An Application of Intuitionistic Fuzzy Dominance Matrix in Real Life Situation

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Abstract— This paper prefaces the idea of intuitionistic fuzzy dominance matrix. Dominance plays a vital role in decision making problems. It gives the optimal solution. In this paper, IFDM has been used to identify the most affected child in a particular family when the family members pamper them.

Key words: **Intuitionistic Fuzzy Dominance Matrix (IFDM)**

I. INTRODUCTION

In 1965, L. A. Zadeh introduced fuzzy set theory. Many researchers have applied fuzzy optimization techniques in decision making based on the mathematical formulation of fuzzy sets. Molodtsov has shown that each of the above topics suffers from some inherent difficulties due to inadequacy of their parametrization tools and introduced a concept of soft set theory. Maji introduced several algebraic operations in soft set theory and published a detailed theoretical study on soft sets. Recently, Cagman introduced soft matrix and applied it in decision making problems. Decision making is the study of identifying and choosing alternatives based on the values and preferences of the decision maker.

II. BASIC PRELIMINARIES OF FUZZY SOFT SETS

A. *Soft Set:*

Let U be an initial set and E be a set of parameters. Let P(U) denotes the power set of U, and let A⊂E. A pair (F,A) is called a soft set over U, where F is a mapping given by
F: A→P(U).

Example:

Let U = $\{u_{1,}u_{2,}u_{3,}u_4\}$ be the set four areas and E = $\{e_1(greenary),\ e_2(hill), e_3(sea\ side)\}$ be the three parameters. If A = $\{e_1, e_2\} \subset$ E. Let $F_A(e_1)$ = $\{u_{1,}u_{2,}u_{3,}u_4\}$ and

$$F_A(e_2) = \{u_{1,}u_{2,}u_3\}$$

then we can write the soft set (F_A, E) = $\{(e_1, \{u_{1,}u_{2,}u_{3,}u_4\}$, $(e_2, \{u_{1,}u_{2,}u_3\})\}$ over U which describes the "quality of areas" which Mrs. going to buy. We may represent the soft set in the following form:

U	e_1	e_2	e_3
$u_{1,}$	1	1	0
$u_{2,}$	1	1	0
$u_{3,}$	1	1	0
u_4	1	0	0

Table 1: Soft Set

B. *Fuzzy Soft Set:*

Let P(U) denotes the set of all fuzzy sets of U. Let $A_i \subset$ E. A pair (F_i, A_i) is called a fuzzy soft set over U, where F_i is a mapping given by F_i: A→ P(U).

Example:

Suppose a fuzzy soft set (F,E) describes attractiveness of the sarees with respect to the given parameters, which the teachers are going to wear.

U = $\{s_{1,}s_{2,}s_{3,}s_{4,}s_5\}$ which is the set of all sarees under consideration.

Let I^U be the collection of all fuzzy subsets of U. Also let E = {e_1= colorful, e_2= bright, e_3=cheap, e_4= dull}

Let F(e_1)= { s_1/0.5, s_2/0.9, s_3/0.0, s_4/0.0, s_5/0.0},
F(e_2)= {s_1/1.0, s_2/0.8, s_3/0.0, s_4/0.0, s_5/0.0},
F(e_3)= {s_1/0.0, s_2/0.0, s_3/0.0, s_4/0.6, s_5/0.0},
F(e_4)= {s_1/0.0, s_2/1.0, s_3/0.0, s_4/0.0, s_5/0.3}

Then the family { F(e_i), i = 1,2,3,4} of I^U is a fuzzy soft set (F,E).

C. *Intuitionistic Fuzzy Soft Set:*

Let U be an initial universal set and let E be set the of parameters. Let P(U) denotes the set of all intuitionistic fuzzy sets of U. A pair (F,A) is called an intuitionistic fuzzy soft set over U if F is a mapping given by F:A→ $P(U)$.

Example:

Suppose that there are four students in the universe given by U = $\{p_1, p_2, p_3, p_4\}$ and
E = where e_1 - excellent , e_2- good, e_3- average.
F(e_1)={($s_{1,}$0.5,0.2),($s_{1,}$0.9,0.1), ($s_{3,}$0.4,0.3)}
F(e_2)={($s_{1,}$0.3,0.2), ($s_{1,}$0.9, 0.7), ($s_{1,}$0.5,0.2)}
F(e_3)s_1,0.4,0.9), ($s_{1,}$0.3,0.7), ($s_{1,}$0.5,0.5)}

Thus Intuitionistic fuzzy soft set is a parameterized family of all Intuitionistic fuzzy set of U.

D. *Fuzzy Soft Matrices:*

Let $((f_A, E)$ be a fuzzy soft set over U. Then the subset of $U \times E$ is defined by $R_A = \{(u, e): e \in A, u \in (f_A(e)$ which is called a relation form of (f_A, E) . the characteristic function of R_A is written by $\mu_{R_A}: U \times E \rightarrow [0.1]$, where $\mu_{R_A}(u, e) \in [0, 1]$ is the membership value of $e \in E$.

If $\mu_{ij} = \mu_{R_A}(u_i, e_j)$, we can define a matrix

$$(\mu_{ij})_{m\times n} = \begin{bmatrix} \mu_{11} & \mu_{12} & \cdots & \mu_{1n} \\ \mu_{21} & \mu_{22} & \cdots & \mu_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ \mu_{m1} & \mu_{m2} & \cdots & \mu_{mn} \end{bmatrix}$$

Which is called $m \times n$ fuzzy soft matrices over the fuzzy soft se over U

E. *Complement of Intuitionistic Fuzzy Soft Matrix:*

Let (x_{ij}) be an $m \times n$ intuitionistic fuzzy soft matrix, where $x_{ij} = (\alpha_{ij}, \beta_{ij})$ for all i, j. Then the complement of (x_{ij}) is defined by $(x_{ij})^c = (y_{ij})$ is also an intuitionistic fuzzy soft matrix of order $m \times n$ and $y_{ij} = (\beta_{ij}, \alpha_{ij})$ for all i, j.

F. *Sum of the Intuitionistic Fuzzy Soft Matrices:*

Two intuitionistic fuzzy soft matrices A and B are said to be comformable for addition, if they be of the same order. The sum of two intuitionistic fuzzy soft matrices (x_{ij}) and (y_{ij}) of arder $m \times n$ is defined by, $(x_{ij}) + (y_{ij}) = (z_{ij})$ is also an $m \times n$ intuitionistic fuzzy soft matrix and $(z_{ij}) = (\max\{\alpha_{x_{ij}}, \alpha_{y_{ij}}\}, \min\{\beta_{x_{ij}}, \beta_{y_{ij}}\})$ for all i, j.

G. Subtraction of the Intuitionistic Fuzzy Soft Matrices:

Two intuitionistic fuzzy soft matrices A and B are said to be comformable for subtraction, if they be of the same order. The sum of two intuitionistic fuzzy soft matrices (x_{ij}) and (y_{ij}) of arder $m \times n$ is defined by, $(x_{ij}) - (y_{ij}) = (z_{ij})$ is also an $m \times n$ intuitionistic fuzzy soft matrix and $(z_{ij}) = \left(\max\{\alpha_{x_{ij}}, \alpha_{y_{ij}{}^c}\}, \min\{\beta_{x_{ij}}, \beta_{y_{ij}{}^c}\}\right)$ for all i, j, where ${y_{ij}}^c$ is the complement of y_{ij}.

H. Intuitionistic Fuzzy Dominance Matrix:

An intuitionistic fuzzy dominance matrix R on a set of alternatives U is a intuitionistic fuzzy set on the product set E × E. it is characterized by a membership function $\mu: E \times E \to [0,1]$, $\vartheta: E \times E \to [0,1]$, when cardinality of U is small, the dominance matrix may be conviently represented by $n \times n$ matrix, R = (r_{ij}), $r_{ij} = \mu(e_i, e_j)$ for all I, j ∈ {1, 2,…k}, i $\neq j$ interpreted as the dominace degree or intensity of the expert e_i over e_j on the set of $(x_i, c_j), i \in \{1,2, \dots m\}, j \in \{1,2, \dots n\}$ where $r_{ij} = 0$ indicates that e_i is preferred to e_j; $r_{ij} < 0$ indicates that e_j is preferred to e_i. Dominance degree of expert e_i over e_j on the set of $(x_i, c_j), i \in \{1,2, \dots m\}, j \in \{1,2, \dots n\}$ can be calculated as ${r_{ij}}^{A,B} = {d_{ij}}^A - {d_{ij}}^B$, $1 \le i \le m, 1 \le n, A, B \in E$, where ${d_{ij}}^A$ and ${d_{ij}}^B$ ar intuitionistic fuzzy decision matrices of experts A and B respectively.

I. Algorithm:

The fuzzy decision matrix has constructed with the alternatives $U = \{x_1, x_2, x_3, x_4, \dots\dots x_m\}, m \ge 2$ and n attributes $C = \{\{c_1, c_2, c_3, \dots\dots, c_n\}, n \ge 2$ and it is represented as

$$D = (d_{ij})_{m \times n} = \begin{bmatrix} d_{11} & d_{12} & \dots & d_{1n} \\ d_{21} & d_{22} & \dots & d_{2n} \\ \dots & \dots & \dots & \dots \\ d_{m1} & d_{m2} & \dots & d_{mn} \end{bmatrix}, d_{ij} \in [0,1]$$

By using the subtraction of fuzzy decision matrix D of individual experts, we can easily construct intuitionistic fuzzy dominance matrix R

$$R = (r_{ij})_{m \times n} = \begin{bmatrix} r_{11} & r_{12} & \dots & r_{1n} \\ r_{21} & r_{22} & \dots & r_{2n} \\ \dots & \dots & \dots & \dots \\ r_{m1} & r_{m2} & \dots & r_{mn} \end{bmatrix} r_{ij} \in [-1,1]$$

${d_{ij}}^A - {d_{ij}}^B$, $1 \le i \le m, 1 \le n, A, B \in E$,

Add all the dominance values corresponding to the alternatives.

$$c_v{}^i = \sum_{j=1}^{n} (r_{ij}), i \in [1,2, \dots, m]$$

The maximum value of $c_v{}^i$ has been selected.

J. Application:

Let $U = \{c_1, c_2, c_3\}$ be the three children in a family and $E = \{e_1 = adament, e_2 = poor\ in\ studies, e_3 = taking\ freedom\}$ be the set of parameters. The problem is to find the most affected child if the persons in the family like grandmother , grandfather and the uncle defining $P = \{P_1, P_2, P_3\}$ pamper the children.

The intuitionistic fuzzy soft decision matrices of the persons P_1, P_2 and P_3 are

$$P_1 = \begin{bmatrix} (0.7, 0.2) & (0.6, 0.1) & (0.9, 0.0) \\ (0.5,0.3) & (0.2, 0.1) & (0.8, 0.1) \\ (0.7, 0.1) & (0.4,0.0) & (0.6, 0.3) \end{bmatrix}$$

$$P_2 = \begin{bmatrix} (0.8, 0.0) & (0.7, 0.2) & (0.8, 0.1) \\ (0.7,0.2) & (0.1, 0.0) & (0.7, 0.1) \\ (0.8, 0.1) & (0.5,0.1) & (0.6, 0.4) \end{bmatrix}$$

$$P_3 = \begin{bmatrix} (0.1, 0.8) & (0.1, 0.1) & (0.9, 0.0) \\ (0.0,0.9) & (1.0, 0.0) & (0.4, 0.5) \\ (0.1, 0.8) & (0.6,0.4) & (0.3, 0.2) \end{bmatrix}$$

Steps to solve the problem:

Step 1: The intuitionistic fuzzy dominance soft matrices of P_1 and P_2 are calculated as

$$\begin{matrix} c_1 \\ c_2 \\ c_3 \end{matrix} \begin{bmatrix} (0.7, 0.2) & (0.6, 0.2) & (0.8, 0.1) \\ (0.5,0.3) & (0.1, 0.1) & (0.7, 0.1) \\ (0.7, 0.1) & (0.4,0.1) & (0.6, 0.3) \end{bmatrix}$$

Step 2: The intuitionistic fuzzy dominance soft matrices of P_2 and P_3 are calculated as

$$\begin{matrix} c_1 \\ c_2 \\ c_3 \end{matrix} \begin{bmatrix} (0.1, 0.8) & (07, 0.2) & (0.8, 0.1) \\ (0.0,0.2) & (0.1, 0.0) & (0.4, 0.5) \\ (0.1, 0.8) & (0.5,0.4) & (0.3, 0.4) \end{bmatrix}$$

Step 3: The intuitionistic fuzzy dominance soft matrices of P_3 and P_1 are calculated as

$$\begin{matrix} c_1 \\ c_2 \\ c_3 \end{matrix} \begin{bmatrix} (0.1, 0.8) & (0.6, 0.1) & (0.9, 0.1) \\ (0.0,0.9) & (0.2, 0.1) & (0.4, 0.5) \\ (0.1, 0.8) & (0.4,0.4) & (0.3, 0.3) \end{bmatrix}$$

Step 4: Aggregated IFDM

	e_1	e_2	e_3	Choice parameter	Choice value
c_1	(0.7, 0.2)	(0.7, 0.1)	(0.9,0.1)	(0.9,0.1)	1.0
c_2	(05,0.2)	(0.6,0.0)	(0.7,0.1)	(0.7,0.0)	0.7
c_3	(0.7,0.1)	(0.5.0.1)	(0.6,0.3)	(0.7,0.1)	0.8

Table 2:

From the table, we have 1.0 is the maximum value corresponding to the alternative c_1. Therefore c_1 is the most affected child.

III. Conclusion

in this paper, the study has introduced intuitionistic fuzzy dominance matrix for solving multi attribute decision making problems in vague situation. some of the definitions on intuitionistic fuzzy dominance matrix has been studied and also found the most affected child in a particular family when some of the family members pamper all the children. The solution of the problem is represented in graphical form to understand more clear.

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