Image Compression using Wavelets

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Abstract— Images require substantial storage and transmission resources, thus image compression is advantageous to reduce these requirements. The report covers some background of wavelet analysis, data compression and how wavelets have been and can be used for image compression. An investigation into the process and problems involved with image compression was made and the results of this investigation are discussed. It was discovered that thresholding was had an extremely important influence of compression results so suggested thresholding strategies are given along with further lines of research that could be undertaken.

Key words: Image Compression using Wavelets, Image Compression

I. INTRODUCTION

Often signals we wish to process are in the time-domain, but in order to process them more easily other information, such as frequency, is required. Mathematical transforms translate the information of signals into different representations. For example, the Fourier transform converts a signal between the time and frequency domains, such that the frequencies of a signal can be seen. However the Fourier transform cannot provide information on which frequencies occur at specific times in the signal as time and frequency are viewed independently. To solve this problem the Short Term Fourier Transform STFT) introduced the idea of windows through which different parts of a signal are viewed. For a given window in time the frequencies can be viewed. However Heisenburg.s Uncertainty Principle states that as the resolution of the signal improves in the time domain, by zooming on different sections, the frequency resolution gets worse. Ideally, a method of multiresolution is needed, which allows certain parts of the signal to be resolved well in time, and other parts to be resolved well in frequency. The power and magic of wavelet analysis is exactly this multiresolution.

Images contain large amounts of information that requires much storage space, large transmission andwidths and long transmission times. Therefore it is advantageous to compress the image by storing only the essential information needed to reconstruct the image. An image can be thought of as a matrix of pixel (or intensity) values. In order to compress the image, redundancies must be exploited, for example, areas where there is little or no change between pixel values. Therefore images having large areas of uniform colour will have large redundancies, and conversely images that have frequent and large changes in colour will be less redundant and harder to compress.

Wavelet analysis can be used to divide the information of an image into approximation and detail subsignals. The approximation subsignal shows the general trend of pixel values, and three detail subsignals show the vertical, horizontal and diagonal details or changes in the image. If these details are very small then they can be set to zero without significantly changing the image. The value below which details are considered small enough to be set to zero is known as the threshold. The greater the number of zeros the greater the compression that can be achieved. The amount of information retained by an image after compression and decompression is known as the .energy retained and this is proportional to the sum of the squares of the pixel values. If the energy retained is 100% then the compression is known as lossless, as the image can be reconstructed exactly. This occurs when the threshold value is set to zero, meaning that the detail has not been changed. If any values are changed then energy will be lost and this is known as lossy compression. Ideally, during compression the number of zeros and the energy retention will be as high as possible. However, as more zeros are obtained more energy is lost, so a balance between the two needs to be found.

The first part of the report introduces the background of wavelets and compression in more detail. This is followed by a review of a practical investigation into how compression can be achieved with wavelets and the results obtained. The purpose of the investigation was to find the effect of the decomposition level, wavelet and image on the number of zeros and energy retention that could be achieved. For reasons of time, the set of images, wavelets and levels investigated was kept small. Therefore only one family of wavelets, the Daubechies wavelets, was used. The images used in the investigation can be seen in Appendix B. The final part of the report discusses image properties and thresholding, two issues which have been found to be of great importance in compression.

II. BACKGROUND

A. The Need for Wavelets

Often signals we wish to process are in the time-domain, but in order to process them more easily other information, such as frequency, is required. A good analogy for this idea is given by Hubbard[2], p14.

The analogy cites the problem of multiplying two roman numerals. In order to do this calculation we would find it easier to first translate the numerals in to our number system, and then translate the answer back into a roman numeral. The result is the same, but taking the detour into an alternative number system made the process easier and quicker. Similarly we can take a detour into frequency space to analysis or process a signal.

1) Fourier Transforms (FT)

Fourier transforms can be used to translate time domain signals into the frequency domain. Taking another analogy from Hubbard[2] it acts as a mathematical prism, breaking up the time signal into frequencies, as a prism breaks light into different colours.

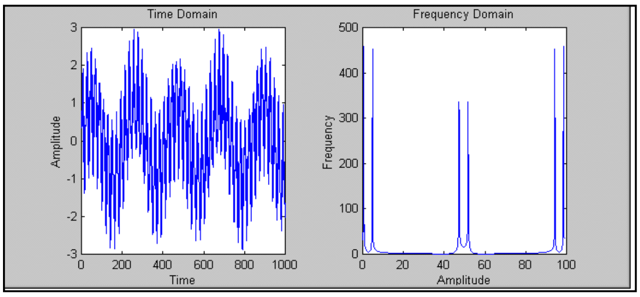

Fig. 1: The left graph shows a signal plotted in the time domain, the right graph shows the Fourier transform of the signal.

2) Multiresolution and Wavelets

The power of Wavelets comes from the use of multiresolution. Rather than examining entire signals through the same window, different parts of the wave are viewed through different size windows (or resolutions). High frequency parts of the signal use a small window to give good time resolution, low frequency parts use a big window to get good frequency information.

An important thing to note is that the 'windows' have equal area even though the height and width may vary in wavelet analysis. The area of the window is controlled by Heisenberg's Uncertainty principle, as frequency resolution gets bigger the time resolution must get smaller.

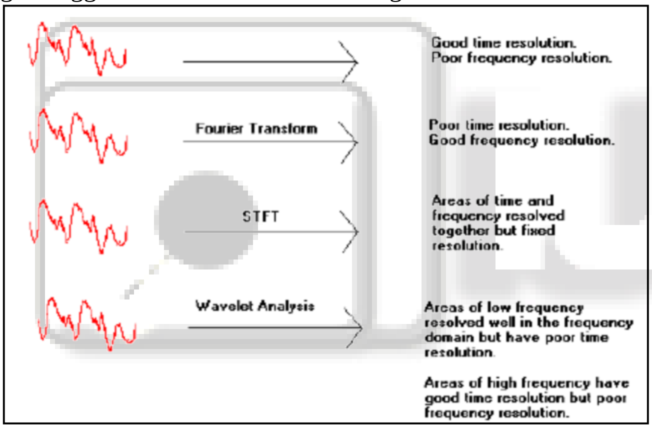

Fig. 2: The different transforms provided different resolutions of time and frequency.

In Fourier analysis a signal is broken up into sine and cosine waves of different frequencies, and it effectively re-writes a signal in terms of different sine and cosine waves. Wavelet analysis does a similar thing, it takes a .mother wavelet., then the signal is translated into shifted and scale versions of this mother wavelet.

3) The Continuous Wavelet Transform (CWT)

The continuous wavelet transform is the sum over all time of scaled and shifted versions of the mother wavelet ψ. Calculating the CWT results in many coefficients C, which are functions of scale and translation.

$$C(s,\tau) = \int_{-\infty}^{\infty} f(t)\psi(s,\tau,t)\,dt$$

The translation, τ, is proportional to time information and the scale, s, is proportional to the inverse of the frequency information. To find the constituent wavelets of the signal, the coefficients should be multiplied by the relevant version of the mother wavelet.

4) Sampling and the Discrete Wavelet Series

In order for the Wavelet transforms to be calculated using computers the data must be discretised. A continuous signal can be sampled so that a value is recorded after a discrete time interval, if the Nyquist sampling rate is used then no information should be lost. With Fourier Transforms and STFT's the sampling rate is uniform but with wavelets the sampling rate can be changed when the scale changes. Higher scales will have a smaller sampling rate. According to Nyquist Sampling theory, the new sampling rate N_2 can be calculated from the original rate N1 using the following:

$$N_2 = \frac{s_1}{s_2} N_1 \qquad N_2 = \frac{s_1}{s_2} N_1$$

Where s_1 and s_2 are the scales. So every scale has a different sampling rate.

After sampling the Discrete Wavelet Series can be used, however this can still be very slow to compute. The reason is that the information calculated by the wavelet series is still highly redundant, which requires a large amount of computation time. To reduce computation a different strategy was discovered and Discrete Wavelet Transform (DWT) method was born.

5) DWT and subsignal encoding

The DWT provides sufficient information for the analysis and synthesis of a signal, but is advantageously, much more efficient.

Discrete Wavelet analysis is computed using the concept of filter banks. Filters of different cut-off frequencies analyse the signal at different scales. Resolution is changed by the filtering, the scale is changed by upsampling and downsampling. If a signal is put through two filters:

– a high-pass filter, high frequency information is kept, low frequency information is lost.
– a low pass filter, low frequency information is kept, high frequency information is lost.

Then the signal is effectively decomposed into two parts, a detailed part (high frequency), and an approximation part (low frequency). The subsignal produced from the low filter will have a highest frequency equal to half that of the original. According to Nyquist sampling this change in frequency range means that only half of the original samples need to be kept in order to perfectly reconstruct the signal. More specifically this means that upsampling can be used to remove every second sample. The scale has now been doubled. The resolution has also been changed, the filtering made the frequency resolution better, but reduced the time resolution.

6) Conservation and Compaction of Energy

An important property of wavelet analysis is the conservation of energy. Energy is defined as the sum of the squares of the values. So the energy of an image is the sum of the squares of the pixel values, the energy in the wavelet transform of an image is the Compaction of Energy An important property of wavelet analysis is the sum of the squares of the transform coefficients. During wavelet analysis the energy of a signal is divided between approximation and details signals but the total energy does not change. During compression however, energy is lost because thresholding changes the coefficient values and hence the compressed version contains less energy.

The compaction of energy describes how much energy has been compacted into the approximation signal during wavelet analysis. Compaction will occur wherever the magnitudes of the detail coefficients are significantly smaller than those of the approximation coefficients. Compaction is important when compressing signals because the more energy that has been compacted into the approximation signal the less energy can be lost during compression.

B. *Image Compression*

Images require much storage space, large transmission bandwidth and long transmission time. The only way currently to improve on these resource requirements is to compress images, such that they can be transmitted quicker and then decompressed by the receiver.

In image processing there are 256 intensity levels (scales) of grey. 0 is black and 255 is white. Each level is represented by an 8-bit binary number so black is 00000000 and white is 11111111. An image can therefore be thought of as grid of pixels, where each pixel can be represented by the 8-bit binary value for grey-scale.

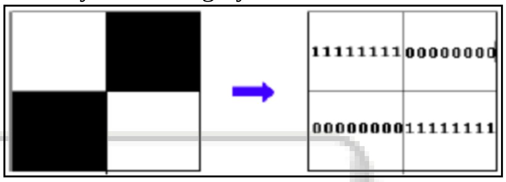

Fig. 3: Grey-Scale

The resolution of an image is the pixels per square inch. (So 500dpi means that a pixel is 1/500th of an inch). To digitise a one-inch square image at 500 dpi requires 8 x 500 x500 = 2 million storage bits.Using this representation it is clear that image data compression is a great advantage if many images are to be stored, transmitted or processed.

There are two parts to the compression:

- Find image data properties; grey-level histogram, image entropy, correlation functions etc..
- Find an appropriate compression technique for an image of those properties.

1) *Image Data Properties*

In order to make meaningful comparisons of different image compression techniques it is necessary to know the properties of the image. One property is the image entropy; a highly correlated picture will have a low entropy. For example a very low frequency, highly correlated image will be compressed well by many different techniques; it is more the image property and not the compression algorithm that gives the good compression rates. Also a compression algorithm that is good for some images will not necessarily be good for all images, it would be better if we could say what the best compression technique would be given the type of image we have. One way of calculating entropy is suggested by [3] :

2) *Compression techniques*

There are many different forms of data compression. This investigation will concentrate on transform coding and then more specifically on Wavelet Transforms.

Image data can be represented by coefficients of discrete image transforms. Coefficients that make only small contributions to the information contents can be omitted. Usually the image is split into blocks (subimages) of 8x8 or 16x16 pixels, then each block is transformed separately. However this does not take into account any correlation between blocks, and creates "blocking artifacts", which are not good if a smooth image is required.

However wavelets transform is applied to entire images, rather than subimages, so it produces no blocking artefacts. This is a major advantage of wavelet compression over other transform compression methods.

3) *Thresholding in Wavelet Compression*

For some signals, many of the wavelet coefficients are close to or equal to zero. Thresholding can modify the coefficients to produce more zeros. In Hard thresholding any coefficient below a threshold λ, is set to zero. This should then produce many consecutive zero's which can be stored in much less space, and transmitted more quickly by using entropy coding compression.

An important point to note about Wavelet compression is explained by Aboufadel[1]:

"The use of wavelets and thresholding serves to process the original signal, but, to this point, no actual compression of data has occurred".

This explains that the wavelet analysis does not actually compress a signal, it simply provides information about the signal which allows the data to be compressed by standard entropy coding techniques, such as Huffman coding. Huffman coding is good to use with a signal processed by wavelet analysis, because it relies on the fact that the data values are small and in particular zero, to compress data. It works by giving large numbers more bits and small numbers fewer bits. Long strings of zeros can be encoded very efficiently using this scheme. Therefore an actual percentage compression value can only be stated in conjunction with an entropy coding technique. To compare different wavelets, the number of zeros is used. More zeros will allow a higher compression rate, if there are many consecutive zeros, this will give an excellent compression rate.

4) *Images in MATLAB*

The project has involved understanding data in MATLAB, so below is a brief review of how images are handled. Indexed images are represented by two matrices, a colormap matrix and image matrix.

- The colormap is a matrix of values representing all the colours in the image.
- The image matrix contains indexes corresponding to the colour map colormap.

A colormap matrix is of size Nx3, where N is the number of different colours in the image. Each row represents the red, green, blue components for a colour.

$$\text{e.g. the matrix} \begin{bmatrix} r1 & g1 & b1 \\ r2 & g2 & b2 \end{bmatrix}$$

Represents two colours, the first have components r1, g1, b1, and the second having the components r2, g2 and b2.

The wavelet Toolbox only supports indexed images that have linear, monotonic colormaps. Often colour images need to be pre-processed into a grey scale image before using wavelet decomposition.

The Wavelet Toolbox User's Guide [4] provides some sample code to convert colour images into grey scale, this will be useful if I need to put any images into MATLAB.

C. Wavelets and Compression

Wavelets are useful for compressing signals but they also have far more extensive uses. They can be
used to process and improve signals, in fields such as medical imaging where image degradation is not
tolerated they are of particular use. They can be used to remove noise in an image, for example if it is of very fine scales, wavelets can be used to cut out this fine scale, effectively removing the noise.

III. Conclusions

Wavelet analysis is very powerful and extremely useful for compressing data such as images. Its power comes from its multiresolution. Although other transforms have been used, for example the DCT was used for the JPEG format to compress images, wavelet analysis can be seen to be far superior, in that it doesn.t create .blocking artefacts. This is because the wavelet analysis is done on the entire image rather than sections at a time. A well-known application of wavelet analysis is the compression of fingerprint images by the FBI.

The project involved writing automated scripts in Matlab which could calculate a great number of results for a range of images, Daubechies wavelets and decomposition levels. The first set of results calculated used global thresholding, however this was found to be an inferior way of calculating threshold values. To improve upon this, a second result set was calculated, using local thresholding.

This second results set proved to be more useful in understanding the effects of decomposition levels, wavelets and images. However, this was still not the optimal thresholding in that it is possible to get a higher energy retention for a given percentage of zeroes, by thresholding each detail subsignal in a different way.

The image itself has a dramatic effect on compression. This is because it is the image.s pixel values that determine the size of the coefficients, and hence how much energy is contained within each subsignal.

Furthermore, it is the changes between pixel values that determine the percentage of energy contained within the detail subsignals, and hence the percentage of energy vulnerable to thresholding. Therefore, different images will have different compressibilities.

There are many possible extensions to this project. These include finding the best thresholding strategy, finding the best wavelet for a given image, investigating other wavelet families, the use of wavelet packets and image denoising.

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