Profit Optimization of CDCC Bank, As LPP

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Abstract— In this paper the Optimal Profit of Coimbatore District Central Co-operative Bank in basis of agri loans such as Kisan Credit Card loan, Kisan Credit Card Jewel loan, Produce Pledge loan, investment loan of at least 200 societies for the period from April 2015 to March 2016 is investigated by using Revised Simplex Method and Sensitivity Analysis. Also this work analyzes the representation of the problem of profit optimization of bank as Linear Programming Problem. Finally, the problem of profit optimization is resolved by Sensitivity Analysis. The problem of profit optimization of Bank modeled as a Linear Programming Problem (LPP). The subsequent LPP then solved by the Revised Simplex Method(RSM). The paper revealed that the Optimal Profit of Bank in the areas of loans of at least 200 societies for the period of 1 year from April 2015 to March 2016 was 358,48.37 (in lakhs). Therefore, for the bank to achieve the Optimal Profit of 358,48.37 (in lakhs), Kisan Credit Card x_1was allocated an amount of 143,39.35 (in lakhs); Kisan Credit Card JL x_2 an amount of 0.00; Produce Pledge Loan x_3an amount of 215,09.02; Investment Credit Loan 0.00. It was also observed that if CDCC Bank does not allocate any amount to Kisan Credit Card and Investment Credit loan, the bank can still achieve the Optimal Profit of 10916.55.

Key words: District Central Co-operative Bank, Profit Optimization, Linear Programming Problem (LPP)

I. Introduction

A. *Co-Operative Banks-Introduction*

Co-operative Banks are organized and managed by the principle of co-operation, self-help and mutual help. They function with the rule of "one member, one vote" and function on "no profit, no loss" basis. Co-operative Banks, as a principle do not pursue the goal on profit maximization. Co-operative Banks provide limited banking products and are functionally specialized in agriculture limited products.

The main aim of the Co-operative Banks is to provide cheaper credit to their members and not to maximize profit they may access money market to improve their income so as to remain viable.

B. *Linear Programming*

Linear Programming is a subset of Mathematical Programming that is concerned with efficient allocation of limited resources to known activities with the objective of meeting a desired goal of maximization of profit or minimization of cost. Linear programming (LP) (also called linear optimization) is a method to achieve the best outcome (such as maximum profit or lowest cost) in a mathematical model whose requirements are represented by linear relationships. Linear programming is a special case of mathematical programming (mathematical optimization).

C. *Revised Simplex Method*

In mathematical optimization, the revised simplex method is a variant of George Dantig's simplex method for linear programming. The revised simplex method is mathematically equivalent to the standard simplex method but differs in implementation.

The Revised Simplex Method is commonly used for solving linear programs. This method operates on a data structure that is roughly of size m by m instead of the whole table. On the other hand, the revised method requires extra computation to generate necessary elements of the table.

In the revised method, the standard form is represented implicitly in terms of the original system together with a functional equivalent of the inverse of the basis B.

D. *Organization of Paper*

The first section focuses on the introduction to the paper and deals with the background to the paper, the statement of the problem, and organization of the paper. The second section deals with the literature review. This provides a theoretical frame work within which the paper is located and some related research findings. The third section highlights on the methodology, this includes modeling the problem of profit optimization of CDCC Bank as Linear Programming Problem; showing the optimal profit using Revised Simplex Algorithm and affording sensitivity analysis of the problem of profit optimization. Section four provides for the results of the data analysis and discussion of the result while the final section provides the conclusion of the paper.

II. Linear Programming Problem with a Constant Term in the Objective Function

Given any Linear Programming Problem:

Max $Z = cx+b$

s.t. $Ax \leq b$ $x > 0$

Where d is a constant term in the objective function. To solve such problem, we keep the constant term aside and apply the appropriate method of solving the LP. When the Objective function is determined, we then add the constant term to objective function to obtain the required optimal objective Function.

E. Revised Simplex Method (RSM)

Original simplex method calculates and stores all numbers in the table. Revised Simplex Method which is more efficient for computing Linear programming problems operates on a data structure that is roughly of size m by m instead of the whole table.

$$\text{Max } Z = cx+b$$
$$\text{s.t. } Ax \leq b$$
$$x > 0$$

Initially constraints become the standard form:

$$[A \quad I] \begin{pmatrix} x \\ x_s \end{pmatrix} = [b]$$

Where x_s = slack variables

Basis matrix: columns relating to basic variables.

$$B = \begin{bmatrix} B_{11} \cdots\cdots B_{1M} \\ B_{21} \cdots\cdots B_{2M} \\ \cdots\cdots \\ \cdots\cdots \\ B_{M1} \cdots\cdots B_{MM} \end{bmatrix}$$

(Initially B = I)

Basic variable values: $X_B = \begin{pmatrix} X_{B1} \\ \cdots \\ \cdots \\ X_{BM} \end{pmatrix}$

At any iteration non-basic variables = 0

$$Bx_B = b$$

$$X_B = B^{-1}b$$

Where B^{-1} is the inverse matrix

At any iteration, given the original b vector and the inverse matrix, X_B(current R.H.S.) can be calculated.

$$Z = c_B x_B$$

Where c_B = objective coefficients of basic variables.

F. Steps in the Revised Simplex Method

1. Determine entering variable, X_j, with associated vector P_j.

Compute Y = $c_B B^{-1}$

Compute Z_j- C_j = Yp_j- c_j for all non-basic variables.

Choose largest negative value (maximization).

If none, stop.

2. Determine leaving variable, X_r , with associated vector P_r.

Compute $X_B = B^{-1}$b (current R.H.S.)

Compute current constraint coefficients of entering variable:

$$\alpha^j = B^{-1}P_j$$

X_r is associated with

$$\theta = \text{Min } \{(x_B)_K / \alpha_K^J, \alpha_K^J > 0\} \text{ i.e.}$$

K minimum ratio rule

3. Determine next basis i.e. calculate B^{-1}.

Go to step 1.

III. STATEMENT OF THE PROBLEM

The data type is secondary data in the areas of loans such as Kisan Credit Card, Kisan Credit Card JL, Produce Pledge Loan, Investment Credit of atleast 200 societies for the period of one year from April 2015 to March 2016 as shown in table 4.1 below.

The bank is to allocate a total fund of 358,48.37(in lakhs) to the various loan products. The bank is faced with following constraints:

Amounts allocated to the various loan products should not be more than the total funds.

Allocate not more than 70% of the total funds to Kisan Credit Card, Produce Pledge Loan. Kisan Credit Card JL , Produce Pledge Loan, Investment Credit should not be more than 60% of the total funds.

The overall bad debts on the Kisan Credit Card, Kisan Credit Card JL, Produce Pledge Loan, Investment Credit should not exceed 0.05 of the total funds.

A. Mathematical Model

The variables of the model are defined as

x_1= Kisan Credit Card (in lakhs)

x_2= Kisan Credit Card JL

x_3= Produce Pledge Loan

x_4= Investment Credit

The objective function is written as

Z = [0.062(0.97)-0.03]+
[0.062(0.98)-0.02]+
[0.105(0.96)-0.04]+
[0.0995(0.95)-0.05]

Z = 0.03014 x_1+0.04076x_2+0.0608x_3+0.044525x_4

The problem has four (4) constraints:

Total funds:

$x_1 + x_2 + x_3 + x_4 \le 3.584837$

Allocate not more than 70% of the total funds to Kisan Credit Card, , Produce Pledge Loans, Investment Credit :

$x_1 + x_3 + x_4 \le 0.7 \times 3.584837$

$x_1 + x_3 + x_4 \le 2.5093859$

Produce Pledge Loan , Investment Credit should not be more than 60% of the total funds:

$x_3 + x_4 \le 2.1509022$

The overall bad debts on the Kisan Credit Card, Kisan Credit Card JL, Produce Pledge Loan, Investment Credit should not exceed 0.05 of the total funds:

$0.03x_1+0.02x_2+0.04x_3+0.05x_4 \le 0.05\times3.584837$

$0.03x_1+0.02x_2+0.04x_3+0.05x_4 \le 0.179242185$

Non-negativity constraints:

$x_1 \ge 0,\ x_2 \ge 0,\ x_3 \ge 0,\ x_4 \ge 0$

Now the LP is

Max Z= 0.03014 x_1+0.04076x_2+0.0608x_3+ 0.044525x_4

s.t $x_1 + x_2 + x_3 + x_4 \le 3.584837$

$x_1 + x_3 + x_4 \le 2.5093859$

$x_3 + x_4 \le 2.1509022$

$0.04x_1 + 0.03x_2 + 0.02x_3 + 0.01x_4 \le 0.17924185$

B. Using the Revised Simplex Method to solve the Lpp

The Linear Programming Problem(LPP) of the problem of Profit Optimization of Coimbatore District Central Co-operative Bank was

Max Z =0.03014 x_1+0.04076x_2+0.0608x_3+ 0.044525x_4

s.t $x_1 + x_2 + x_3 + x_4 \le 3.584837$

$x_1 + x_3 + x_4 \le 2.5093859$

$x_3 + x_4 \le 2.1509022$

$0.04\ x_1 + 0.03\ x_2 + 0.02x_3 + 0.01x_4 \le 0.17924185$

Standard form of constraints is:

$x_1 + x_2 + x_3 + x_4 + s_1 = 3.584837$

$x_1 + x_3 + x_4 + s_2 = 2.5093859$

$x_3 + x_4 + s_3 = 2.1509022$

$0.04x_1+0.03x_2+0.02x_3+0.01x_4 + s_4=0.17924185$

$x_1, x_2, x_3, x_4, s_1, s_2, s_3, s_4 \ge 0$

$$x_B = B^{-1}b = \begin{pmatrix} 3.584837 \\ 2.5093859 \\ 2.1509022 \\ 0.17924185 \end{pmatrix}^T$$

$Z = c_B x_B = 0$

C. First Iteration

Step 1:

Determine entering variable, x_j , with associated vector p_j.

Compute Y = $c_B B^{-1}$

Compute Z_j- C_j = Yp_j- c_j for all non-basic variables.
Y = (0 0 0 0)

NUMBER OF CUSTOMERS	TYPE OF PRODUCTS	INTEREST RATE	PROBABILITY OF BAD DEBT	PROBABILITY OF NO BAD DEBT
130	Kisan Credit Card	5.2%+1% =6.2%	3%	=1-0.03=0.97
120	Kisan Credit Card JL	5.2%+1% =6.2%	2%	=1-0.02=0.98
100	Produce Pledge Loan	10.50%	4%	=1-0.04=0.96
110	Investments Credit	9.95%	5%	=1-0.05=0.95

Z_1- C_1 = Yp_1- c_1 = -0.03014
Z_2- C_2 = Yp_2- c_2 = -0.04076
Similarly for, Z_3- C_3 = Yp_3- c_3 = -0.0608
Z_4- C_4 = Yp_4- c_4 = -0.044525
Therefore x_3 is the entering variable.
Step 2:
Determine leaving variable, x_r , with associated vector p_r.
Compute $x_B = B^{-1}$b (current RHS)
Compute current constraint coefficients of entering variable

$$\alpha^j = B^{-1}P_j$$

x_r is associated with,

$$\theta = \underset{K}{\text{Min}} \{(x_B)_K / \alpha_K^J , \alpha_K^J > 0\}$$

α^3 = (1 1 1 0.04)^T
θ = Min {3.584837, 2.5093859, 2.1509022, 0.17924185/0.04} = 2.1509022
Therefore S_3 leaves the basis.
Step 3:
Solution after one iteration:

$$x_B = B^{-1} b = \begin{pmatrix} 1.4339348 \\ 3.9433207 \\ 2.1509022 \\ 0.09320576 \end{pmatrix}$$

Go to step 1

D. Second Iteration

Step 1:
Determine entering variable, x_j , with associated vector p_j.
Compute Y = $c_B B^{-1}$
Compute Z_j- C_j = Yp_j- c_j for all non-basic variables
Y = (0 0 0.0608 0)
x_1: Z_1-C_1= Yp_1- c_1 = -0.03014
x_2: Z_2-C_2= Yp_2- c_2 = 0.02004
S_3: Z_3-C_3= Yp_3- c_3 = 0.0608
x_4: Z_4-C_4= Yp_4- c_4= 0.016275
Therefore x_1 is the entering variable
Step 2:
Determine leaving variable, x_r , with associated vector p_r.
Compute $x_B = B^{-1}$b (current RHS)
Compute current constraint coefficients of entering variable

$$\alpha^j = B^{-1}P_j$$

x_r is associated with

$$\theta = \underset{K}{\text{Min}} \{(x_B)_K / \alpha_K^J , \alpha_K^J > 0\}$$

$$x_B = \begin{pmatrix} 1.4339348 \\ 3.9433207 \\ 2.1509022 \\ 0.09320576 \end{pmatrix}$$

α^1 = (1 1 0 0.03)^T
θ = Min {1.4339348, 3.9433207, 0.09320576/0.03} = 1.4339348
Therefore S_1 leaves the basis.
Step 3:
Solution after one iteration:

$$x_B = B^{-1}b = \begin{pmatrix} 1.433935 \\ 2.509386 \\ 2.15090 \\ 0.0501878 \end{pmatrix}$$

Go to step 1
THIRD ITERATION
Step 1:
Determine entering variable, x_j , with associated vector p_j.
Compute Y = $c_B B^{-1}$
Compute Z_j- C_j = Yp_j- c_j for all non-basic variables.
Y = (0.03014 0 0.03066 0)
S_1 : Z_1- C_1 = Yp_1- c_1 = 0.03014
x_2 : Z_2- C_2= Yp_2 - c_2 = 0.02004
S_3 :Z_3- C_3 = Yp_3- c_3 = 0.03066
x_4 : Z_4- C_4= Yp_4- c_4 = 0.016275
No negatives. Therefore, stop.
Optimal Solution:
x_1 = 1.433935, x_2 = 0
x_3 = 2.1509, x_4 = 0
Z^* = $c_B x_B$ = 0.1091655
The optimum objective function value
= 0.1091655(in lakhs)
The optimum solution is:
x_1 = 143,39.35(in lakhs)
x_2 = 0
x_3 = 2.150902
x_4 = 0

However, the optimum objective function value of 0.1091655 an Optimal Profit of 0.1091655 (in lakhs). i.e. the Optimal Profit of Coimbatore District Central Co-operative Bank in the areas of loans such as Kisan Credit Card, Kisan Credit Card JL, Produce Pledge Loan, Investment Loans of atleast 200 societies for the period of one year from April 2015 to March 2016 was 10916.55

IV. SENSITIVITY ANALYSIS

Now the Linear Programming model of the problem of Profit Optimization of CDCC Bank was
Max Z = 0.03014 x_1+0.04076 x_2+0.0608x_3+ 0.044525x_4
The solution to this problem was:
x_1 = 1.433935, x_2 = 0, x_3 = 2.1509022, x_4 = 0

A. Changing Objective Function

While changing the objective function it turns out that there are two cases to consider. The first case is the change in a non-basic variable (a variable that takes on the value zero in the solution). What happens to the solution if the coefficient of a non-basic variable decreases? For example, suppose that the coefficient of x_1 in the objective function above was reduced from 0.03014 to 0.021 so that the objective function is:

$$\text{Max } Z = 0.021\,x_1+0.04076x_2+0.0608x_3+0.044525x_4$$

By taking the variable that didn't need to be used in the first place and then made it less profitable (lowered its coefficient in the objective function). Still not going to use it. The solution does not change.

1) Observation

If the objective function is lower coefficient of a non-basic variable, then the solution does not change. Automatically, raising the coefficient a lot might induce to change the value of x_1 in a way that makes (x_1>0). So, for a non-basic variable, should expect a solution to continue to be valid for a range of values for coefficients of non-basic variables. The range should include all lower values for the coefficient and some higher values. If the coefficient increases enough and putting the variable into the basis is feasible, then the solution changes. The change makes the variable contribute less to profit. For example, if the coefficient of x_4 in the objective function in the example was 0.02259 instead was 0.044525(so that the objective was

Max Z = $0.021\,x_1+0.04076\;x_2+0.0608x_3+0.02259x_4$, will change the solution since the reduction in the coefficient of x_4 is large. On the other hand, a small reduction in x_4 objective function coefficient would classically not cause to change the solution. So, intuitively, there should be a range of values of the coefficient of the objective function in which the solution of the problem does not change. Outside of this range, the solution will change. The value of the problem always changes while changing the coefficient of a basic variable.

B. Changing a Right-Hand Side (RHS) Constant of a Constraint

While changing the amount of resource in a non-binding constraint, i.e. small increases will never change our solution and small decreases will also not change anything. However, if decreasing the amount of resource enough to make the constraint binding, the solution could change. But, changes in the right-hand side of binding constraints always change the solution.

C. Adding a Constraint

If adding a constraint to a problem, two things can happen.

1) Original solution satisfies the constraint or it doesn't. If it does, then it is done.
2) If we had a solution before and the solution is still feasible for the new problem, then we must still have it. If the original solution does not satisfy the new constraint, then possibly the new problem is infeasible. If not, then there is another solution.

V. Conclusion

Based on our analysis, it is concluded that the Profit Optimization in CDCC Bank can be achieved through LP modeling of the problem and using the Revised Simplex Method to solve the L.P. THE BANK TO ACHIEVE THE OPTIMAL PROFIT OF 10916.55, Kisan Credit Card x_1 must be allocated an amount of 143,39.35, Kisan Credit Card JL x_2 an amount of 0.00, Produce Pledge Loan x_3 an amount of 215,09.02 investment Loans x_4 an amount of 0.00.

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