# FE Analysis of Stationary and Propagating Cracks for The Determination of SIF and Crack Initiation Angle

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*Abstract*— Determination of the behaviour of a crack, both stationary as well as propagating, in many situations plays a vital role for satisfactory and non-catastrophic performance of components, especially in Aviation Industries, Automobile Industries, Power-Plant Industries etc. In many situations, it is very difficult to determine the crack behaviour analytically and hence cracks are analysed using Finite Elements Analysis. In the present work, both the stationary and propagating cracks are analysed using Finite Elements Analysis and results obtained from aforementioned methods are then compared with the analytical results. Stationary cracks are compared using Stress Intensity Factors and propagating cracks are compared for the crack propagation paths, both obtained from Finite Elements Analysis and Analytical calculations. ABAQUS analysis software tool is used for the Finite Elements Analysis and Analytical computations are done using MATLAB software tool.

***Keywords:*** SIF, ABAQUS, MATLAB. MTS, SED

## I. Introduction

Cracks in engineering components are characterized by the parameters such as Stress Intensity Factor (SIF), Energy Release Rate (ERR), J-integral etc. For Linear Elastic Fracture Mechanics (LEFM) approach, which deals with the linear elastic behaviour of the materials with crack when subjected to remote stress, SIF is mostly used as a parameter to evaluate the state of stress near the crack tip. Analysis of crack can be done for isotropic materials and composite materials with simple geometries and simple load cases using analytical calculations but in case of complex geometries and complex load cases, it is very difficult to analyse the crack using analytical methods. Hence Finite Elements Analysis is done using commercially available packages. For the validation of the results obtained from Finite Elements Analysis, the element size of the cracked component is varied from coarse mesh to fine mesh and the SIFs are obtained. Proper element size is validated when SIF does not change much with the change in the element size. In the present work the SIFs are determined for various crack cases, which are obtained by changing a/b ratios for stationary straight and angle crack, and the crack initiation angle is determined by increasing the crack increment in case of propagating crack, using Finite Elements Analysis and these results are compared with the results obtained from Analytical Calculations. The effect of variation of plate thickness on SIF is also analysed along with the analysis of straight propagating cracks. For stationary straight cracks the mode I SIFs are calculated and for angle crack, which is a case of mixed mode fracture, both mode I and mode II SIFs are calculated. Here the mode I and mode II SIFs are obtained by varying a/b ratio. The inclined propagating cracks are analysed for the direction of propagation of cracks. This is done using both Finite Elements Analysis and Analytical calculations and both the results are compared. The Analytical computations for the determination of direction of propagation of crack are performed using MATLAB, which is a very powerful software tool for many computational works such as numerical computations, solving algebraic and differential equations, data and image processing, curve fitting, plotting data and many other works involved in science and engineering. The Finite Elements Analysis is done using ABAQUS analysis software tool, which has integrated pre-processor, solver and postprocessor, and is very popular in solving finite elements problems in various fields such as structural mechanics, fluid mechanics, fracture mechanics, thermal analysis etc.

## II. Analytical Calculations

### A. *Analysis of Stationary Crack:*

#### 1) *SIF for Straight, Edge Crack:*

Consider a rectangular plate of dimensions ‘2h’ by ‘b’ with straight edge crack of length ‘a’ subjected to a remote loading ‘σ’ in a direction perpendicular to the crack as shown in fig. 1. The stress intensity factor for this case, which is mode I SIF, is determined for various cases using the expressions given below:

For ≥ 1 and ≤ 0.6

$$K_I = \sigma\sqrt{\pi a}\left[1.12 - 0.23\left(\frac{a}{b}\right) + 10.6\left(\frac{a}{b}\right)^2 - 21.7\left(\frac{a}{b}\right)^3 + 30.4\left(\frac{a}{b}\right)^4\right]$$

For ≥ 1 and ≥ 0.3

$$K_I = \sigma\sqrt{\pi a}\left[\frac{1 + 3\left(\frac{a}{b}\right)}{2\sqrt{\pi\left(\frac{a}{b}\right)}\left(1 - \frac{a}{b}\right)^{\frac{3}{2}}}\right]$$

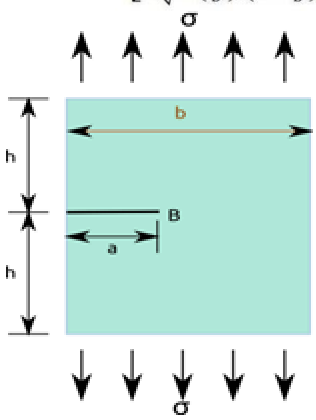


Fig. 1: Plate with Edge Crack Subjected to Uniaxial Load

#### 2) *SIF For Inclined, Edge Crack:*

Consider a rectangular plate of dimensions ‘2h’ by ‘b’ with centre crack of length ‘a’ inclined by an angle ‘β’ with abscissa, subjected to a stress of ‘σ’ along the ordinate and a stress of ‘α σ’ along the abscissa as shown in the fig. 2. For simplicity the stress along the abscissa is not applied and since the crack is inclined, it is a mixed mode problem. Hence

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it is analysed for mode I and mode II SIFs using the expressions given below:

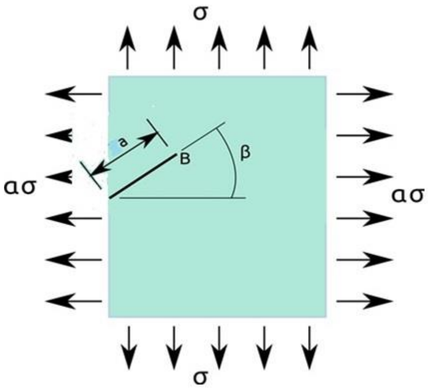


Fig. 2: Plate with Slanted Edge Crack under Biaxial Load

The SIFs for mode I and mode II cases are given below:

$$K_I = K_0\,(\cos^2\beta + \alpha \sin^2\beta)$$

$$K_{II} = K_0\,(1-\alpha)\sin\beta\cos\beta$$

where,

$$K_0 = \sigma\sqrt{\pi a}\left[1.12 - 0.23\left(\frac{a}{b}\right) + 10.6\left(\frac{a}{b}\right)^2 - 21.7\left(\frac{a}{b}\right)^3 + 30.4\left(\frac{a}{b}\right)^4\right]$$

*3) SIF For Inclined, Centre Crack*

Again, considering the same plate as discussed in the previous section but with a central, inclined crack as shown in the fig. 3. For simplicity, the stress along the abscissa is not applied and since the crack is inclined, it is a mixed mode problem. Hence it is also analysed for mode I and mode II SIFs using the expressions given below:

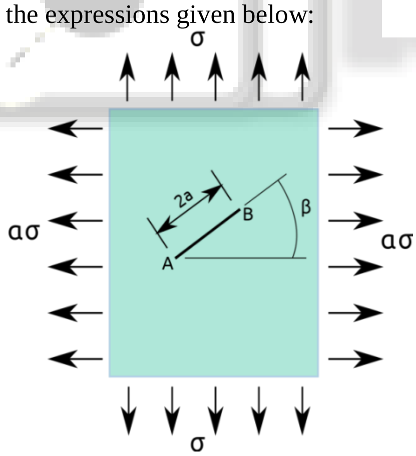


Fig. 3: Plate with Slanted Central Crack under Biaxial Load

The mode I and mode II SIFs are given below:

$$K_I = \sigma\sqrt{\pi a}\,(\cos^2\beta + \alpha \sin^2\beta)$$

$$K_{II} = \sigma\sqrt{\pi a}\,(1-\alpha)\sin\beta\cos\beta$$

*B. Analysis Of Propagating Crack*

1) *Maximum Tensile Stress (MTS) Criterion:* This criterion considers a certain component of stress state and it states that the crack will propagate only if this component reaches a critical condition. Again consider the fig. 3, the components of stress near the crack tip in polar coordinate are:

$$\sigma_{rr} = K_I f_{11}(r,\theta) + K_{II} f_{12}(r,\theta)$$

$$\sigma_{\theta\theta} = K_I f_{21}(r,\theta) + K_{II} f_{22}(r,\theta)$$

$$\tau_{r\theta} = K_I f_{31}(r,\theta) + K_{II} f_{32}(r,\theta)$$

Where $f_{ij}(r,\theta)$ are given by

$$f_{11}(r,\theta) = \frac{1}{\sqrt{2\pi r}}\left(\frac{5}{4}\cos\frac{\theta}{2} - \frac{1}{4}\cos\frac{3\theta}{2}\right)$$

$$f_{12}(r,\theta) = \frac{1}{\sqrt{2\pi r}}\left(-\frac{5}{4}\cos\frac{\theta}{2} + \frac{1}{4}\sin\frac{3\theta}{2}\right)$$

$$f_{21}(r,\theta) = \frac{1}{\sqrt{2\pi r}}\left(\frac{3}{4}\cos\frac{\theta}{2} + \frac{1}{4}\cos\frac{3\theta}{2}\right)$$

$$f_{22}(r,\theta) = \frac{1}{\sqrt{2\pi r}}\left(-\frac{3}{4}\sin\frac{\theta}{2} - \frac{3}{4}\sin\frac{3\theta}{2}\right)$$

$$f_{31}(r,\theta) = \frac{1}{\sqrt{2\pi r}}\left(\frac{1}{4}\sin\frac{\theta}{2} + \frac{1}{4}\sin\frac{3\theta}{2}\right)$$

$$f_{32}(r,\theta) = \frac{1}{\sqrt{2\pi r}}\left(\frac{1}{4}\cos\frac{\theta}{2} + \frac{3}{4}\cos\frac{3\theta}{2}\right)$$

Now substituting $f_{21}(r,\theta)$ and $f_{22}(r,\theta)$ in equation of $\sigma_{\theta\theta}$ and maximizing $\sigma_{\theta\theta}$ using equations

$$\frac{\partial\sigma_\theta}{\sigma\theta} = 0$$

$$\frac{\partial\sigma_\theta^2}{\partial\theta} < 0$$

Now simplifying and replacing θ by crack extension direction $\theta_c$ we get the following expression:

$$K_I\sin(\theta_c) + K_{II}(3\cos(\theta_c) - 1) = 0$$

Solving the above equation for $\theta_c$ gives multiple values of crack propagation angle of which one that satisfies the above inequality is considered as appropriate crack propagation angle.

## III. Finite Elements Analysis

*A. Mesh Convergence Study:*

The analysis for stationary straight and angle crack is done on a plate with width of 1m, length of 2m and thickness of 0.1m. The CAD model of the plate is created using Abaqus/CAE is shown in fig. 4. The material assigned is Aluminium with Young's modulus E=70GPa and Poisson's ratio υ=0.33 and Static, General Step is created and XFEM edge crack is assigned in interaction module. A pressure load of 1MPa is applied at top and bottom surfaces along with the constraint in the movement of top right edge in all directions except in the y-direction and the movement of bottom right edge is constrained in all directions. It should be ensured that the PHILSM and PSILSM under Failure/Fracture and STATUS XFEM under State/Field/User/Time are checked in Field Output Request. History Output Request is created for Crack domain and Energy Release Rate is selected under Stress

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Intensity Factor. The mesh is varied from coarse mesh to finer mesh as shown in fig. 4. This is obtained by selecting element sizes in a decreasing magnitude so that the number of elements per unit length of the component is increased in magnitude. The results in the form of SIFs are obtained and the proper mesh will be the one for which the result is not much varied even when the mesh is further refined. The results (fig. 4) are tabulated in table 1 and plotted as shown in the graph 1.

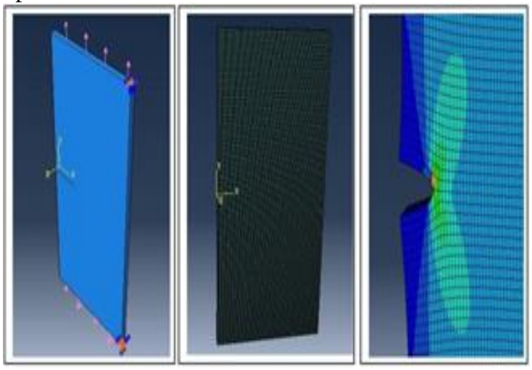

Fig. 4: Plate with Straight Edge Crack Along with Load, BC, Mesh and Analysis Results in Abaqus

| Element Size | No. of Elements per Unit Length | SIF |
|---|---|---|
| 0.075 | 13.333 | 1.820 |
| 0.05 | 20 | 1.763 |
| 0.025 | 40 | 1.711 |
| 0.02 | 50 | 1.692 |
| 0.0175 | 57.143 | 1.665 |
| 0.0125 | 80 | 1.661 |
| 0.01 | 100 | 1.623 |

TABLE 1: SIFs for various Number of Elements per Unit Lengths

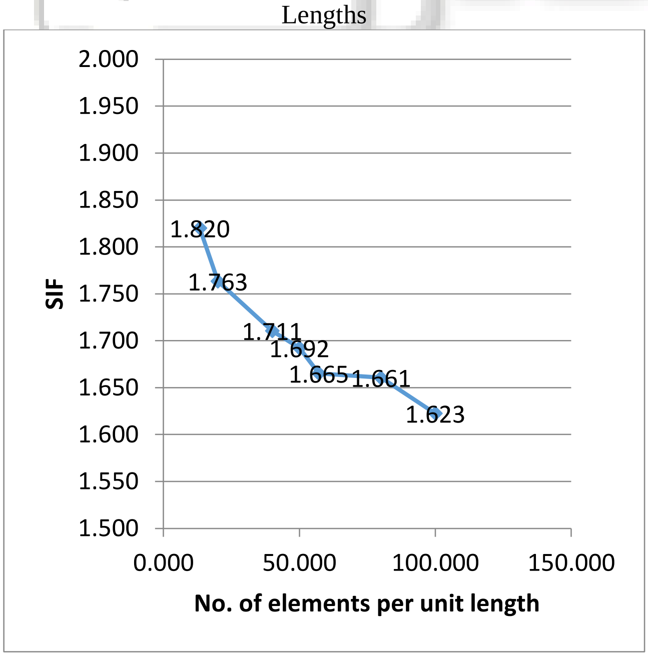


Fig. 5: Graph 1. SIFs for various Number of Elements per Unit Length

*B. FE Analysis of Stationary Crack:*

*1) FE Analysis for Straight Crack:*

The stationary cracks are analysed for various plate thicknesses and various crack length to plate width ratios. This is done on the same plate as mentioned in the previous section along with material properties, mesh, load and boundary conditions. The crack lengths are increased for each thickness of the plate and the results are tabulated. This is repeated for various thicknesses.

*2) FE Analysis for Inclined Crack:*

Angle cracks, when subjected the loading, will be in a mixed mode loading scenario and hence they need to be evaluated using both mode I and mode II SIFs. They are modelled in Abaqus/CAE and are given the same properties, mesh, step, loading and boundary conditions as they were for straight crack as shown in the fig. 5 and fig. 6. The crack length to plate width ratio is varied for each crack angle and the results are tabulated. This is again repeated for various crack angles. The t/b ratio for inclined edge cracks is 0.10 and for inclined centre cracks, it is 0.05.

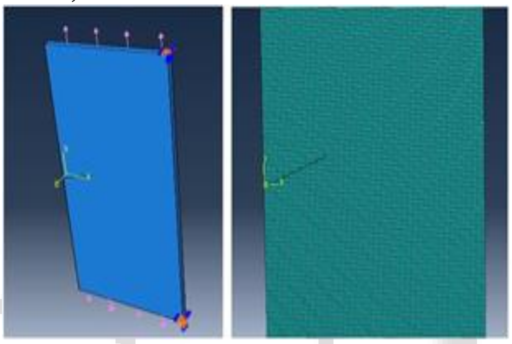

Fig. 6: Plate with Inclined Edge Crack with Load, BC and Mesh in Abaqus

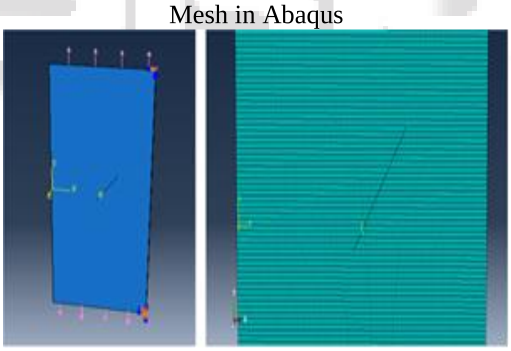

Fig. 7: Plate with Inclined Central Crack with Load, BC and Mesh in Abaqus

| $\sigma_{ys}$ | $\varepsilon_{plastic}$ |
|---|---|
| 111.0 | 0 |
| 112.6 | 0.0019 |
| 113.6 | 0.0038 |
| 116.1 | 0.0067 |
| 116.9 | 0.0086 |
| 118.7 | 0.0135 |
| 120.7 | 0.0183 |
| 122.5 | 0.0231 |
| 124.6 | 0.0279 |
| 127.5 | 0.0326 |
| 130.0 | 0.0421 |
| 133.3 | 0.0514 |
| 136.1 | 0.0607 |
| 139.3 | 0.0699 |
| 141.8 | 0.079 |
| 144.4 | 0.0881 |
| 146.3 | 0.0971 |

TABLE 2: Plasticity Data for material of CT Specimen

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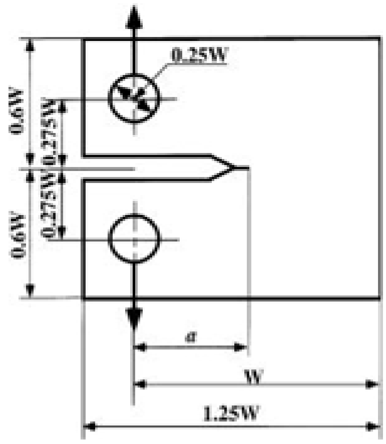


Fig. 8: Compact Tension (CT) Specimen

Propagating Straight Crack is analysed using standard CT specimen for the variation of strain energy and plastic dissipation energy as crack propagates for an initial crack length of 5 mm and applied displacements at each hole in y-direction is 0.5mm which will cause the propagation of crack. Propagating Inclined Crack is analysed using standard CT specimen, with initial crack length of 5mm; inclination of 30 degrees w.r.t. x-axis and applied displacements at each hole in y-direction is 6.9mm, to determine the path of crack. Standard CT specimen is modelled in Abaqus/CAE and is applied with the material properties of Aluminium with elasticity property as E=66.36 GPa, v=0.33 and plasticity property as shown in TABLE 2. Maximum Principal Stress Damage Criterion is used as Damage for Traction Separation Laws with maximum principal stress of 146.3 MPa along with Energy as damage evolution type with fracture energy of 24.2 MPa and viscosity coefficient of 10-6 for damage stabilization cohesion. Static, General Step is created with non-linear geometry turned on and maximum number of increments set to 10000 along with initial, minimum and maximum increment sizes of 0.02, 10-40and 0.1 respectively. XFEM crack is assigned in the interaction module. Displacement controlled analysis is done by applying a displacement of 6.9mm each at upper and lower portion of the plate. Free mesh technique is used for meshing the specimen with 4-noded bilinear plane stress quadrilateral element. It should be ensured that the PHILSM and PSILSM under Failure/Fracture and STATUS XFEM under State/Field/User/Time are checked in Field Output Request. General Solutions Control under "Other" tab in the main menu should be edited for Step created so that the Time Incrimination is specified for Discontinuous Analysis and IA under "More" tab is set to 50. The result of the analysis then obtained is shown in figure 4.5.

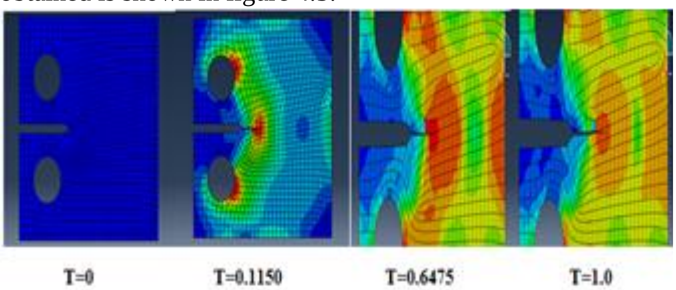


Fig. 9: Straight Propogating Crack in CT SPECIMEN with Mesh for various step Times in Abaqus

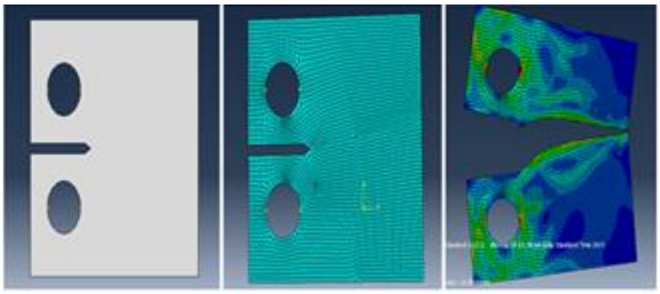

Fig. 10: Inclined Crack in CT Spicemen with Mesh and Crack Propogation path of in Abaqus

## IV. Results

### *A. Stationary Crack:*

#### *1) Straight Crack:*

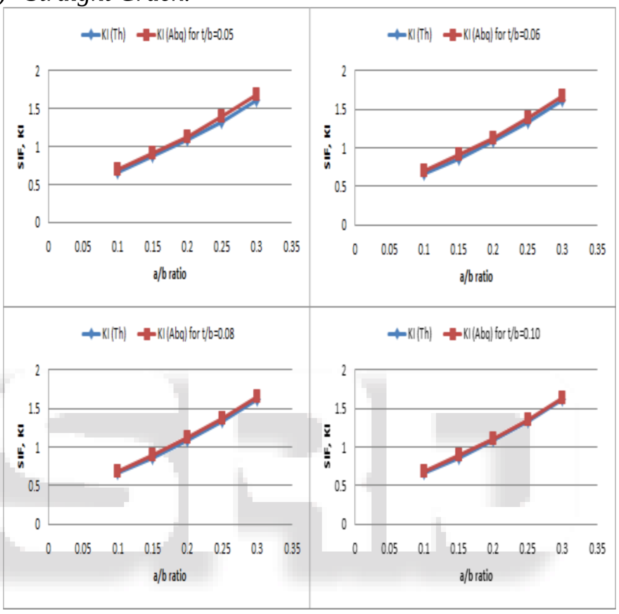


Fig. 11: Graph 2. SIF Analysis Results for various t/b ratios (Straight Edge Crack)

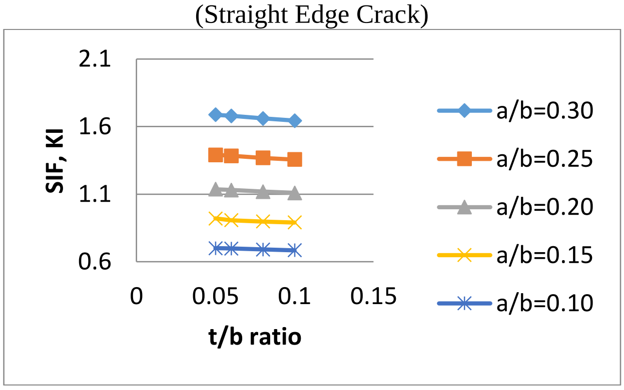


Fig. 12: Graph 3. Variation of SIF with t/b ratios for various a/b ratios

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*2) Inclined Edge Crack:*

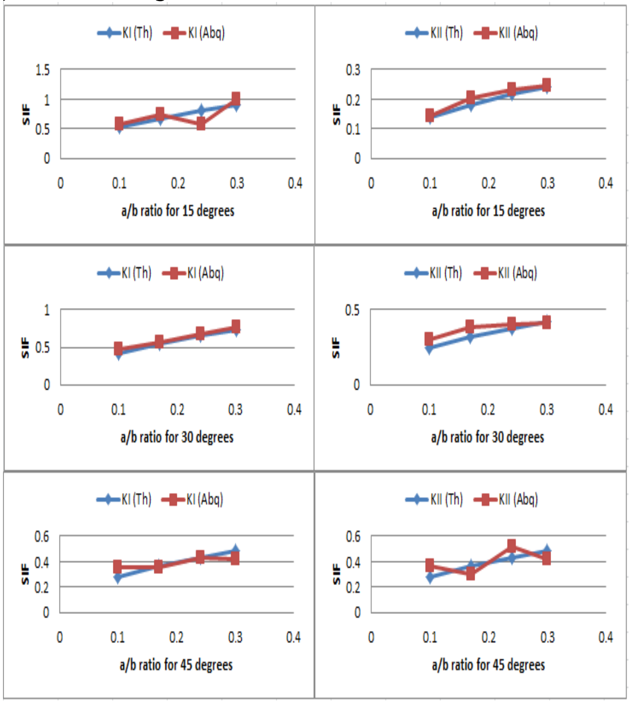


Fig. 13: Graph 4. SIFs for various angle cracks

*3) Inclined Central Crack:*

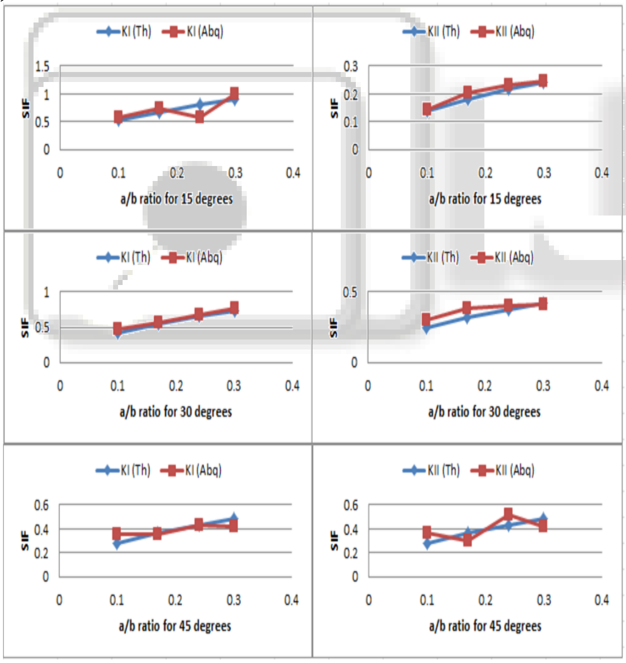


Fig. 14: Graph 5. SIFs for various Inclined Central Crack

*B. Propagating Crack:*

*1) Straight Propagating Crack:*

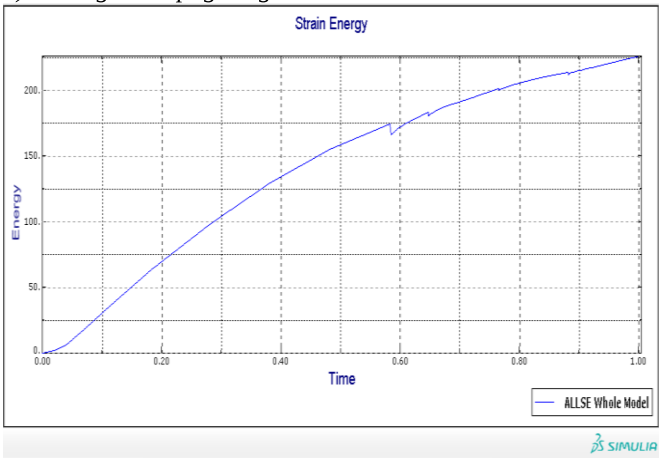


Fig. 15: Graph 6. Variation of Strain Energy with Step Time

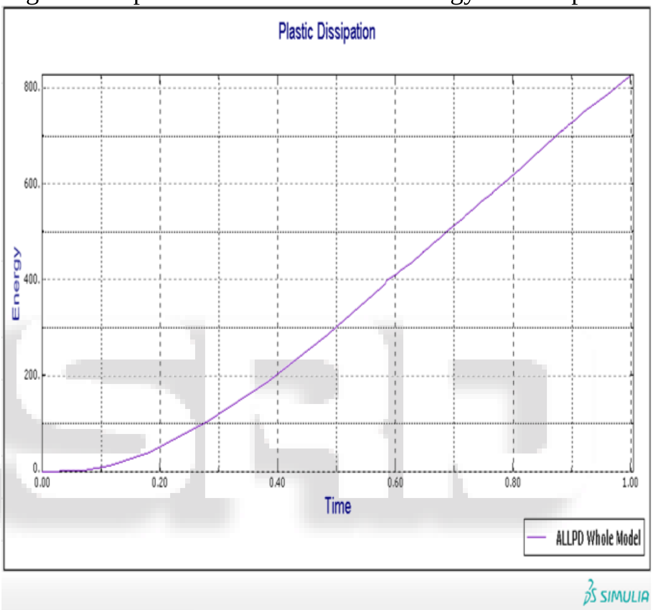


Fig. 16: Graph 7. Variation of Plastic Dissipation Energy with Step Time

*2) Inclined Propagating Crack:*

The path of crack propagation of inclined crack is given below:

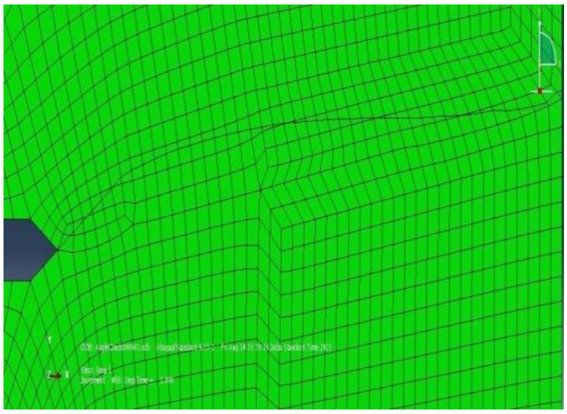

Fig. 17: Crack propagation path using Abaqus

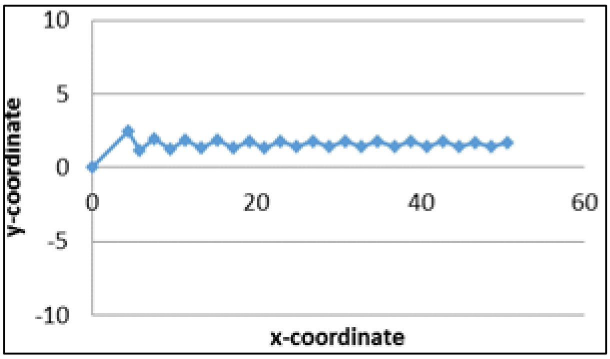


Fig. 18: Graph 8. Crack propagation path by Analytical Calculations using Matlab

| x | y |
|---|---|
| 0 | 0 |
| 4.3301 | 2.5 |
| 5.788 | 1.131 |
| 7.614 | 1.947 |
| 9.489 | 1.252 |
| 11.391 | 1.87 |
| 13.31 | 1.308 |
| 15.241 | 1.828 |
| 17.181 | 1.341 |
| 19.127 | 1.801 |
| 21.079 | 1.365 |
| 23.035 | 1.782 |
| 24.995 | 1.383 |
| 26.958 | 1.767 |
| 28.924 | 1.397 |
| 30.892 | 1.755 |
| 32.862 | 1.409 |
| 34.834 | 1.743 |
| 36.807 | 1.418 |
| 38.782 | 1.735 |
| 40.758 | 1.426 |
| 42.735 | 1.727 |
| 44.713 | 1.432 |
| 46.692 | 1.721 |
| 48.672 | 1.438 |
| 50.653 | 1.715 |

TABLE 3: Crack Propagation Path

## V. CONCLUSION

SIFs are determined for straight cracks by varying a/b ratios for various t/b ratios. It is found that as the thickness of the plate increases from t/b of 0.05 to 0.10 the results of SIFs obtained from analytical calculations and FE analysis converge. The SIFs of inclined edge and centre cracks are also determined by varying a/b ratios for different inclinations of cracks. The results of analysis obtained from analytical calculations and Abaqus analysis tool are in close agreement. Furthermore, it is found that the SIFs calculated from Abaqus by using SED and MTS criteria are essentially the same. Straight propagating cracks are analysed using Abaqus on standard CT specimen for the variation of Strain Energy and Plastic Dissipation Energy. It is found that there is a sudden drop in strain energy when crack incrimination takes place and at the same time there is an increase in plastic dissipation energy. This observation is in agreement with the findings mentioned in [28]. For inclined propagating cracks in CT specimen, the crack propagation path is analysed using MTS criterion. The propagation path is obtained by developing a program in MATLAB numerical software tool using MTS criterion and the result is compared with the propagation path obtained from Abaqus analysis software tool. The results are in close agreement.

## REFERENCES

[1] J. Ahmed and F.T. Loo, “Solution of plate bending problems in fracture mechanics using specialized FEA”, Engineering Fracture Mechanics 661-672.
[2] Barna. A. Szabo and Glenn J. Sharmann, 1988, “International Joual for Numerical Methods”, Vol. 26, 1855-1881.
[3] Ang D.D., Williams. M.L., 1961, “Combined Stresses in an Orthotropic plate having a finite crack”, ASME Jounal of Applied Mechanics 28, 372-378.
[4] Yuan. F.G., Yang, S., 2000, “Asymptotic crack-tip field in an anisotropic plate subjected to bending, twisting moments and transverse shear loads”, Composites Science and Technology 60, 2489-2502.
[5] Nakamura, T., 1991, “Three-dimensional stress fields of elastic interface cracks”, Journal of Applied Mechanics 58, 939-946.
[6] A. A. Griffith, “The Phenomena of Rupture and Flow in Solids”, Philos. Trans., R. Soc. Lond., Ser. A., Vol. 221, 1920, p.163
[7] G.R. Irwin, “Analysis of Stresses and Strains Near the End of a Crack Traversing a Plate”, J. Appl. Mech., Vol. 24, 1957, p.361.
[8] V. Weiss and S. Yukawa, “Critical Appraisal of Fracture Mechanics”, Fracture Toughness Testing and Its Applications, ASTM STP 381, ASTM, West Conshohocken, PA, 1965, p.1.
[9] F. W. Smith and D. R. Sorenson, “The Semi-Elliptical Surface Crack: A Solution by the Alternating Method”, Int. J. Fract., Vol. 12, No. 1, 1976, p. 47.
[10] T. A. Cruse and G. J. Meyers, “Three-Dimensional Fracture Mechanics analysis”, J. Struct. Div, ASCE, Vol.103, No. ST2, 1977, p. 309.
[11] J. C. Newman, Jr., and I. S. Raju, “Stress-Intensity Factor Equations for Cracks in three-Dimensional Finite Bodies”, Fracture Mechanics, “Fracture Mechanics, Vol.I: theory and Analysis, ASTM STP 791, J. C. Lewis and G. Sines, eds., ASTM, West Conshohocken, PA, 1983, p. 238.
[12] Y. Murakami, ed. In chief, Stress Intensity Factor Handbook, Vol. 2, Pergamon Press, oxford, 1987.
[13] C. W. Smith, “Experimental Techniques in Fracture Mechanics, Vol. 2, A. S. Kobayashi, ed., Iowa State University Press, Ames, 1975, p.3.
[14] D. Broek, Elementary Engineering Fracture mechanics, 4th ed., Kluwer Academic Publications, Dordrecht, the Netherlands, 1986.

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[15] R. W. Hertzberg, Deformation and Facture Mechanics of Engineering Materials, 4th ed., John Wiley and Sons, New York, 1996.
[16] H. L. Ewalds and R. J. H. Wanhill, Fracture Mechanics, Edward Arnold Publishers, Baltimore, 1984.
[17] T. L. Anderson, Fracture Mechanics: Fundamentals and Applications, 2nd ed., CRC Press, Boca Raton, FL, 1995.
[18] S. Suresh, Fatigue of Materials, 2nd ed., Cambridge University Press, Cambridge, 1998.
[19] H. Tada, P. C. Paris, and G. R. Irwin, The Stress Analysis of Cracks Handbook, 2nd ed., Paris Productions, St. Louis, MO, 1985.
[20] G. C. Sih, Handbook of Stress Intensity Factors, Institute of Fracture and Solid Mechanics, Lehigh University, Bethleham, PA, 1973.
[21] D. P. Rooke and D. J. Cartwright, Compendium of Stress Intensity Factors, Her Majesty's Stationery Office, London, 1976.
[22] D. P. Rooke and D. J. Cartwright, Compendium of Stress Intensity Factors, Her Majesty's Stationery Office, London, 1976.
[23] Petrasinovic, D., Bosko, R., Petrasinovic, N., (2012), "Extended finite elements method (XFEM) applied to aircraft duralium spar fatigue life estimation", Technicki Vjesnik – Technical Gazzete, Vol. 19, no. 3, p.557-562.
[24] Luigi Gigliotti, "Assessment of the applicability of XFEM in Abaqus for modelling crack growth in rubber", Master Thesis, Royal Institute of Technoloty, Stockholm-Sweden.
[25] Boljanovic, S., Maksimovic, S. (2011), "Analysis of the crack growth propagation process under mixed-mode loading", Engineering Fracture Mechanics, Vol. 78, no. 8, p.1565-1576; DOI: 10.1016/j.engfracmech.2011.02.003.
[26] Shafique, Khan, S. M. A., Marwan, K. K. (2000), "Analysis of mixed mode crack initiation angles under various loading conditions", Engineering Fracture Mechanics, Vol. 67, no. 5, p.397-419, DOI:10.1016/soo13-7944(00)00068-0.
[27] P.O. Kettunen, T.K. Lepisto, M.E. Lehtonen, "Strength of Metals and Alloys", Pergamon Press, ICSMA8, Vol. I, Tempere Finland, 1988.
[28] L. Karlsson, L.E. Lindgren, M. Jonsson, "mechanical Effects of Welding", Springer-Verlag, IUTAM Symposium, 1991, p.172.

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