Review Paper on Screw Theory and its Application in the Field of Parallel Robotics

M.S.Arora
Student
Department of Mechanical Engineering
A. D. Patel Institute of Technology, New Vallabh Vidyanagar, Gujarat, India

Abstract— Screw theory is an emerging approach in the field of robotics. It has been proven screw theory is an efficient approach for kinematic and dynamic analysis of parallel mechanism. In this paper detailed description of reciprocal screw is described. Reciprocal screws plays important role in robotics. In inverse kinematic analysis passive joint velocity eliminated using normalized reciprocal screws.

Key words: Screw theory, Reciprocal screws, parallel manipulator, robotics

I. INTRODUCTION

Mathematic model of screw theory has been developed by Sir Robert Stawell Ball in 1876 for application in kinematics and statics of mechanisms (rigid body mechanics). [1] A screw is a six-dimensional vector constructed from a pair of three-dimensional vectors. The components of the screw define the Plücker coordinates of a line in space and the magnitudes of the vector along the line and moment about this line. The instantaneous motion of a rigid body can be represented by a six-component vector called twist. Similarly, all the forces acting on a rigid body can be described by another six-component vector called wrench. Although twist and wrench have different physical meanings, they have the same mathematical representation called screws. Concept of screw theory and reciprocal screws and its application are defined in next section.

II. SCREW THEORY

A. The Screw

The concept of screws was first studied by Ball (1900), followed by Waldron(1969), Hunt (1978), Roth (1984), and others. A unit screw $\$$ is defined by a pair of vectors:

$$\hat{\$}=\begin{bmatrix} S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \\ S_6 \end{bmatrix} = \begin{cases} \begin{bmatrix} \hat{\mathbf{s}} \\ \hat{\mathbf{s}} \times \mathbf{r} + \mathbf{h}\hat{\mathbf{s}} \end{bmatrix} & \textbf{if h is finite} \\ \begin{bmatrix} \mathbf{0} \\ \hat{\mathbf{s}} \end{bmatrix} & \textbf{if } \mathbf{h} = \infty \end{cases} \quad \ldots\ldots(1)$$

where $\hat{\mathbf{s}}$ is a unit vector along the direction of the screw axis, $\mathbf{r}$ is the position vector of any point on the screw axis with respect to a reference frame, and $\mathbf{h}$ is called the pitch.

A screw of intensify ρ can be written as $\mathbf{\$} = \rho\hat{\mathbf{\$}}$. We call the screw a twist if it represents an instantaneous motion of a rigid body, and a wrench if it represents a system of forces and couples acting on a rigid body. In this regard, the first three components of a twist represent the angular velocity and the last three components represent the linear velocity of a point in the rigid body which is instantaneously coincident with the origin of the reference frame. On the other hand, the first three components of a wrench represent the resultant force and the last three components represent the resultant moment about the origin of the reference frame.

B. Screw Algebra

Screws obey the following algebraic operations [2] , and these operations have special meanings.

1) Screw Sum

The sum of two screws $\$_1 = (S_1; S_1^0)$, and $\$_2 = (S_2; S_2^0)$ is defined as follows

$$\$_1 + \$_2 = (S_1 + S_2; S_1^0 + S_2^0)\ldots..(2)$$

2) Product Of A Scalar And A Screw

The product of a scalar λ and a screw $\$$ is defined by

$$\lambda\$ = (\lambda S; \lambda S^0),\ldots..(3)$$

3) Reciprocal Product

The reciprocal product of two screws, say $\$_1 = (S_1; S_1^0)$, $\$_2 = (S_2; S_2^0)$, is defined by

$$\$_1 \circ \$_2 = \boldsymbol{S_1}.\boldsymbol{S_2^0} + \boldsymbol{S_2}.\boldsymbol{S_1^0}\ldots..(4)$$

where the symbol ○ denotes the reciprocal product of two screws.

C. Reciprocal Screws

Two screws, $\$$ and $\$r$, are said to be reciprocal if they satisfy the condition:

$$\$_r^T\$ = 0 \ldots..(5)$$

where the transpose of a screw is defined as

$$\$^T = [S_4\ S_5\ S_6\ S_1\ S_2\ S_3]$$

such that the orthogonal product of the two screws is given by:

$$\$_r^T\$ = S_{r4}S_1 + S_{r5}S_2 + S_{r6}S_3 + S_{r1}S_4 + S_{r2}S_5 + S_{r3}S_6 \ ..(6)$$

If one screw is twist and the other is wrench, the physical meaning for reciprocity between these two screws is that the instantaneous work for the wrench along the twist is zero. The reciprocity condition can be stated as the virtual work between a wrench and a twist is equal to zero. From the geometry of the lines associated with the two screws, the reciprocal condition can be derived as:

$$(\lambda + \lambda r) \cos \alpha - a \sin \alpha = 0,\ldots..(7)$$

where a is the offset distance along the common perpendicular leading from the screw axis of $\$$ to $\$r$, and α is the twist angle between the axes of $\$$ and $\$r$, measured from $\$$ to $\$r$ about the common perpendicular according to the right-hand screw rule.

D. Reciprocity Condition Of Two Screws

Reciprocity condition of two screws is described by Huang et al. as described in table below [2].

No.	Reciprocity conditions of two screws
1.	The necessary and sufficient condition for the reciprocity of two line vectors is they should be coplanar
2.	Two couples are always reciprocal
3.	Line vector and couple are reciprocal only when they are perpendicular

Table 1: Reciprocity conditions of two screws

III. RECIPROCAL SCREWS OF SOME KINEMATIC CHAINS [3]

Using the above information, reciprocal screws associated with a kinematic chain can be obtained by an intersection of the systems of reciprocal screws associated with the joints in the kinematic chain. In what follows, the reciprocal screws associated with several frequently used kinematic chains are developed.

Universal-Spherical Dyad. The joint screws associated with a universal-spherical dyad form a five-system. Hence, the reciprocal screw is a one-system. Because of the presence of a spherical joint and a universal joint, the reciprocal screw is a zero-pitch screw passing through the centers of the two joints.

Revolute-Spherical Dyad. The joint screws associated with a revolute-spherical dyad form a four-system. The reciprocal screws form a two-system. Because of the presence of a spherical joint and a revolute joint, all reciprocal screws are zero-pitch screws forming a planar pencil. Specifically, all reciprocal screws pass through the center of the spherical joint and lie on a plane which contains both the axis of the revolute joint and the center of the spherical joint as shown in Fig. 1(a).

Prismatic-Spherical Dyad. The joint screws associated with a prismatic-spherical dyad form a four-system. The reciprocal screws form a two-system. Because of the presence of a spherical joint and a prismatic joint, all reciprocal screws are zero-pitch screws forming a planar pencil. Specifically, all reciprocal screws pass through the center of the spherical joint and lie on a plane which is perpendicular to the axis of the prismatic joint as shown in Fig. 1(b).

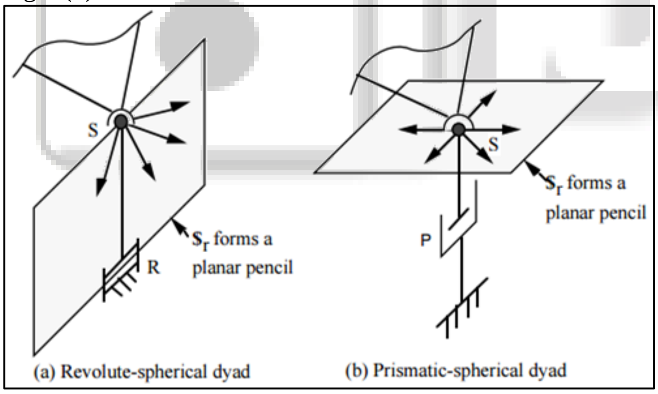

Fig. 1: Reciprocal screw systems of two dyads

IV. GEOMETRICAL METHOD TO DETERMINE THE RECIPROCAL SCREWS [4]

Two screws are said to be reciprocal to each other if their reciprocal product equals to zero. If one screw is twist and the other is wrench, the physical meaning for reciprocity between these two screws is that the instantaneous work for the wrench along the twist is zero. Hence if a screw system of twists represents the degrees of freedom (DOF) of a rigid body, the reciprocal system of wrenches is the constraint forces acting on it. The dual proposition also holds: When a screw system of wrenches act on a rigid body, then the reciprocal system of twists is the DOF of the body. Therefore, the reciprocity property makes it possible to obtain motion information from the corresponding constraint counterpart and vice versa, and it is necessary to study how to find the reciprocal screw system for a given one.

Traditionally, the reciprocal screw system is obtained by algebraic approach. First of all, a Cartesian coordinate frame is established and the Pl¨ucker coordinates for screws in the frame are derived. Then a set of homogeneous equations is formed by the reciprocal condition. Finally, various methods are applied to obtain null space of the equations. Different from conventional method, we try to propose a geometrical method to determine the reciprocal screw systems. Compared with the algebraic approach, one only needs to implement three simple observations developed in the paper. Simply by inspecting the joint axes and applying the observations, one can easily obtain a basis of the reciprocal screw system.

A. Basic Observations and Steps to Obtain Reciprocal System

J.S.Zhao et al. suggested method to obtain reciprocal screws using geometric method based on these three observations [4].

- Observation 1: Two line vectors
  Two line vectors are reciprocal to each other if and only if they are coplanar.
- Observation 2: Two couple vectors
  Two couple vectors are always reciprocal to each other.
- Observation 3: Line vector and couple vector
  A line vector is reciprocal to a couple vector if and only if they are perpendicular to each other.

B. Steps To Obtain Reciprocal System

A screw system is a span of k ($k \leq 6$) linearly independent screws, and is often called a k system. For this screw system, any k linearly independent screws can be a basis of the system. The reciprocal screw system for the given one is a $6 - k$ system in which any one screw is reciprocal to all the screws in the given system. Similarly, any $6 - k$ linearly independent screws form a basis of the reciprocal system.6 Hence we only need to find $6 - k$ convenient linearly independent screws to represent the reciprocal system. For traditional Pl¨ucker coordinate method, the steps to obtain the reciprocal screws are outlined as follows:

1) Establish a convenient coordinate frame;
2) Write the Pl¨ucker coordinate for each of the screws;
3) Use linear algebra to check the linear dependence of the screws and find one basis of the screw system;
4) Use the reciprocal condition to establish linear equations and solve the reciprocal screws.

In contrast, the steps for proposed geometrical method are as follows:

1) Use Grassmann geometry to determine the dependence of the screws by inspection7 and find one basis of the screw system;
2) Use the three observations to find the reciprocal screws and reciprocal screw system.

Simply from the steps of the two methods, we can see that the geometrical method is simpler than the Pl¨ucker coordinate method.

V. Applications

A. Mobility Analysis of A Rigid Body

A free rigid body has six DOF. Analysing the motion twists for a rigid body under given constraint wrenches is called the exact constraint. It is useful in the work piece localization and parts assembly. Blading discusses the R and C patterns for the motion and constraint of a rigid body. However, he only analyses the pure force constraint, and he also treats the translation as rotation about an axis at infinity which is not intuitive. In this section, more general results will be obtained based on the three observations.

The procedure to obtain the exact DOF of parallel manipulator is listed as follows and outlined in Fig. 2.

1) Find a basis for each limb motion screw system;
2) Solve limb constraint screw system for each limb by the reciprocal condition and find a basis for the system;
3) Find the union of all limb constraint screw systems to get the platform constraint screw system. Determine a basis for the system;
4) Solve platform motion screw system using the reciprocal condition again and find a basis for the system. It is the desired DOF of the manipulator.

Other than this J.S. Zhao et al. also developed theory od degree of freedom for mechanism based applying screw theory [5]. On later stage J.S. Zhao et al. also worked on computation of configuration degree of freedom based on screw theory [6].

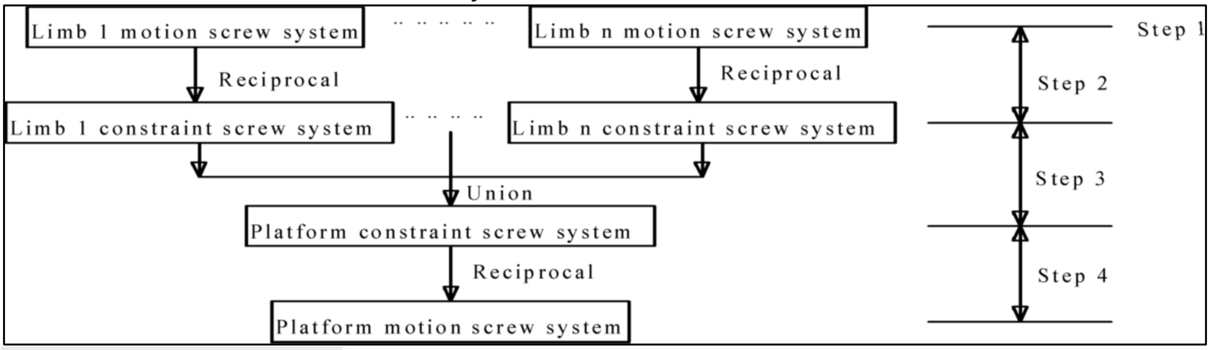

Fig. 2: Steps to obtain the exact DOF of parallel manipulators

B. Jacobian Formulation for Parallel Manipulators

Jacobian relates the actuators' velocity to end-effector's velocity which is crucial for velocity analysis. For parallel manipulator, various methods can be used to obtain the Jacobian matrix. Since the inverse kinematics relates the end-effector's position to the actuators' displacement, the most intuitive method is the direct derivation method which differentiates the inverse kinematic equations. Though straightforward, it is rather cumbersome for complex manipulators. Another method is the velocity vector loop method that forms velocity equations for different loops in the manipulator. Then the passive joints' velocity will be eliminated by equation manipulation. This method is also very complicated when applying to complex manipulators.

The screw-based method is the most efficient one. As mentioned before, all the joints in common manipulator can be either one-DOF joints or can be decomposed into several one-DOF joints. Without loss of generality, suppose the manipulator has n limbs and λ one-DOF joints in each limb. Each limb has only one actuator; thus the number of limbs is equal to the DOF of the manipulator. Then for every limb, the twist of the moving platform is the linear combination of the twist associated with every joint.

C. Singularity Analysis for Parallel Manipulators

In the workspace of parallel manipulator, there exist some configurations called singular configurations where the manipulator cannot move in certain directions or have extra uncontrollable DOF. Moreover, the manipulator also exhibits bad performance near singular configurations. Thus it is necessary to study the singularities in order to keep the manipulator far away from them. J.S Zhao et al. Two types of singularities, DKS and CS for parallel manipulator, are analysed based on the three observations [4]. Many researcher has been working on this research area like Gallardo et al. worked on singularity analysis of 4-DOF parallel manipulator [5]. Xiang et al. worked on singularity analysis of lower-mobility parallel manipulator using screw theory based on force-motion transmissibility [6].

VI. Conclusion

In this paper, screw theory and its applications is reviewed. The concept of unit screws, reciprocal screws, twist and wrench is described. A geometrical approach to determine the reciprocal screws for a given screw system is shown. First of all, three basic geometrical observations are discussed which describe the reciprocity between line vectors and couple vectors. Based on the observations, reciprocal screws for common kinematic elements are first illustrated which pave way for the kinematic analysis for parallel manipulator. Different aspects of mechanism kinematics demonstrate the usefulness of the geometrical approach. The mobility for a rigid body subjected to different constraints was deliberated. It can be stated that one can directly obtain the mobility by examining the structure of the manipulator. In addition to mobility analysis, Jacobian matrix formulation for parallel manipulator is also discussed by the geometrical approach. The novel elimination screw is proposed for the screw-based approach. Finally, singularity analysis of parallel manipulator using screw theory is discussed. By applying the geometrical method to parallel manipulator analysis, we conclude that this method has many advantages such as intuitive and simple compared with the traditional Pl¨ucker coordinate method. Therefore, it can be used to facilitate the parallel manipulator design process and give insights to existing manipulators.

REFERENCES

[1] R. S. Ball, A Treatise on the theory of screw, Cambridge university, 1900.
[2] Z. Huang, . Q. Li and H. Ding, Theory of Parallel Mechanisms.
[3] L.-W. Tsai, Robot Analysis: The Mechanics of Serial and Parallel Manipulator.
[4] J. Zhao, B. Li, X. Yung and H. Yu, "Geometrical method to determine the reciprocal screws and applications to parallel manipulators," Robotica, 2009.
[5] J. s. Zhao, k. Zhou and Z. J. Feng, "A theory of degrees of freedom for mechanisms," Mechanism and Machine Theory, 2004.
[6] J. S. Zhao, Z. j. Feng and J. x. dong, "Computation of the configuration degree of freedom of a spatial parallel mechanism by using reciprocal screw theory," Mechanism and Machine Theory, 2006.
[7] J. Gallardo, J. M. Rico and G. Alici, "Kinematics and singularity analyses of a 4-dof parallel manipulator using screw theory," Mechanism and Machine Theory, 2006.
[8] X. Chen, X. J. Liu and F. Xie, "Screw Theory Based Singularity Analysis of Lower-Mobility Parallel Robots considering the Motion/Force Transmissibility and Constrainability," Mathematical Problems in Engineering, 2014.
[9] H. J. Su, D. V. Dorozhkin and J. M. Vance, "A Screw Theory Approach for the Conceptual Design of Flexible Joints for Compliant Mechanisms," Journal of Mechanisms and Robotics, 2009.

