Unsteady MHD Flow of a Non-Newtonian Fluid under Effect of Couple Stresses between Two Parallel Plates

Arjumand Sattar[1] S. M. Subhani[2]
[1]Associate Professor [2]Assistant Professor
[1,2]Department of Mathematics
[1,2]Deccan College of Engineering & Technology, Darussalam, Hyderabad – 500001, Telangana, India

Abstract— In this paper the MHD of a Non-Newtonian unsteady flow of an incompressible fluid under the effect of couple stresses and a uniform magnetic field is analysed by using Finite Element Method. The solution is obtained by using Galerkin Method with assuming a pulsatile pressure gradient in the direction of the motion. The Galerkin Method endowed with distinct features that account for its superiority over competing methods. The effect of different parameters are discussed with the help of graphs.

Key words: Pulsatile Flow, Magnetohydrodynamic (MHD), Couple Stresses, Non-Newtonian, Porous Plates, Hartmann Number, Reynolds Number

I. INTRODUCTION

A fluid which differ from Newtonian fluid in that the relationship between the shear stress and the flow field is more complicated. Such fluid is Non-Newtonian. Examples coal-water, polymer solutions, etc. Studies of unsteady magnetohydrodynamic (MHD) flows of Non-Newtonian fluids have been made for planar porous walls and in the zero-induction approximation. MHD flows require time to obtain he steady velocity distribution.

The influence of the magnetic field on the starting phase was demonstrated for the Hartmann flow [1], assuming a small magnetic Reynolds number and a constant pressure gradient in the direction of the flow. Approximate solutions were obtained by Yen [2], when the MHD Hartmann flow is affected by a periodic change of the pressure gradient. Shakadz and Megahed [3] studied the problem of an unsteady MHD flow assuming constant pressure, and an exact solution was obtained when the upper wall moving with a time dependent velocity, where the two walls were not porous. In [4] an unsteady MHD Non-Newtonian flow between parallel fixed porous walls was studied using the Eyring Powell model [5]. A Non-Newtonian fluid flow between two parallel walls, one of them moving with a uniform velocity under the action of a transverse magnetic field, was studied in [6].

Previously this paper is solved by numerically. This problem used to land in complicated situations with complex arguments etc. obtaining the solution in such cases was tedious job and truncation errors were causing a lot of problems. These difficulties can be overcome by use of finite element techniques. Most practical problems involve complicated domains, load and nonlinearities that forbid the development of analytical solutions. Therefore, the only alternative is to find approximate solutions using finite element techniques. A finite element techniques, with advent of a computer, can be used to investigate the effect of various parameters of the system on its response to gain a better understanding of the system being analyzed. It is cost effective, saves time and material resources compared to the multitude of physical experiments needed to gain the same level of understanding. Because of the power of finite element techniques, it is possible to include all relevant features in a mathematical model of a physical process without worrying about its solution by exact mean.

The present paper treats the flow of a pulsatile Non-Newtonian incompressible and electrically conducting fluid in a magnetic field. Couple stresses are the consequence of assuming that the mechanical action of one part of a body on another across a surface is equivalent to a force and moment distribution. In classical non-polar theory, moment distributions are not considered and the mechanical action is assumed to be equivalent to a force distribution only. The state of stress is measured by a stress tensor τ_{ij} and a couple stress tensor M_{ij} . The purpose of the present paper is to investigate the effect of couple stress parameter besides other parameters entering the problem on the velocity distribution using Galerkin Method.

The continuity equation $\rho' + \rho v_{i,i} = 0$, Cauchy's first law of motion $\rho a_i = T_{ji,i} \rho f_i$, and Cauchy's second law of motion $M_{ji,j} + \rho l_i + e_{ijk} T_{jk} = 0$ $M_{ji,j} + \rho l_i + e_{ijk} T_{jk} = 0$, where ρ is the density of the fluid, v_i are the velocity components, a_i the components of the acceleration, T_{ij} is the second order stress tensor, M_{ij} the second order couple stress tensor f_i, the body force per unit volume, l_i the moment per unit volume and e_{ijk} the third order alternating pseudo tensor, which is equal to +1 or -1 if (i,j,k) is an (even or odd) permutation of (1,2,3), and is equal to zero if two or more of indices i,j,k are equal. Mindlin and Tiersten [8] obtained the constitutive equations for a linear perfectly elastic solid in the form

$$T_{ji,j} = T^{s}_{ji,j} + T^{A}_{ji,j}$$

Where T^s_{ji} is the symmetric part of the stress tensor

$$T^{s}_{ji,j} = -P_{,i} + (\lambda + \mu) v_{j,ji} + \mu v_{i,ji} = -P_{,i} + (\tau_{ji})_j$$

$$T^{A}_{ji,j} = -2\eta \omega_{ji,jll} + \frac{1}{2} e_{ijk} (\rho_{lk})_{,j}$$

Where τ_{ij} is the stress tensor in the nonpolar classical theory, ω_{ij} is the spin tensor, which is considered as a measure of the rates of rotation of elements in a certain average sense

$$\omega_{ij} = e_{ijk} \omega_k$$

Where ω_i is the vorticity vector

Since $\omega_{ij} = \frac{1}{2} e_{ijk} v_{k,j}$, one has $\omega_{ij} = \frac{1}{2}(\nu_{j,i} - \nu_{i,j})$

and we can write

$$T^{A}_{ji,j} = -\eta \nu_{i,jjll} + \eta \nu_{j.jill} + \frac{1}{2} e_{ijk} (\rho_{lk})_{,j}$$

and the equation of motion becomes

$$\rho a_i = T^{s}_{ji,j} + \eta(\nu_{j,j})_{,ikk} - \eta \nu_{i,jjkk} + \frac{1}{2} e_{ijsk} (\rho_{lk})_{,j} + \rho f_i$$

For incompressible fluids and if the body force and body moment are absent, the equations of motion reduce to

$$\rho a_i = T^{s}_{ji,j} - \eta \nu_{i,jjkk}$$

Which, in vector notation, can be written as

$$\rho a_i = -\nabla P + \nabla.(\tau_{ij}) - \eta \nabla^4 \nu$$

The last term in this equation gives the effect of couple stresses. Thus, for the effect of couple stresses to be present, $\nu_{i,rrss}$ must be nonzero. $\tau_{i,j}$ represents the stress tensor in the case of the nonpolar theory of fluids. The Eyring-Powell model for describing the shear of a Non-Newtonian flow is derived from the theory of rate processes. This model can be used in some cases to describe the viscous behavior of polymer solutions and viscoelastic suspensions over a wide range of shear rates. The stress tensor in the Eyring-Powell model for Non-Newtonian fluids takes the form

$$\tau_{i,j} = \mu \frac{\partial u_i}{\partial x_j} + \frac{1}{\beta} \sinh^{-1}\left(\frac{1}{c} \frac{\partial u_i}{\partial x_j} \right)$$

Where μ is the viscosity coefficient. β and c are the charactericties of the Eyring-Powell model

In the present paper we study the problem of an unsteady MHD Non-Newtonian flow between two parallel fixed porous plates, where the x and y axes are taken along and perpendicular to the parallel walls, respectively, and for the velocity $v = (u(y,t), v(y,t), 0)$ in this case the continuity equation becomes

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

Which gives $\frac{\partial v}{\partial y} = 0, i.e. v = f(t)$. The function f(t) is taken as a constant velocity v_0, which represents the velocity of suction or injection through the plate. Then the velocity tends to u(y) only.

In hydromagnetic parallel flows of the type where the velocity is of the form $v_1 = u(y)$, $v_2 =$ constant, $v_3 = 0$, and these flows are assumed to be subjected to a uniform magnetic field B0 in the positive y direction, thr equation of motion becomes [7]

$$\rho\left(\frac{\partial u}{\partial t} + V_0 \frac{\partial u}{\partial y} \right) = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial y}(\tau_{ij}) + \sigma B_0 (E - u B_0) - \eta \frac{\partial^4 u}{\partial y^4}$$

Neglecting the electric currents, i.e. E, the above equation becomes

$$\rho\left(\frac{\partial u}{\partial t} + V_0 \frac{\partial u}{\partial y} \right) = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial y}(\tau_{ij}) + \sigma B_0^2 u - \eta \frac{\partial^4 u}{\partial y^4}$$

The solution obtained by using Galerkin Method with assuming a pulsatile pressure gradient in the direction of the motion. The effect of parameters of different parameters of the problem are discussed with help of graphs.

II. MATHEMATICAL MODEL

Consider a Non-Newtonian unsteady electrically conducting incompressible flow between two parallel porous walls situated a distant L apart under the effect of couple stresses. We take the x and y axes along and transverse to the parallel walls and assume a uniform magnetic field B acting along the y-axis. The fluid is injected into the lower wall at y = 0 and is sucked through the upper wall at y = L with the uniform velocity V_0. The electric field is assumed to be zero. The induced magnetic field is assumed to be very small, and the electric conductivity σ of the fluid is sufficiently large. The governing equations are

$$\rho\left(\frac{\partial u}{\partial t} + V_0 \frac{\partial u}{\partial y} \right) = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial y}(\tau_{xy}) + \sigma B_0^2 u - \eta \frac{\partial^4 u}{\partial y^4}, \quad (1)$$

$$\frac{\partial p}{\partial y} = 0 \quad (2)$$

Where u = u(y,t) is the velocity components of the fluid in the x-direction, p the fluid pressure, τ_{xy} the stress tensor in the classical nonpolar theory, β the coefficient of couple stresses and B_0 the external magnetic field. For a Non-Newtonian fluid obeying the Eyring-Powell model we have

$$\tau_{xy} = \mu \frac{\partial u}{\partial y} + \frac{1}{\beta} \sinh^{-1}\left(\frac{1}{c} \frac{\partial u}{\partial y} \right) \quad (3)$$

$$\sinh^{-1}\left(\frac{1}{c} \frac{\partial u}{\partial y} \right) \cong \frac{1}{c} \frac{\partial u}{\partial y} - \frac{1}{6}\left(\frac{1}{c} \frac{\partial u}{\partial y} \right)^3, \quad \left| \frac{1}{c} \frac{\partial u}{\partial y} \right| \prec 1 \quad (4)$$

Then (1) will be reduced to

$$\frac{\partial u}{\partial t} + V_0 \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \frac{\mu}{\rho} \frac{\partial^2 u}{\partial y^2} + \frac{1}{\rho \beta c} \frac{\partial^2 u}{\partial y^2} + \frac{1}{\rho \beta c}\left(\frac{\partial u}{\partial y} \right)^2 \frac{\partial^2 u}{\partial y^2} + \frac{\sigma}{\rho} B_0^2 u - \frac{\eta}{\rho} \frac{\partial^4 u}{\partial y^4} \quad (5)$$

The boundary conditions are

$$u = 0, \quad u'' = 0; \quad at \quad y = 0$$
$$u = 0, \quad u'' = 0; \quad at \quad y = L$$
$$0 < y < 1, \quad u = V_0 \sin\left(\frac{\pi y}{L} \right), \quad t \le 0 \quad (6)$$

Where a dash means differentiation with respect to y

III. METHOD OF SOLUTION

Let us introduce non-dimensional quantities

$$x^* = \frac{x}{L}, \; y^* = \frac{y}{L}, \; t^* = \frac{V_0}{L} t, \; u^* = \frac{u}{V_0},$$
$$p^* = \frac{p}{\rho V_0^2} \quad (7)$$

Substituting from (7) into (5) (* is dropped) we get

$$\frac{\partial u}{\partial t} + \frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \frac{N^*}{\mathrm{Re}} \frac{\partial^2 u}{\partial y^2} - D^*\left(\frac{\partial u}{\partial y} \right)^2 \frac{\partial^2 u}{\partial y^2} - \frac{H_a^2}{\mathrm{Re}} u - \frac{1}{a^2 \mathrm{Re}} \frac{\partial^4 u}{\partial y^4} \quad (8)$$

Subjected to the boundary conditions

$$u = 0,\quad u'' = 0;\quad at\quad y = 0$$
$$u = 0,\quad u'' = 0;\quad at\quad y = L$$
$$0 < y < 1,\quad u = \sin(\pi y),\quad t \le 0 \qquad (9)$$

where

$$\mathrm{Re} = \frac{V_0 L}{\nu} \equiv \text{Reynolds number},$$
$$H_a = B_0 L\sqrt{\frac{\sigma}{\rho\nu}} \equiv \text{Hartmann number, and}$$
$$\nu = \frac{\mu}{\rho}, N^* = 1 + M, M = \frac{1}{\mu c \rho},$$
$$D^* = \frac{V_0}{2\rho\beta c^3 L^3}, a^2 = \frac{L^2}{h^2}, h^2 = \frac{\eta}{\mu} \qquad (10)$$

Suppose a pulsation pressure gradient of the form

$$-\frac{\partial p}{\partial x} = \left(\frac{\partial p}{\partial x}\right)_s + \left(\frac{\partial p}{\partial x}\right)_0 e^{i\omega t} = P_s + P_0 e^{i\omega t} \qquad (11)$$

Where P_s and P_0 are constants. P_0 is the pulsation of the pressure parameter.

Then (8) is reduced to

$$\frac{\partial u}{\partial t} + \frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \frac{N^*}{\mathrm{Re}}\frac{\partial^2 u}{\partial y^2} - \frac{H_a^2}{\mathrm{Re}}u - \frac{1}{a^2}\frac{\partial^4 u}{\partial y^4} \qquad (12)$$

To obtain the solution of (12) we use perturbation technique as follows :

$$u = u_0 + u_1 e^{i\omega t} \qquad (13)$$

Substituting (11) and (13) into (12) we get the two ordinary differential equations

$$\frac{1}{a^2}\frac{d^4 u_0}{dy^4} - \frac{N^*}{\mathrm{Re}}\frac{d^2 u_0}{dy^2} + \frac{du_0}{dy} + \frac{H_a^2}{\mathrm{Re}}u_0 = P_s \qquad (14)$$
$$\frac{1}{a^2}\frac{d^4 u_1}{dy^4} - \frac{N^*}{\mathrm{Re}}\frac{d^2 u_1}{dy^2} + \frac{du_1}{dy} + \frac{H_a^2}{\mathrm{Re}}u_1 = P_0 \qquad (15)$$

Subjected to the boundary conditions

$$u_0 = u_1 = 0,\quad u_0'' = u_1'' = 0;\quad at\quad y = 0$$
$$u_0 = u_1 = 0,\quad u_0'' = u_1'' = 0;\quad at\quad y = 1 \qquad (16)$$

The solutions of (14) and (15) under the boundary conditions (16) are evaluated by using Galerkin Method, $u_0(y)$ and $u_1(y)$ are given by

$$u_0(y) = \left(\frac{P_s}{10 d_{06}}\right)(2y^3 - y^4 - y) \qquad (17)$$

$$u_1(y) = \frac{(d_{14} + i d_{15})}{2}(2y^3 - y^4 - y) \qquad (18)$$

Substituting (17) and (18) into (13), we get
The velocity distribution

$$u(y) = (2y^3 - y^4 - y)\left\{\begin{matrix}\left[\dfrac{P_s}{10 d_{06}} + \dfrac{d_{14}}{2}\cos\omega t + \dfrac{d_{15}}{2}\sin\omega t\right] + \\ i\left[\dfrac{d_{14}}{2}\sin\omega t + \dfrac{d_{15}}{2}\cos\omega t\right]\end{matrix}\right\} \qquad (19)$$

The real part of the velocity of the fluid is given by

$$u_{11}(y) = (2y^3 - y^4 - y)\left[\begin{matrix}\dfrac{P_s}{10 d_{06}} + \dfrac{d_{14}}{2}\cos\omega t + \\ \dfrac{d_{15}}{2}\sin\omega t\end{matrix}\right] \qquad (20)$$

where

$$d_{01} = \frac{H_a^2}{4\mathrm{Re}},\quad d_{02} = \frac{H_a^2 + 2\mathrm{Re}}{2\mathrm{Re}},\quad d_{03} = \frac{6N^* + 3\mathrm{Re}}{2\mathrm{Re}},$$
$$d_{04} = \frac{12N^* + H_a^2}{4\mathrm{Re}},\quad d_{05} = \frac{24 + a^2}{4a^2},$$
$$d_{06} = -\frac{d_{01}}{36} + \frac{11 d_{02}}{280} + \frac{5 d_{03}}{84} - \frac{d_{04}}{10} - \frac{d_{05}}{5},$$
$$d_{11} = \frac{d_{01}}{36} - \frac{11 d_{02}}{280} - \frac{5 d_{03}}{84} + \frac{d_{04}}{10} + \frac{d_{05}}{5},$$
$$d_{12} = \frac{\omega}{4}\left(\frac{-11}{140} + \frac{1}{18} + \frac{1}{10}\right),\quad d_{13} = d_{11}^2 + d_{12}^2,$$
$$d_{14} = \frac{-p_0 d_{11}}{10 d_{13}},\quad d_{15} = \frac{p_0 d_{12}}{10 d_{13}}$$

The numerical values of $u_{11}(y)$ are carried out using C++ language.

IV. RESULTS AND DISCUSSION

For calculation of velocity distribution for the Non-Newtonian unsteady flow of an incompressible fluid under the effect of couple stresses and a uniform magnetic field is analyzed by using Finite Element Method, Galerkin Method. Influence of parameters on velocity distribution are studied in the present investigation.

It is observed from Fig : 1 that velocity $u_{11}(y)$ decreases with increasing Non-Newtonian fluid effect M, at $a = 0.5$, $\mathrm{Re} = 0.1$, $H_a = 3$ & $P_0 = 5$, which is the first approximatiom parameter of Non-Newtonian effect. It is observed from Fig : 2 that velocity $u_{11}(y)$ increases with increasing couple stresses parameter a^2, at $M = 5$, $\mathrm{Re} = 0.1$, $H_a = 3$ & $P_0 = 5$, which is the inverse of the couple stresses, which shows that the velocity of the fluid decreases with increasing of couple stresses. It is observed from Fig : 3 that velocity $u_{11}(y)$ increases with increasing Reynolds number Re, at $a = 0.5$, $M = 5$, $H_a = 3$ & $P_0 = 5$. It is observed from Fig : 4 that velocity $u_{11}(y)$ decreases with increasing Hartmann number H_a, at $a = 0.5$, $\mathrm{Re} = 0.1$, $M = 5$ & $P_0 = 5$. It is observed from Fig : 5 that velocity $u_{11}(y)$ increases with increasing Pulsation of the pressure parameter P_0, at $a = 0.5$, $\mathrm{Re} = 0.1$, $H_a = 3$ & $M = 5$.

V. CONCLUSIONS

In this paper we have studied the effect of couple stresses on an unsteady MHD Non-Newtonian flow between two parallel fixed porous plates under a uniform external magnetic in models like the Eyring Power model. We conclude that the flow is damping with increasing effect of couple stresses. This result may be very useful in many cases, like the discussion of some diseases of the blood. Couple stresses, for the model under consideration, depend on vorticity gradients. Since vorticity gradients are known to be large in hydromagnetic flows of nonpolar fluids, couple stress effects may be expected to be large in electrically conducting polar fluids also, which may be useful in studying the blood flow in the arteries.

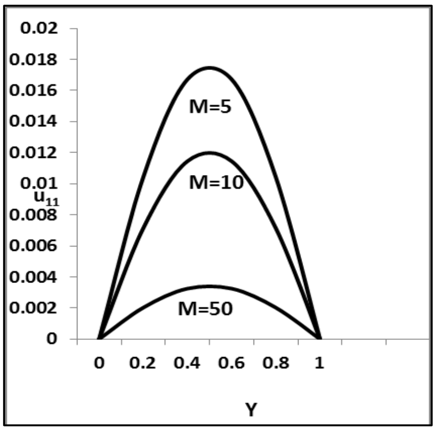

Fig. 1: Velocity distribution u11 versus y when a = 0.5, Re = 0.1, Ha = 3 & P_0 = 5

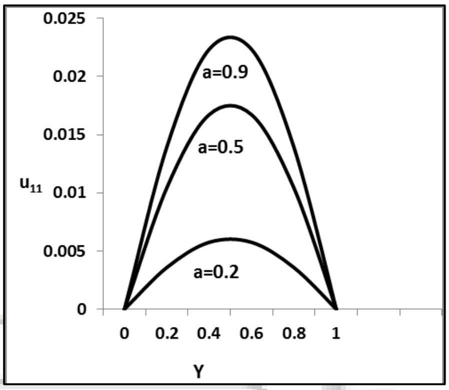

Fig. 2: Velocity distribution u_{11} versus y when M = 5, Re = 0.1, H_a = 3 & P_0 = 5

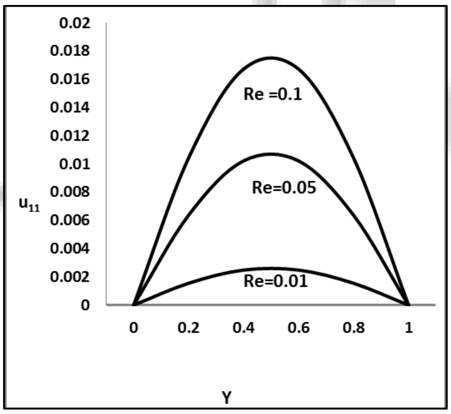

Fig. 3: Velocity distribution u_{11} versus y when M = 5, a = 0.5, H_a = 3 & P_0 = 5

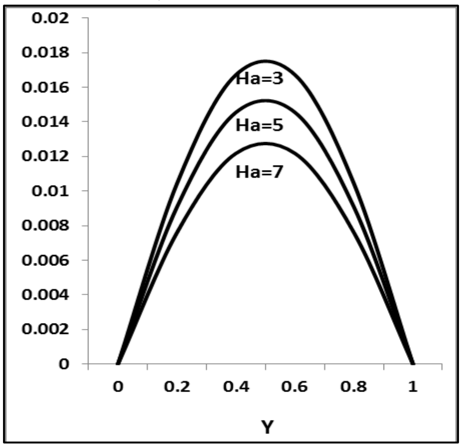

Fig. 4: Velocity distribution u_{11} versus y when M = 5, a = 0.5, Re = 0.1 & P_0 = 5

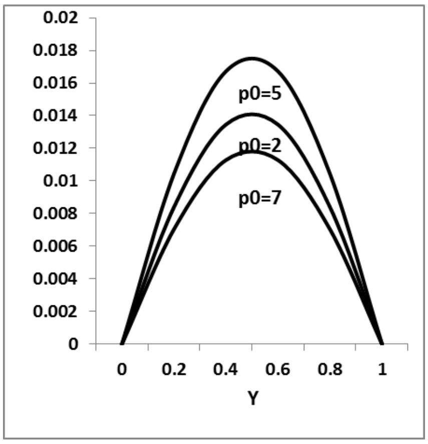

Fig. 5: Velocity distribution u_{11} versus y when M = 5, a = 0.5, Re = 0.1 & H_a = 3

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