and different relative positions are still hugely unexplored. Another concept to reduce gear size without reducing its torque transmitting capability is designing of asymmetric gear. Asymmetric gears are mainly used in those vehicles where there is no back gear concept the power transmission system. Aviation vehicles do not have back gears. So concept of asymmetric gear is very useful for aviation industry.

Russian scientists first started work on this concept of asymmetric gear. In 1970s and 80s many researches were done by many Russian scientists and engineers. E. B. Vulgakov [1] published a paper on improvement of the characteristics of gear design in the year of 1974. Then in the year of 1984 I. A. Bolotovsky, O. F. Vasil'eva and V. P.Kotelnikov[2] did a work on asymmetric involute gear and published a paper. Work of A. L. Kapelevich[3] on asymmetric is also very remarkable which he did and published in the year of 1987. There is a fabulous work in gear manufacturing done by Mabie H. H., Rogers C. A., and Reinholtz C. F [4] and they published it in the year of 1990. In the year of 1993 N. Canesan and S. Vijayarangan [5] did an analysis on composite spur-gear with three-dimensional finite element method. Another beautiful work on analysis of asymmetric gear tooth has been done by G.DiFrancesco and S.Marini[6] in the year of 1997 and was published in 'Gear Technology'. Kapelevich A.L. [7] did a remarkable work on involute spar gear with asymmetric tooth profile in the year of 2000 and published it in the renowned journal named "Mechanism and Machine Theory". In the same year another important work was done on the process of noise reduction of asymmetric gear drive during gear meshing. That was done by Litvin, F.L., Q. Lian and A.L. Kapelevich[8] and they published their work as a research paper in the journal "Computer Methods in Applied Mechanics and Engineering". In the year of 2001 so many works were done on the analysis of asymmetric gear. G. Gang and T. Nakanishi [9] did a work on "Enhancement of Bending load Carrying Capacity of Gears Using an Asymmetric Involute Tooth," and published their work in "The JSME International Conference on Motion and Transmissions (MPT2001-Fukuoka), Fukuoka, JAPAN". Yeh,T.,Yang and D. Tong [10] did a work on "Design of new tooth profiles for high-load capacity gears" which was published in the journal "Mechanism and Machine Theory". Another work was done in that same year that is in 2001 on

Tooth contact analysis of an asymmetric gear jointly by US Army Research Laboratory and NASA [11]. The document number of that research work is NASA/TM-2001-210614, ARL-TR-2373. In next two years that is in 2002 and 2003 few works were done on direct gear design method to design spar and helical gear. A.L. Kapelevich and R.E. Kleiss [12] did a work "Direct Gear Design for Spur and Helical Gears" and published in journal Gear Technology (September/October 2003, 29-35). Kapelevich A.L. and Shekhtman Y.V. [13] did a work on "Direct Gear Design: Bending Stress Minimization" and that was published in Gear Technology, September/October 2003, 44-49. In the year of 2005 K. Cavdar, F. Karpat and F.C. Babalik [14] performed a fabulous work on "Computer aided analysis of bending strength of involute spur gears with asymmetric profile" and published their work in Journal of Mechanical Design 127 (3) (2005) 477-484. In the year of 2007 and 2008 many works on asymmetric gear were done. Flavia CHIRA, VasileTisan and Anamaria Dascalescu [15] did a work on "Modelling of the Asymmetric Gears using Applications in Matlab and Autolisp" and published in ANNALS Of THE ORADEA UNIVERSITY, Fascicle of Management and Technological Engineering, Volume VI (XVI), 2007. Kapelevich A.L. [16] did a work on "Direct Design Approach for High Performance Gear Transmissions" and was published in Gear Solutions, January 2008, 22-31. This article was presented at the Global Powertrain Congress 2007 June 17-19, 2007, Berlin, Germany and published in the Global Powertrain Congress Proceedings, Vol. 39-42, 66-71. A.S. Novikov, A.G. Paikin, V.L. Dorofeyev,V.M. Ananiev and

A.L. Kapelevich [18] did a work on "Application of Gears with Asymmetric Teeth in Turboprop Engine Gearbox" and published his work in Gear Technology, January/February, 2008.

Few recent work have been done in the field of gear design with stress relieving features in India. These works have contributed remarkably in the asymmetric gear design with stress relieving features. Mr. Sumit Agrawal and Dr. R. L. Himte have done a very good work on analysis of an asymmetric spur gear tooth with a circular stress relieving feature. Their work was published in International Journal of Computer Applications (0975 – 8887) in the year of 2012 as a work title "Evaluation of Bending Stress at Fillet Region of an Asymmetric Gear with a Hole as Stress Relieving Feature using a FEA Software ANSYS"

Another very recent work done by Mr. Satvinder Singh Bhatia and Mr. Hari Ram Chandrakar on investigation of the influence of elliptical hole as stress relieving feature in an asymmetric gear which they published as a title of "Analysis of an Asymmetric Spur Gear with an Elliptical Hole as Stress Relieving Feature Using ANSYS APDL as Tool" in the journal named International Journal for Research in Applied Science & Engineering Technology (IJRASET) in the year of 2015.

Above mentioned two papers have been referred in this work as a base paper. In these papers author have done remarkable work but influence of multiple type of holes have not been investigated. In the present work effect of one circular as well as one elliptical hole on the fillet stress in an asymmetric spur gear has been investigated.

II. Design Parameters of an Asymmetric Gear

Work of authors in references [24] and [25] have been reproduced first in this work. Following are the dimensions referred from the reference [24] and [25].

Parameters	Symmetric Toothed Gear	Asymmetric toothed Gear
Number of Tooth (N)	32	32
Diametral Pitch (p)	0.21	0.21
Drive Pressure Angle (ϕd)	25°	35°
Coast Pressure Angle (ϕc)	25°	15°
Pitch Diameter (Dp)	152.4mm	152.4mm
Drive Base Diameter (Dbd)	138.12mm	124.84mm
Coast Base Diameter	138.12mm	147.21mm
Outside Diameter	162.56mm	162.21mm
Root Diameter	141.5034mm	141.1732mm
Fillet Radius	1.8796mm	1.9812mm
Face width	9.525mm	9.525mm
Torque	564923.94N-mm	564923.94N-mm
Load Application Radius	81.28mm	81.28mm

Table 1: Parameters of the Asymmetric Gear from Ref. [24] & [25]

An asymmetric gear has been modelled in PTC Creo Parametric 2.0 software with help of the above mentioned dimensions. To create parametric models following parameters have been adopted.

- Diametral Pitch (P)
- Number of teeth (N)
- Pressure Angle (PHI)
- Pitch Diameter (DP)
- Addendum (A)
- Duodenum (B)
- Addendum circle diameter (DA)
- Root circle diameter (DR)
- Base circle diameter (DB)
- Fillet radius (FR)
- Face width (F)

Among the above mentioned parameters the user inputs are only Diametral Pitch (P), Number of teeth (N) and Pressure Angle (PHI). Other parameters have been calculated as per the following equations.

Pitch Circle Diameter (DP) = Number of Teeth (N)/ Diametral Pitch (P)
Addendum (A) = 1/Diametral Pitch (P)
Duodenum (B) = 1.157/ Diametral Pitch (P)
Addendum Circle Dia (Da) = DP + 2*A
Duodenum Circle Dia or Root Circle Dia (Dr) = DP – 2*B
Base Circle Dia (Db) = DP * Cos(PHI)
Fillet radius (r) = 0.4 *A
Face Width (F) = 0.0625*DP

With help of the above equations a parametric involute profile has been generated then using the following equations an asymmetric involute profile has been generated. These equations have been referred from reference [24] and [25]

For Drive direction

```
todeg=180/pi
alpha=t*sqrt((Da/Dbd)^2-1)
alpha2=sqrt((Dp/Dbd)^2-1)
r=0.5*Dbd*sqrt(1+alpha^2)
theta=alpha*todeg-atan(alpha)-(alpha2*todeg-atan(alpha2))-(90/n)-1
z=0
```

For Coast direction

```
todeg=180/pi
alpha= t*sqrt((Da/Dbc)^2-1)
alpha2=sqrt((Dp/Dbc)^2-1)
r=0.5*Dbc*sqrt(1+alpha^2)
theta=alpha*todeg-atan(alpha)-(alpha2*todeg-atan(alpha2))-(90/n)+.4
z=0
```

Using the above equations involute curve for asymmetric gear has been generated in Pro-Engineer software and using the involute curves thus generated, a profile of involute asymmetric gear with one gear tooth has been generated. Finally a 3-D model of asymmetric gear tooth has been created using the profile as mentioned above.

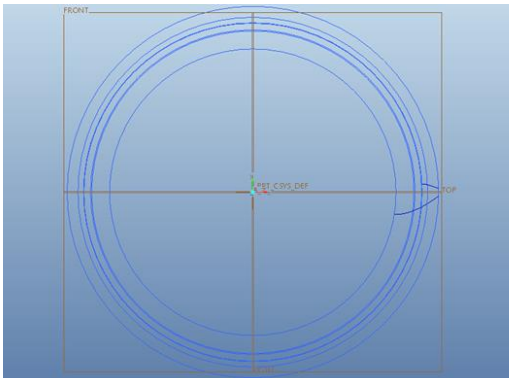

Fig. 1: Involute curves for asymmetric gear.

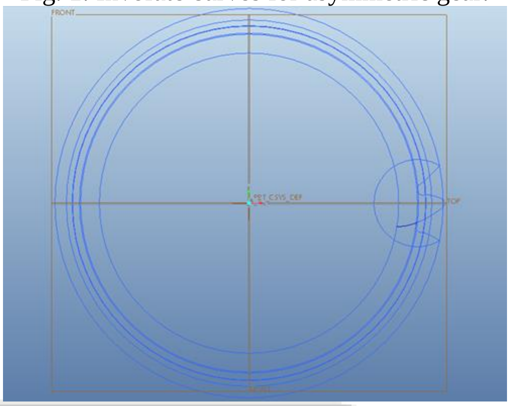

Fig. 2: Partial profile of asymmetric involute gear tooth.

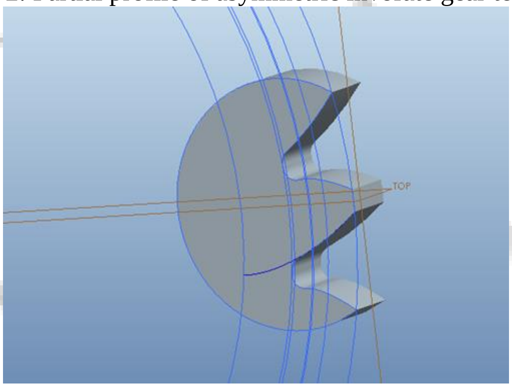

Fig. 3: 3-Dimensional model of an asymmetric gear tooth

III. STRUCTURAL ANALYSIS OF THE ASYMMETRIC GEAR WITH CIRCULAR HOLE

After generation of asymmetric gear Mr. S. Agrawal et. al. introduced a circular hole as a stress relieving feature. They introduced the hole at different positions and measured the fillet region stress.

To calculate load on the gear work of Frederick W. Brown et. al. (Reference [17]) has been referred in the base papers. As per Frederick W. Brown et. al.[17] torque value of 564923.94N-mm has been applied on the gear with a torque radius of 81.28mm. This gives the tangential force of 6950.334 N on the gear tooth.

In reality load is actually exerted on a line of contact passing through a point near pitch circle. But it is not possible as the mashing is unstructured and so a series of nodes cannot be available long a line near pitch circle. To avoid this problem load in the simulation is imposed at tip of the gear model with a modified pressure angle. To make the above consideration or assumption effective following equation has been implemented or used. The equation is:

$$\emptyset_m = \emptyset - \frac{s_a}{2r_a} \qquad (1)$$

Where-
ϕm is modified pressure angle.
ϕ is actual pressure angle.
sa is tooth thickness at addendum circle.
ra is addendum circle radius.

From the above equation the modified pressure angle has been calculated as 24.05°.

After importing the 3-D gear model in ANSYS software it has been meshed in finite elements. After that simulation has been done with the above mentioned loading and boundary conditions.

Contour plot of Von-Misses stress has been derived for whole the tooth. Figure below shows the Von-Misses stress of gear tooth.

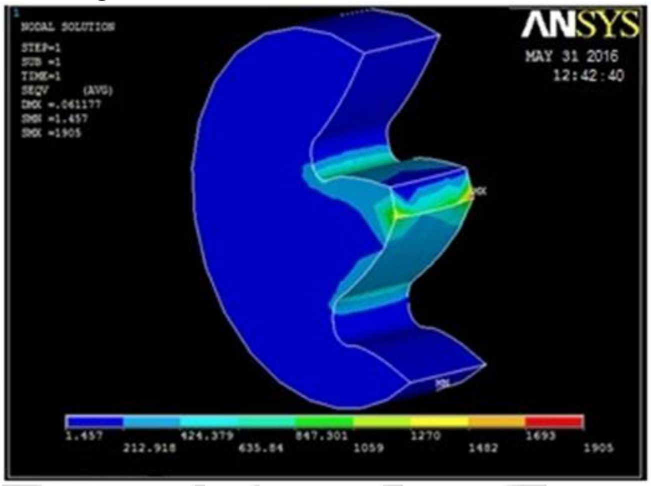

Fig. 4: Von-Misses Stress Distribution of asymmetric gear.

From the above figure it is clear that maximum stress occurs at the tip of the gear tooth. But here bending stress is needed to be calculated. To calculate bending stress at the fillet region a graph between Von-Misses stress of each point at the edge of fillet region and their distance from a reference has been plotted. Figure below shows the graph.

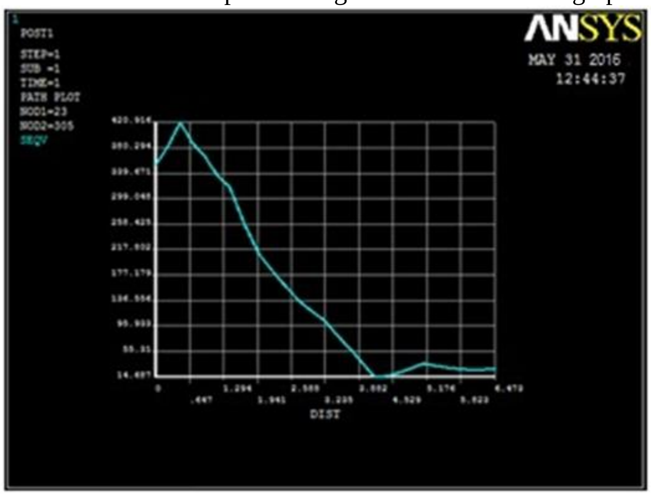

Fig. 5: Graph of Von-Misses Stress Distribution at fillet area of symmetric gear.

The above graph shows that the maximum bending stress at the fillet region is 420.916 N/mm2 which agrees the result found by Frederick W. Brown et al (Reference [17]) in their work. So, it can be said mathematical model of asymmetric gear tooth with involute profile has been validated for further FEA analysis.

Now to improve the stressed fillet region the modification which has been adopted by S. Agrawal et. al. [24] has been discussed. In their work S Agrawal et. al have introduced a circular hole on the face of the asymmetric gear in the vicinity of the fillet area and have found an beneficial position and size of the hole after few trials. Below is the FEA analysis of asymmetric gear with circular hole as mentioned in Reff [24].

S. Agrawal et al [24] has controlled the dimension and position of the circular hole in their work through few parameters like 'HR' representing radius of hole, 'HPR' pitch circle radius of hole-center and 'THETA' representing angular position of hole with respect to the axis of gear.

Here hole's radius 'HR' and hole's pitch circle radius 'HPR' have been parameterized with radius if fillet of gear tooth 'r' and diameter of pitch circle of gear tooth 'DP' respectively like following. THETA is different for different trials.

$$HR = r \times p_1$$
$$HPR = D_p \times p_2$$

Where, p1 and p2 are parameters control the configuration and position of hole.

In the final trial of the work of S. Agrawal et al [24] p1 value has been taken 0.5 and p2 has been considered as 0.445. Using these parameters a three dimensional model of asymmetric gear with a hole has been created in the same way as mentioned for trial one above. Figures below show two view of three dimensional model of asymmetric gear tooth with hole created in second trial.

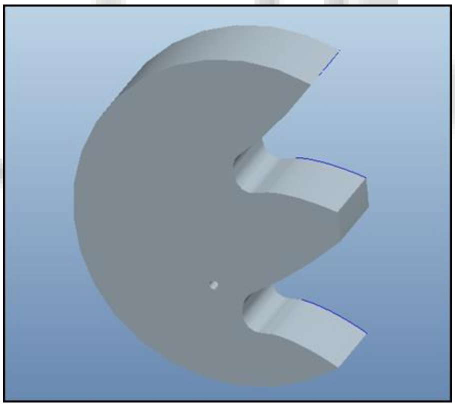

Fig. 6: Isometric view of 3-D asymmetric gear with hole in 3rd trial

From the above figure it is clearly depicted that maximum stress occurs at tooth tip but bending stress occurs at the fillet area. The software calculates all the stresses but on contour plot it only shows the magnitude of maximum stress. To find out bending stress at the fillet region it is needed to plot a graph between Von-Misses stress at different nodes of any section and distance of those nodes from a reference point. Figure below shows the graph of Von-Misses stress at different point on the edge of fillet at section of gear tooth versus distances of those points from a reference.

After generating three dimensional model of asymmetric gear with a hole for third trial position and size in PTC Creo 2.0 (as show in figure above), it has been imported in ANSYS for meshing. After doing Meshing and imposing same boundary conditions and loading conditions following result are derived.

Figures below show the contour plotting for Von-Misses stress on whole the gear tooth and stress around the hole.

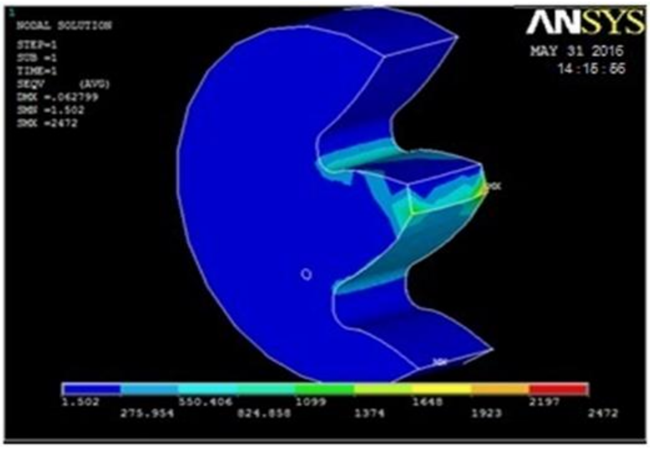

Fig. 7: Contour plotting of stress on whole the gear tooth and near hole.

From the above figure it is clearly depicted that maximum stress occurs at tooth tip but bending stress occurs at the fillet area. The software calculates all the stresses but on contour plot it only shows the magnitude of maximum stress. To find out bending stress at the fillet region it is needed to plot a graph between Von-Misses stress at different nodes of any section and distance of those nodes from a reference point. Figure below shows the graph of Von-Misses stress at different point on the edge of fillet at section of gear tooth versus distances of those points from a reference.

From the above analysis it has been shown that result for maximum bending stress occurred at the fillet region of the gear when circular hole as stress reliving feature is not considered is well agreed with the result found by Frederick W. Brown, Scott R. Davidson, David B. Hanes and Dale J. Weires in their work (Ref. [17]). Whatever discrepancies have been raised between the result of this work and work of Frederick W. Brown et al [17] is due to the fact that Frederick W. Brown et al considered 2-Dimensional model of gear in their analysis and in the present work 3-Dimensional model has been created and analysed.

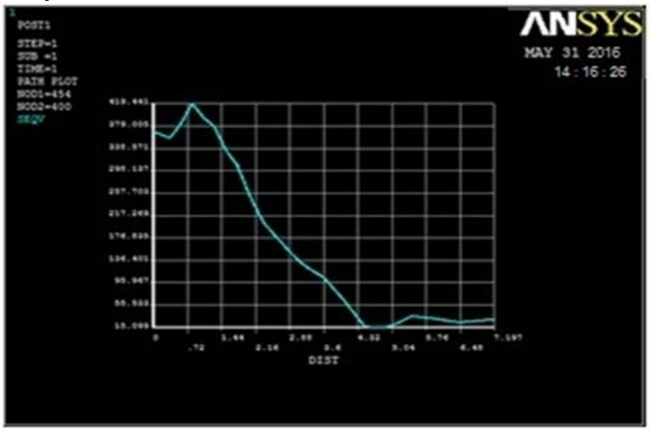

Fig. 8: Graph of Von-Misses Stress Distribution at fillet area of asymmetric gear with hole at third trial.

After validating the work of Frederick et al [17], work of S. Agrawal et al [24] has also been validated considering circular hole as a stress reliving feature.

IV. VALIDATION OF A WORK WITH ELLIPTICAL STRESS RELIEVING FEATURE

In this section work of Mr. Satvinder Singh Bhatia and Mr. Hari Ram Chandrakar has been reproduced who considered shape of the hole as elliptical. The dimension and position of the elliptical hole were found out by trial and error method using FEA software ANSYS for further reduced bending stress. A 3-dimensional model of asymmetric gear tooth with elliptical hole was created in PTC Creo Parametric software parametrically. Dimension of hole were manipulated by parameters 'HMJD' and 'HMND' which represented elliptical hole's major dia and minor dia. Position was manipulated by two parameters 'HPD' & 'HPANG' which represented hole's pitch-circle diameter and angular position of hole with respect to x-axis. Another parameter named 'HAANG' was introduced to specify the angle between the axis of hole-center and hole's major axis.

Here hole's major dia 'HMJD', minor dia 'HMND' and hole's pitch circle Dia 'HPD' were parameterized with radius if fillet of gear tooth 'FR' and diameter of pitch circle of gear tooth 'Dp' respectively like following. 'HPANG' and 'HAANG' were different for different trials.

$$HPD = D_p \times par_1$$

$$HMJD = FR \times par_2$$

$$HMND = FR \times par_3$$

Where, par1, par2 and par3 are parameters were used to control the configuration and position of hole.

Parameter 'par1' defined radial distance of hole-center from the center of gear. Parameter 'par2' and 'par3' defined major axis and minor axis of elliptical hole.

Mr. Satvinder Singh Bhatia et. al. adopted three trials with three set of different above mentioned parameter values. For the first trial 'par1' value was taken 0.85, 'par2' and 'par3' were considered as 0.8 and 0.4 respectively. In the next trial 'par1' value was taken 0.88, 'par2' and 'par3' were considered as 0.8 and 0.4 respectively. HPANG and HAANG were been taken as 7º and 90º respectively. In the last trial they considered 'par1' value as 0.9, 'par2' and 'par3' were considered as 0.8 and 0.4 respectively. HPANG and HAANG considered as 7º and 120º respectively.

In this present work only last trial values have been reproduced. Flowing figures shows the physical model of gear tooth and its FEA model with last trial values.

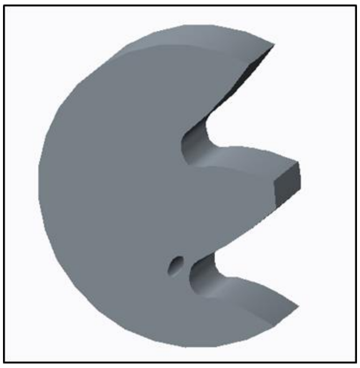

Fig. 9: Isometric view of 3-D asymmetric gear with hole in 3rd trial.

After imposing loads on each node boundary conditions have been fixed. The curved part of the gear tooth has been held fixed by assigning all degree of freedom to zero. Now FEA model of asymmetric gear tooth with second trial hole has been solved in ANSYS. After solution bending stress has been derived from post-processing. Figures below shows the contour plotting for Von-Misses stress on whole the gear tooth and stress around the hole.

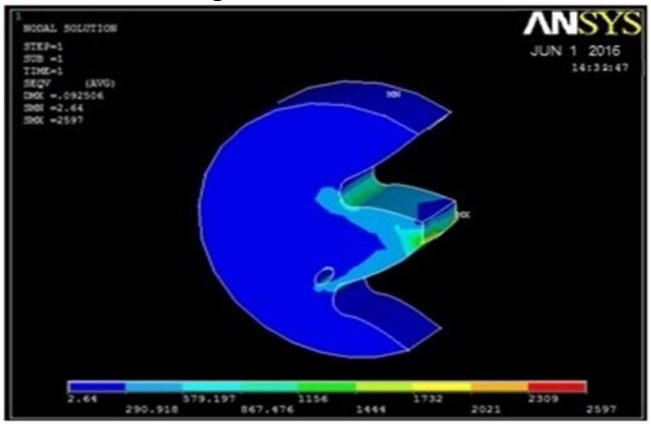

Fig. 10: Contour plotting of stress on whole the gear tooth and near hole.

Figure below shows the graph of Von-Misses stress at different point on the edge of fillet at section of gear tooth versus distances of those points from a reference.

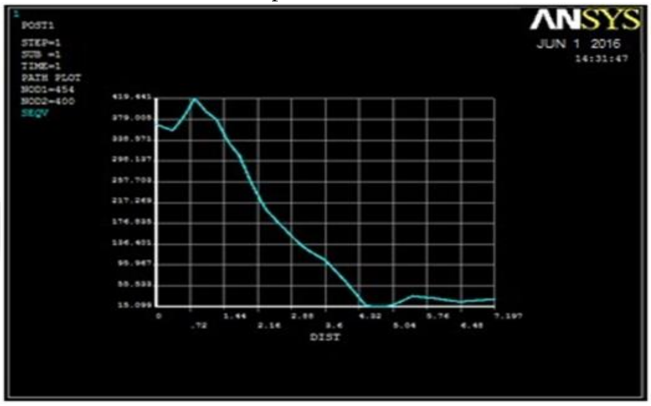

Fig. 11: Graph of Von-Misses Stress Distribution at fillet area of asymmetric gear with elliptical hole for its third position

Results which have been shown above are in complete compliance with the work done by Mr. Satvinder Singh Bhatia et. al. as mentioned in reference [25].

V. ASYMMETRIC GEAR WITH COMBINATION OF CIRCULAR AS WELL AS ELLIPTICAL STRESS RELIEVING FEATURES

In the previous sections it has been shown that Mr. S Agrawal et al [24] and Mr. Satvinder Singh Bhatia et. al.[25] considered a circular hole and an elliptical hole respectively to reduce Von-Misses stress at the fillet region of an asymmetric gear. As they have only tried with single type of hole with different dimensional values and different positional values, scope of work to investigate influence of multiple holes was unexplored. Here in the present work an effort has been made to investigate the influence of a combination of an elliptical hole as well as a circular hole of different sizes and orientations on the Von-Misses stress at fillet region of the gear

In the present work two types of stress relieving features have been introduced in an asymmetric gear to investigate the influence of different geometrical and positional parameter combinations of the composite stress relieving feature on Von-Misses stress at fillet region of the asymmetric gear. In this work combination of circular and elliptical holes have been considered as elements of the composite stress relieving feature. The geometrical as well as positional parameters have been adopted from the work of Mr. S. Agrawal [24] and from the work of Mr. Satvinder Singh Bhatia [25]. The effort which has been put in this present work to combine those parameters from reference [24] and reference [25]. Three types of modifications adopted in this work and these have been presented in the table below

Parameters	Description	Values in three modifications		
		Mod1	Mod2	Mod3
par1	To set the radial position of the elliptical hole with respect to the center of the gear. EHPCD = D_p × par1	0.93	0.93	0.9
par2	To identify the Major Axis length of the elliptical hole. HMJD = FR × par2	0.8	0.8	0.8
par3	To identify the Minor Axis length of the elliptical hole. HMND = FR × par2	0.4	0.4	0.4
HPANG	Angle between the horizontal axis of the gear and axis between center of the gear and hole	3°	4.5°	7°
HAANG	Angle between the major axis of the elliptical hole and axis between center of the gear and the elliptical hole	60°	45°	60°
par4	To set the radial position of the circular hole with respect to the center of the gear. CHPCD = D_p × par4	0.88	0.88	0.92
pr1	To set the radius of the hole CHR = FR × pr1	0.5	0.5	0.5
CHPANG	Angle between the horizontal axis of the gear and axis between center of the gear and the circular hole	5°	5°	5°

Table 2: Parameters of Asymmetric Gear with Composite Holes as Stress Relieving Features adopted in Modification1

Now a 3-dimensional model of asymmetric gear with composite stress relieving features as per the dimensions mentioned in table 2.

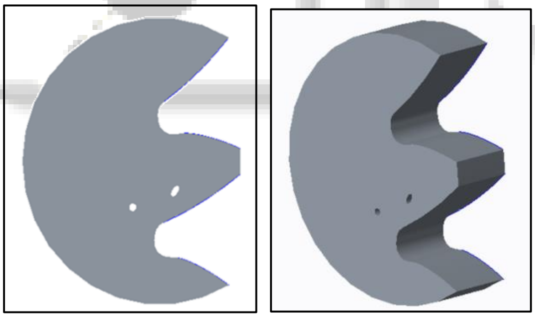

Fig. 12: A & B: Front view and isometric view of asymmetric gear with composite holes as stress relieving feature as per Modification-1

Procedure for solving this model in ANSYS is as described before and the boundary conditions as well as loading will also be same. So after solving it in ANSYS the Stress distribution and graph of stress at fillet region will be as follows.

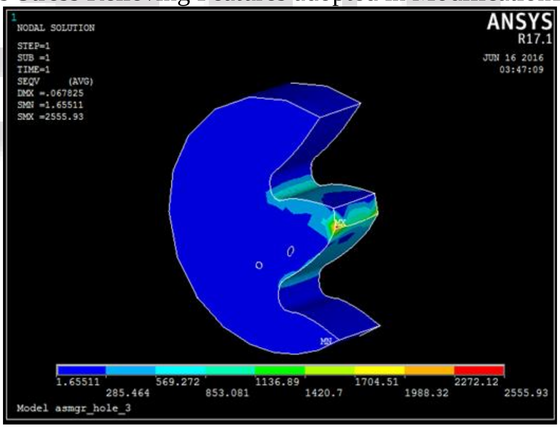

Fig. 13: Contour Plot of Von-Misses Stress Distribution at fillet area of asymmetric gear with composite holes as stress relieving feature as per Modification-1

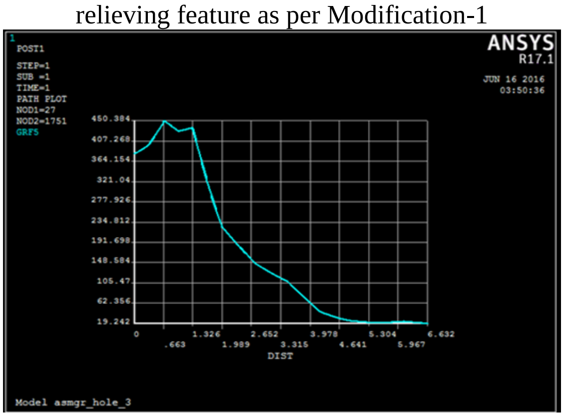

Fig. 14: Graph of Von-Misses Stress Distribution at fillet area of asymmetric gear with composite holes as stress relieving feature as per Modification-1

In modification 2 the parameter values which have been set have been mentioned in table 2. A model has been generated as per the above mentioned parameters in PTC Creo and then the model has been imported in the ANSYS software. The simulation gives the following result.

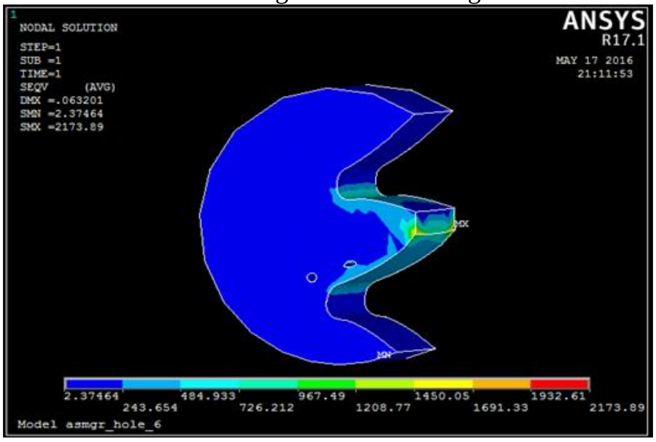

Fig. 15: Contour Plot of Von-Misses Stress Distribution at fillet area of asymmetric gear with composite holes as stress relieving feature as per Modification-2.

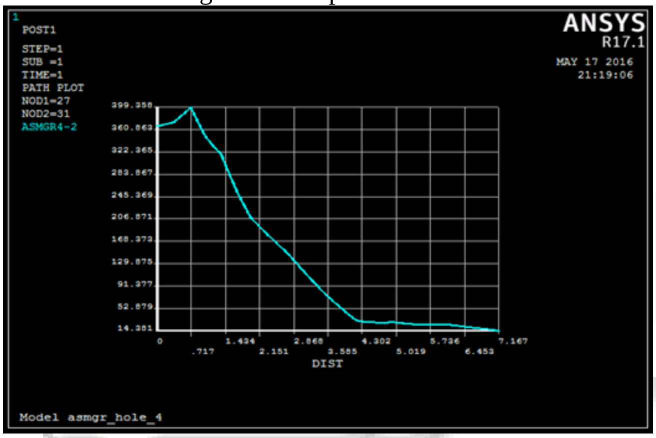

Fig. 16: Graph of Von-Misses Stress Distribution at fillet area of asymmetric gear with composite holes as stress relieving feature as per Modification-2

In modification 3 the parameter values which have been set have been mentioned in table 2. A model has been generated as per the above mentioned parameters in PTC Creo and then the model has been imported in the ANSYS software. The simulation gives the following result.

A model has been generated as per the above mentioned parameters in PTC Creo and then the model has been imported in the ANSYS software. The simulation gives the following result.

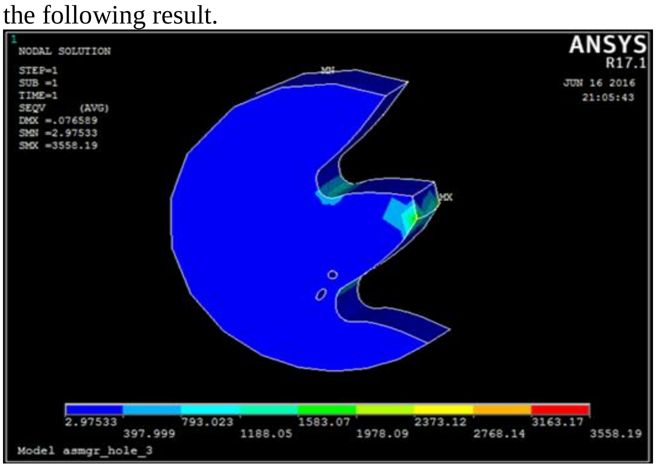

Fig. 17: Contour Plot of Von-Misses Stress Distribution at fillet area of asymmetric gear with composite holes as stress relieving feature as per Modification-3

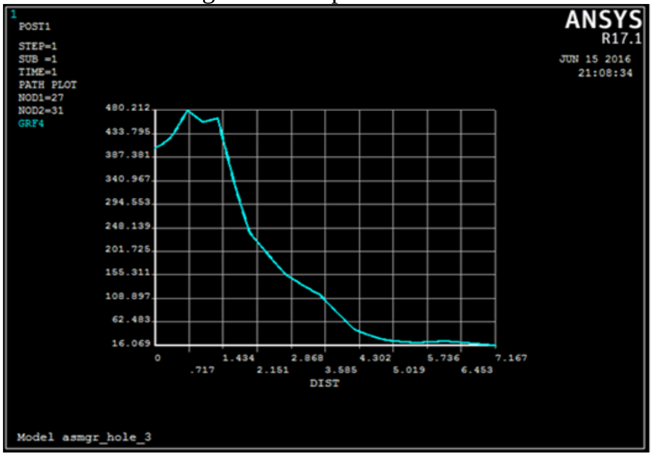

Fig. 18: Graph of Von-Misses Stress Distribution at fillet area of asymmetric gear with composite holes as stress relieving feature as per Modification-3

VI. RESULT AND DISCUSSION

The modifications which have been adopted in the present work have been summarized below.

Work Details	Maximum Von-Misses stress at fillet region
Work done by Mr. S. Agrawal et. al. (as per reference [24]	419.441 MPa
Work done by Mr. Satvinder Singh Bhatia et. al. (as per reference [25])	419.441 MPa
Modification-1 adopted in present work	450.384 MPa
Modification-2 adopted in present work	399.358 MPa
Modification-3 adopted in present work	480.212 MPa

Table 3: Summary of results

From the above result summery it is quite clear that imposing multiple stress relieving features improve the stress concentration problem at gear fillet region by reducing the Von-misses stress. But it worthy to mention here that the Ultimate tensile stress of the gear material is around 600 MPa and if a Factor of Safety of value minimum 1.5 is imposed the safe value becomes 400 MPa. So further doing some fine tuning on the parameter values as mentioned above we can reduce the stresses further keeping the actual gear dimensions as small as possible.

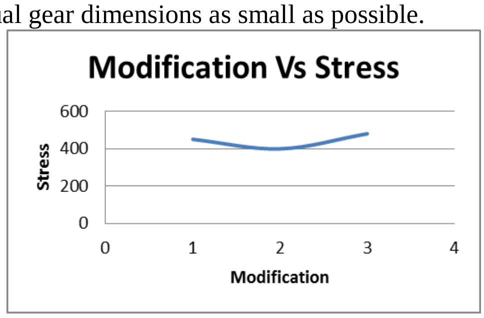

Fig 19: Variations of maximum Von-Misses stress at fillet region of asymmetric gear with multiple holes as per the dimensions mentioned in modifications adopted

Following graph shows the trend of the change in values of the stresses with modifications.

From the above graph it can be concluded that as the graph has an asymptote so optimization on the parameter values is quite possible and this optimization work may be considered as a very promising future scope of the present work.

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