Linear Matrix Inequality based Supplementary Controller for TCSC

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Abstract— The paper discusses the design of a robust controller using LMI for TCSC to damp out the oscillations in a power system. The focus being on robustness and better performance, the pole placement constraints and H∞ performance are formulated into LMI. These LMIs are solved using optimization method with minimization of objective function. The weight functions for conventional H∞ controller are designed for single machine infinite bus system (SMIBS). The responses of both controllers are compared. The electromechanical oscillations present in system are identified using eigenvalues of the system. The simulation results of both controllers at different loading conditions are found satisfactory in damping power system oscillations.

Key words: Electromechanical Oscillations, TCSC Control, H-Infinity Control, Eigenvalues, Linear Matrix Inequality, Power System Stability

I. INTRODUCTION

The electromechanical oscillations are present in power system due to large interconnection of system and inadequate damping torque. These have been the prime cause of power system collapse, when system is perturbed [1]. These oscillations are local mode oscillations (having frequency range 1.0-2.0 Hz) and/or inter mode oscillations (frequency range 0.0-1.0 Hz) [1]. Such oscillations may cause vibrations in rotor of generators, change in operational limits (amount of power transferred across the lines) and discontinuity in power supply (loss of synchronism). Reliability and performance of the system is reduced in the presence of these oscillations. So suitable damping control is necessary for eliminating the effects of electromechanical oscillations [1]-[2].

Power system stabilizers (PSSs) are used conventionally to damp out these oscillations [3]. PSS is an additional control of the generator excitation system which produced additional electrical torque on the rotor in phase with variation in speed., PSSs are not sufficient for interconnected power system with multiple swing modes, especially for inter-area mode oscillations. Development of FACTS devices has increased the existing transmission capabilities of the lines. Generally FACTS devices are installed on transmission lines or bus bars which provided reactive power compensation, voltage stability and increased transmission capacity [4]-[7]. Thyristor Controlled Series Compensation (TCSC) is one of the FACTS devices which is used to increase power transmission capacity [8]. It is also used to damp out oscillations with proper controllers. Lag-lead, H_∞ Controller for TCSC is used already in [9]-[11].

Linear Matrix Inequality (LMI) approach to design controllers in linear control problem is a powerful technique, because LMI problem is solved by using numerically convex optimization technique [12]-[13]. One of the major advantages of LMI is that two or more LMIs can be formulated into single LMI. Therefore, different control system problems are solved by single controller. Formulating a control design problem in LMI can be considered as a solution to this problem. In pole placement problem, poles are placed at specific locations in the complex plane [14]-[15]. Real systems, however, always deal in some amount of uncertainty. Hence the robust pole clustering is required. i. e, when the system is perturbed, poles should remain in the prescribed region. In LMI both H_∞ performance and pole placement technique is reduced by single LMI. Robust controller design using LMI is proposed in [16]-[19].

The design of output feedback controller is discussed in the paper. The controller is designed for TCSC by using LMI approach. Since the robustness of the system depended upon H_∞ performance and system response depended upon the location of closed loop poles, the H_∞problem and pole-placement problem is formulated into LMI problem. The designed controller satisfied constraints on closed loop pole placement in the complex plane with the objective being minimization ofH_∞ norm. The H_∞ controller using mix sensitivity approach is also designed with proper weight selection. The step response of both controllers is compared.

The paper is organized as follow, section II discussed the modelling of power system including TCSC. Description of H_∞ controller design for robust performance is entailed in section III. LMI approach to design output feedback controller is enunciated in section IV. The efficacy of LMI controller using simulation result is depicted in section V. section VI discusses conclusion of the paper.

II. POWER SYSTEM MODELLING INCLUDING TCSC

Single machine infinite bus system is taken as test system as shown in Fig. 1. The generator having constant mechanical power input is connected to infinite bus through transmission having line impedance $Z = R + jX$ and the TCSC. The middle bus supplied a local load having admittance $YL = G + jB$as shown in Fig. 1.

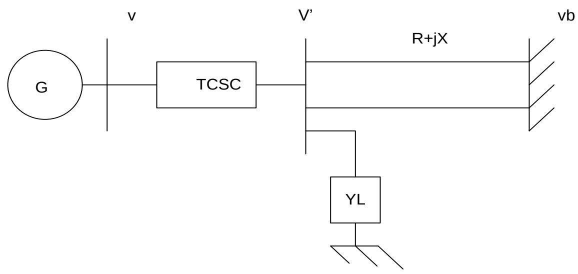

Fig. 1: Single machine infinite bus system with TCSC

A. Generator

The swing equation and terminal voltage of generator is given as [17]:

$$\rho\delta = \omega_b(\omega - 1) \quad (1)$$

$$\rho\omega = \frac{(P_m - P_e - D)(\omega - 1)}{M} \quad (2)$$

$$\rho E'_q = \frac{(E_{fd} - (x_d - x'_d) i_d - E'_q)}{T'_{do}} \quad (3)$$

Where, P_m and P_e are the input and output powers of the generator respectively; M and D are inertia constant and damping coefficient respectively; δ and ω are rotor angle and speed respectively; ω_b is synchronous speed. E_{fd} is the field voltage; T'_{do} is the open circuit fieldtime constant; x_d and x_q are d-axis reactance and q-axis transient reactance of the generator respectively.

B. Excitation System Model

The simplified form of IEEE type ST1A excitation system is shown in Fig. 2. The excitation can be expressed as

$$\rho E_{fd} = \frac{(K_A(v_{ref} - v) - E_{fd})}{T_A} \quad (4)$$

Where K_A and T_A are gain and time constant of voltage regulator; V is terminal voltage; v_{ref} is reference voltage [23].

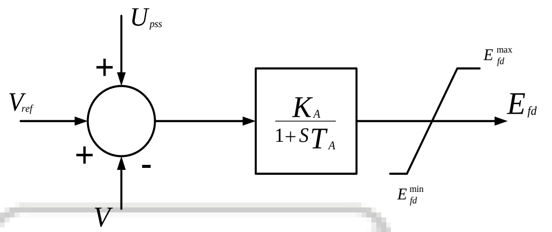

Fig. 2: Employed IEEE type ST1A excitation system

C. Linearizing the Power System

The system required state space model for designing the controller for damping out electromechanical oscillations of the system. The disturbance being small, the model is linearized around the initial equilibrium point. The linearized state space model of power system with TCSC is shown in equation (5). In this model the control input is ΔX_{TCSC} [9]-[10].

$$\begin{bmatrix} \rho\Delta\delta \\ \rho\Delta\omega \\ \rho\Delta E'_q \\ \rho\Delta E_{fd} \end{bmatrix} = \begin{bmatrix} 0 & 2*pi*f & 0 & 0 \\ \frac{-K_1}{M} & \frac{-D}{M} & \frac{-K_2}{M} & 0 \\ \frac{-K_4}{T'_{do}} & 0 & \frac{-K_3}{T'_{do}} & \frac{1}{T'_{do}} \\ \frac{-K_A K_5}{T_A} & 0 & \frac{-K_A K_6}{T_A} & \frac{-1}{T_A} \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta\omega \\ \Delta E'_q \\ \Delta E_{fd} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{-K_p}{M} \\ \frac{-K_q}{T'_{do}} \\ \frac{-K_A K_v}{T_A} \end{bmatrix} [\Delta X_{TCSC}] \quad (5)$$

III. H∞ CONTROLLER DESIGN USING MIXED-SENSITIVITY

The block diagram of the closed loop system with Mixed-Sensitivity damping control problem is shown in Fig. 3.

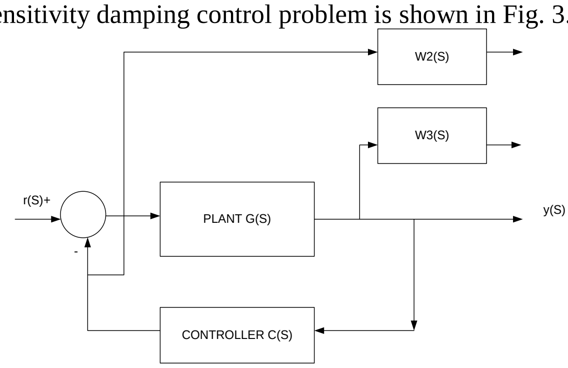

Fig. 3: Augmented plant with weighting functions

$W_3(S)$ is the weight or penalty that is placed on the closed loop transfer function to achieve the desired damping and $W_2(S)$ is the weight that determine the robustness of the closed loop system against model variations. The objective of the controller is to reduce resonance peak of the closed loop transfer function which is defined as

$$V(S) = \frac{y(S)}{r(S)} = \frac{G(S)}{1 + C(S)G(S)} \quad (6)$$

For input output system, the peak value of the transfer function is equal to the H_∞ norm of the transfer function. W(S) is a frequency dependent function whose magnitude is maximum at resonance frequency. If a controller is designed such that the closed loop system satisfied the system inequality $\|W(S)V(S)\|_\infty < 1$, the peak value of the closed loop transfer function is smaller than that of the open loop system, signifying that damping of the system is improved. The selection process of weight functions is given in [11], [21].

To improve damping and robustness of the system, the transfer function of the closed loop system satisfied the following norm inequality

$$\left\| \begin{matrix} W_2(S)V(S)C(S) \\ W_3(S)V(S) \end{matrix} \right\| < 1 \quad (7)$$

The selection of weight function W_2 and W_3 for H_∞ controlleris given in [18]. For this power system model weight functions are

$$W_2 = \frac{0.00045(s + .001)(s + 100)}{(s + .08)(s + 1.5)}$$

$$W_3 = \frac{13(s + .0001)(s + 4)}{(s + 0.002)(s + 500)}$$

In the mix sensitivity approach, the MATLAB command mixsyn is used to calculate the controller matrix. After getting controller matrix, it is connected in feedback path to get better response.

IV. FEEDBACK CONTROLLER DESIGN USING LMI

A. Output feedback controller with LMI

The numerical calculations based on the well-developed convex optimization method are effective in solving variables that satisfied LMI problem. Different control problems (pole placement, H_∞ performance, H_2 performance etc.) can be formulated into LMI problem. The state space representation of LTI system is given as:

$$\dot{x} = Ax + B_1 w + B_2 u$$
$$z = C_1 x + D_{11} w + D_{12} u$$
$$y = C_2 x + D_{21} w + D_{22} u \quad (8)$$

Where u is control input, w is a vector of exogenous inputs (such as reference signals, disturbance signals, sensor noise), y is the measured output, and z is a vector of output signals related to the performance of the control system. The state space form of the controller is considered as:

$$\dot{x}_k = A_K x_K + B_K y$$
$$u = C_K x_K + D_K y \quad (9)$$

Where x_K is state of the controller. The closed loop system is given as:

$$\dot{x}_{cl} = A_{cl} x_{cl} + B_{cl} w$$
$$w = C_{cl} x_{cl} + D_{cl} w \quad (10)$$

Where

$$A_{cl} = \begin{bmatrix} A + B_2 D_K C_K & B_2 C_K \\ B_k C_2 & A_K \end{bmatrix}; B_{cl} = \begin{bmatrix} B_1 + B_2 D_K D_{21} \\ B_K D_{21} \end{bmatrix};$$

$$C_{cl} = [C_1 + D_{12}D_KC_2 \quad D_{12}C_k]; D_{cl} = [D_{11} + D_{12}D_KD_{21}].$$

B. H_∞ Norm (γ) in LMI

H_∞Norm is the supreme value of the gain in frequency domain. In case of open loop system H_∞ norm gave a particular value giving information about stability margin. But for closed loop system it depended upon controller matrices. H_∞Norm for a closed loop system in terms of LMI is written as [16]-[19]:

$$\|H\|_\infty < \gamma = \begin{bmatrix} {A_{cl}}^T P + PA_{cl} & PB_{cl} & {C_{cl}}^T \\ {B_{cl}}^T P & -\gamma I & {D_{cl}}^T \\ C_{cl} & D_{cl} & -\gamma I \end{bmatrix} < 0 \quad (11)$$

C. Constraints on eigenvalues in LMI

The Eigenvalues of the closed loop system is placed in convex region and these convex regions are formulated into LMI with the help of Lyapunov matrix P. LMI for any region D of the complex plane is defined as

$$D = \{z \in C: L + z\,M + \bar{z}M^T < 0\} \qquad (12)$$

Where L^T= L, and M is real matrix. In this paper different LMI regions are discussed as:

Half plane: Mathematical inequality expression for half plane is

$$R\,(z) < \mp\alpha \qquad (13)$$

Where α is any value and sign decide the left half plane or right half plane. It can be formulated into LMI as

$$M_D(A,P) = PA_{cl} + {A_{cl}}^T P + 2\alpha P < 0 \quad (14)$$

Disk centred at (0, q) and radius r: Expression for disk and corresponding LMI formulation is given as:

$$(x+q)^2 + (y)^2 = r^2 \qquad (18)$$

$$M_D(A,P) = \begin{bmatrix} -rP & qP + PA_{cl} \\ qP + {A_{cl}}^T P & -rP \end{bmatrix} \qquad (19)$$

Conic sector with inner angle 2θ: LMI formulation for conic sector is given as

$$\begin{bmatrix} \sin\theta(A_{cl}P + P{A_{cl}}^T) & \cos\theta(A_{cl}P - P{A_{cl}}^T) \\ \cos\theta(P{A_{cl}}^T - A_{cl}P) & \sin\theta(A_{cl}P + P{A_{cl}}^T) \end{bmatrix} > 0 \quad (17)$$

The major advantage using LMI is the combining of two or more LMIs in single LMI. These three LMI regions are combined into single LMI. The convex region shown in Fig. 4 is the graphical representation of three LMIs.

D. Linearizing change of variables

Since LMI becomes nonlinear after putting the closed loop matrix values, controller matrices and Lyapunov matrix is variable matrix. Hence LMI is made by linearizing change of variables.

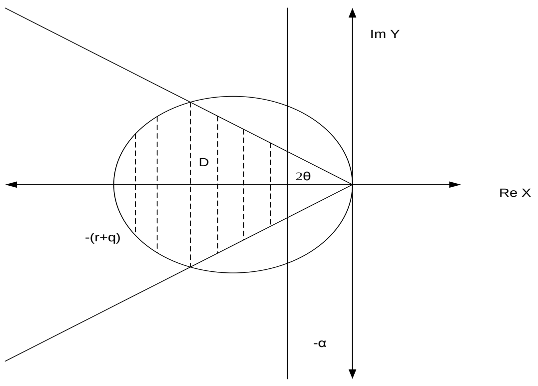

Fig. 4: Pole area constraints

Let P and P^{-1} be partitioned as

$$P = \begin{bmatrix} R & M \\ M^T & U \end{bmatrix}, \quad P^{-1} = \begin{bmatrix} S & N \\ N^T & V \end{bmatrix} \qquad (18)$$

Where R and S are $n \times n$ symmetric matrices, such that

$$P\Pi_2 = \Pi_1 \qquad (19)$$

$$\Pi_1 = \begin{bmatrix} R & I \\ M^T & 0 \end{bmatrix}, \Pi_2 = \begin{bmatrix} I & S \\ 0 & N^T \end{bmatrix} \qquad (20)$$

Let us define new change of variable as

$$\begin{aligned} Ahat &= NA_KM^T + NB_KC_2R + SB_2C_KM^T + S(A + B_2D_KC_2)R \\ Bhat &= NB_K + SB_2D_K \qquad (21) \\ Chat &= C_KM^T + D_KC_2R \\ Dhat &= D_K \end{aligned}$$

Now the Lyponuv matrix is pre and post multiplied with Π_2^T andΠ_2. Similarly the expression obtained in equation (11) and (17) is pre and post multiplied with diag(Π_2^T, I, I)and diag(Π_2, I, I), respectively; and after putting the values of new changed variables, following equations are obtained [22]:

$$\begin{bmatrix} \phi + \phi^T & \phi - \phi^T \\ \phi^T - \phi & \phi + \phi^T \end{bmatrix} < 0 \quad (22)$$

$$\begin{bmatrix} \psi_{11} & \psi_{21}^T \\ \psi_{21} & \psi_{22} \end{bmatrix} < 0 \qquad (23)$$

$$\begin{bmatrix} R & I \\ I & S \end{bmatrix} > 0 \qquad (24)$$

Where

$$\psi_{11} = \begin{bmatrix} AR + RA^T + B_2Chat + Chat^TB_2^T & B_1 + B_2DhatD_{21} \\ CB_1 + B_2DhatD_{21} & -rI \end{bmatrix}$$

$$\psi_{21} = \begin{bmatrix} Ahat + (A + B_2DhatC_2)^T & SB_1 + BhatD_{21} \\ C_1R + D_{12}Chat & D_{11} + D_{12}DhatD_{21} \end{bmatrix}$$

$$\psi_{22} = \begin{bmatrix} A^TS + SA + BhatC_2 + C_2^TBhat^T & (C_1 + D_{12}DhatC_2)^T \\ C_1 + D_{12}DhatC_2 & -rI \end{bmatrix}$$

$$\emptyset = \begin{bmatrix} AR + B_2(C_KM^T + D_KC_2R) & A + B_2D_KC_2 \\ Ahat & SA + (NB_K + B_2D_K)C_2 \end{bmatrix}$$

Finally three LMIs are obtained. Now these LMIs can be solved by using any of the optimization method. In MATLAB different solvers are available to solve these LMIs. Here half plane is shifted by $\alpha = 1$and inner angle for conical sector $2\theta = 2 * 1.42$ rad

V. SIMULATION RESULTS

The effectiveness and robustness of the proposed controlled is checked at different loading conditions and parameter variations as shown in Table I.

Operating condition	P(p.u.)	Q(p.u.)	X_{TCSC}(p.u.)
Nominal	1.00	0.015	0.0
Light	0.7	0.300	0.2
Heavy	1.1	0.400	-0.2

Table 1: Operating Conditions

The step response of the system with controller is illustrated in Fig. 5, 6, 7 at different loading condition. Here, dotted line shows H_∞controller designed by mix sensitivity approach and solid line H_∞ controller designed by LMI approach. From the Fig.5, 6 and 7, it is clear that response obtained by LMI is better with respect to performance and robustness. The controller performance at normal loading condition is shown in Table 2.

Parameter	First peak (p.u.)	Settling time(sec)
$\Delta\omega$(H-infinity)	0.015	6.0
$\Delta\omega$(LMI approach)	0.012	3.5

Table 2: Controller Performance Parameter at Normal Loading Condition

The modes of oscillations and damping ratio at different loading condition are demonstrated in the Table III, IV, and V. obtained eigenvalue of closed loop system with both controller are analysed.

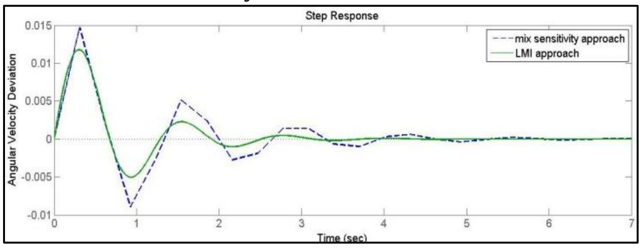

Fig. 5: Step response at Normal loading condition

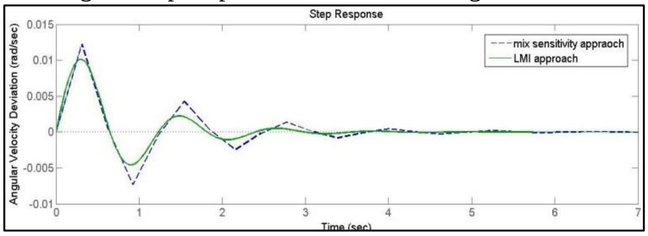

Fig. 6: Step response at lightly loading condition

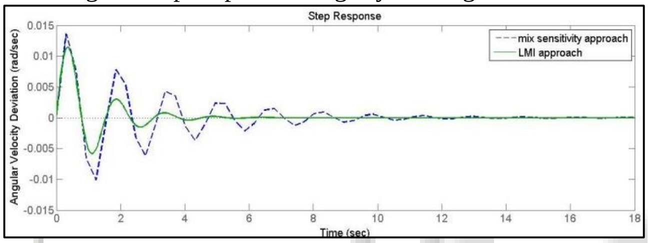

Fig. 7: Step response at heavy loading condition

System & controller	EM mode	Eigenvalue	Damping ratio
Open loop & no controller	Inter-area 0.80	0.303 + 5.025i	-0.060
	Local 1.73	-10.402 + 3.245i	0.955
Closed loop & H-infinity using mix sensitivity approach	Inter-area 0.77	-0.788 + 4.773i	0.163
	Local 1.73	-10.415 + 3.107i	0.958
Closed loop & H-infinity using LMI approach	Inter-area 0.84	-1.301 + 5.116i	0.246
	Local 1.74	-10.443 + 3.302i	0.953

Table 3: Electromechanical Oscillation Modes and Damping Ratio at Normal Loading Condition

System & controller	EM mode	Eigenvalue	Damping ratio
Open loop & no controller	Inter-area 0.67	0.329 + 4.206i	-0.078
	Local 1.79	-10.423 + 4.150i	0.929
Closed loop & H-infinity using mix sensitivity approach	Inter-area 0.64	-0.337 + 3.990i	0.084
	Local 1.84	-10.466 + 4.963i	0.904
Closed loop & H-infinity using LMI approach	Inter-area 0.68	-0.888+ 4.182i	0.208
	Local 1.75	-9.915 + 4.718i	0.903

Table 4: Electromechanical Oscillation Modes and Damping Ratio at Heavy Loading Condition

System & controller	EM mode	Eigenvalue	Damping ratio
Open loop & no controller	Inter-area 0.84	0.157 + 5.259i	-0.029
	Local 1.69	-10.263 + 2.760i	0.966
Closed loop & H-infinity using mix sensitivity approach	Inter-area 0.82	-0.886 + 5.084i	0.172
	Local 1.62	-9.426 + 3.821i	0.927
Closed loop & H-infinity using LMI approach	Inter-area 0.88	-1.246 + 5.385i	0.225
	Local 1.66	-10.169 + 2.442i	0.972

Table 5: Electromechanical Oscillation Modes and Damping Ratio at Lightly Loading Condition

VI. CONCLUSIONS

The robust controller for TCSC has been designed to damp out power system oscillations. The damping control problem has been formulated as pole-placement problem for single machine infinite bus system. H_∞Controller designed using mix sensitivity approach is also illustrated. But filters are required in this approach. Selection of proper filters made it complex and order of closed loop system increased. Hence proper model reduction is required. The H_∞problem and pole-placement problem has been formulated into LMI problem. These two LMIs have been solved by using convex LMI solver in MATLAB. The designed controller satisfied constraints on closed loop pole placement in the complex plane with the objective being minimization of H_∞ norm.From the simulation results, it is clear that the controller designed for TCSC effectively suppressed electromechanical oscillations in power system and improved system damping.

APPENDIX

Generator data: M=9.26, T'do = 7.76, D = 0.0, x_d = 0.973, x'_d = 0.19, x_q = 0.55.

Excitation System data: K_A = 50, T_A = 0.05.

Line and Load data: R = -0.034, X = 0.997, G = 0.249, B = 0.262.

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