Analysis of Orthotropic Skew Plate using FSDT and RBF Collocations

Kumari Shipra Suman[1] Rahul Kumar[2] Jeeoot Singh[3]
[1,2,3]Department of Mechanical Engineering
[1,2,3]Birla Institute of Technology, Ranchi, 835215, India

Abstract— Present paper deals with bending analysis of orthotropic skew plate. Governing differential of the plate is obtained using energy principle. A polynomial radial basis function base meshfree method is used to discretize the partial differential equations in displacement form. A MATLAB code is developed incorporating to obtain the solution. Numerical results related to flexure analysis of skew plates are presented. Results are compared with the other published results to compare present solution methodology. Effect of orthotropic ration, skew angle and span to thickness ratio is presented.

Key words: Skew Plate, FSDT, Orthotropic, Meshfree

I. INTRODUCTION

Plates are defined as plane structural elements with a small thickness compared to the planar dimensions. The typical thickness to width ratio of a plate structure is less than 0.1. The analysis of skew plates is more complicated than the analysis of rectangular ones due to the presence of singularity at the obtuse corners. Skew plates have quite a good number of applications in various mechanical, civil and aero structures. Specific applications of skew plates are floors in bridges, ship hulls, buildings, etc. The present paper deals with the skew plates under uniform pressure loading. In current date analysis of skew panels are much more in demand, as per analysis of skew plates already done by several researchers such as the analysis of isotropic thick skew plates had been carried out by various researchers (Liew and Han, [1]; Muhammad and Singh, [2]). A review on the work done on the bending analysis of skew plates before 1989 has been carried out by Butalia et al. [3].The effect of changing the orientation of the principal orthotropic axes on the stability of clamped skew plates of constant thickness done by Srinivasan and Ramachandran [4]. Daripa and Singha [5] studied the influence of corner stresses on the stability behavior of composite skew plates. Ferreira et al. [6] use the first-order shear deformation theory in the multiquadric radial basis function (MQRBF) procedure for predicting the free vibration behavior of moderately thick symmetrically laminated composite plates. Analysis for critical buckling loads of angle-ply and cross-ply skew laminates with various lamination parameters done by Babu and Kant [7].

II. MATHEMATICAL FORMULATION

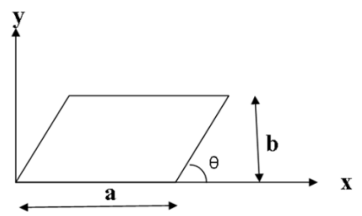

Fig. 1: Geometry of skew plate

The geometry of plate is shown in Fig. 1. Thickness h is along z axis whose mid plane is coinciding with x-y plane of the coordinate system is considered.

The displacement field at any point in the plate is expressed as ignoring initial displacements in X and Y direction:

$$u = -z\phi_x \qquad v = -z\phi_y \qquad w = w_0 \tag{1}$$

The strain-displacement relations can be written as:

$$\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial v}{\partial y} \\ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \end{Bmatrix} \tag{2}$$

$$\begin{Bmatrix} \gamma_{yz} \\ \gamma_{zx} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \\ \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \end{Bmatrix}$$

From eq. (1) and (2)

$$\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} -z\frac{\partial \phi_x}{\partial x} \\ -z\frac{\partial \phi_y}{\partial y} \\ -z\frac{\partial \phi_x}{\partial y} - z\frac{\partial \phi_y}{\partial x} \end{Bmatrix} \tag{3}$$

$$\begin{Bmatrix} \gamma_{yz} \\ \gamma_{zx} \end{Bmatrix} = \begin{Bmatrix} -\phi_y + \frac{\partial w_0}{\partial y} \\ -\phi_x + \frac{\partial w_0}{\partial x} \end{Bmatrix} \tag{4}$$

The constitutive stress strain relation can be written as:

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \\ \sigma_{yz} \\ \sigma_{zx} \end{Bmatrix} = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & 0 & 0 & 0 \\ \bar{Q}_{12} & \bar{Q}_{22} & 0 & 0 & 0 \\ 0 & 0 & \bar{Q}_{66} & 0 & 0 \\ 0 & 0 & 0 & \bar{Q}_{44} & 0 \\ 0 & 0 & 0 & 0 & \bar{Q}_{55} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{Bmatrix} \tag{5}$$

The governing differential equations of plate are obtained using Hamilton's principle and expressed as:

$$\frac{\partial M_{xx}}{\partial x} + \frac{\partial M_{xy}}{\partial y} - Q_x = 0$$
$$\frac{\partial M_{xy}}{\partial x} + \frac{\partial M_{yy}}{\partial y} - Q_y = 0 \tag{6}$$
$$\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} - q_z = 0$$

Where, $M_{xx} = D_{11}\frac{\partial \phi_x}{\partial x} + D_{12}\frac{\partial \phi_y}{\partial y}$

$$M_{yy} = D_{12}\frac{\partial \phi_x}{\partial x} + D_{22}\frac{\partial \phi_y}{\partial y}$$

$$M_{xy} = D_{16}\frac{\partial \phi_x}{\partial x} + D_{26}\frac{\partial \phi_y}{\partial y}$$

$$Q_x = kA_{55}\phi_x,\ Q_y = kA_{55}\phi_y$$

The boundary conditions for an arbitrary edge with simply supported conditions are as follows:

$$\phi^s, w, M_{nn} = 0 \quad (7)$$

Where

$$\phi^s = -n_y.\phi^x + n_x.\phi^y$$

$$M_{nn} = n_x^2 M_{xx} + 2n_x n_y M_{xy} + n_y^2 M_{yy}$$

$$n_x = \cos(\theta), \quad n_y = \sin(\theta)$$

If θ=900, plate will be rectangular.

III. SOLUTION METHODOLOGY

The governing differential equations (6) are expressed in terms of displacement functions. Radial basis function based formulation works on the principle of interpolation of scattered data over entire domain. A 2D rectangular domain having N_B boundary nodes and N_D interior nodes is shown in Figure-2.

The variable $w_0, \phi_x \; and \; \phi_y$ can be interpolated in form of radial distance between nodes. The solution of the linear governing differential equations is assumed in terms of polynomial radial basis function for nodes 1:N, as;

$$w_o, \phi_x, \phi_y = \sum_{j=1}^{N} (\alpha_j^w, \alpha_j^{\phi_x}, \alpha_j^{\phi_y}) \; g\left(\left\|X - X_j\right\|, m\right)$$

Where, N is total numbers of nodes which is equal to summation of boundary nodes NB and domain interior nodes ND. $g\left(\left\|X - X_j\right\|, m\right)$ is polynomial radial basis function expressed as $g = r^m$, $\delta = \alpha_j^w, \alpha_j^{\phi_x}, \alpha_j^{\phi_y}$ are unknown coefficients. $\left\|X - X_j\right\|$ is the radial distance between two nodes.

Where, $r = \left\|X - X_j\right\| = \sqrt{(x - x_j)^2 + (y - y_j)^2}$ and m is shape parameter. The value of 'm' taken here is 5.

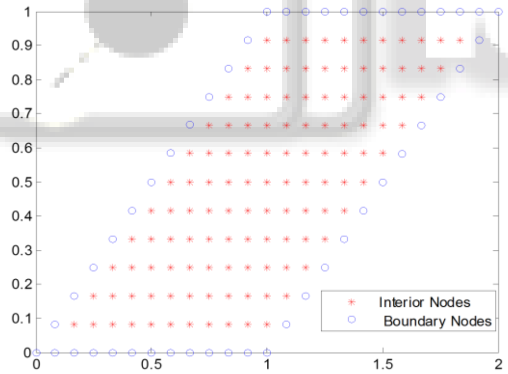

Fig. 2: An arbitrary two dimensional domains

Polynomial radial basis function becomes singular, when r = 0 i.e. for zero distance. In order to eliminate the singularity, an infinitesimally small value is added into the r^2 or zero distance. Mathematically it is explained as; $r^2 = r^2 + \mu^2$ when r = 0 or i = j; μ^2 is small numerical value of the order 10^{-10}.

The discretized governing equations for linear flexural analysis can be written as:

$$\begin{bmatrix} [K]_L \\ [K]_B \end{bmatrix}_{3N \times 3N} \{\delta\}_{3N \times 1} = \begin{Bmatrix} [F]_L \\ 0 \end{Bmatrix}_{3N \times 1} \quad (8)$$

The unknown coefficients {δ} are calculated from equation (8).

IV. NUMERICAL RESULTS AND DISCUSSIONS

In order to demonstrate the accuracy and applicability of present formulation, a RBF based meshless code in MATLAB is developed following the analysis procedure as discussed above. Several examples have been analyzed and the computed results are compared with the published results. Based on convergence study, a 13x13 node is used throughout the study. For orthotropic material.

The deflection and moments are normalized as:

$$\bar{w} = w_{c\max}.100.h^3/(qa^4) \qquad \bar{M} = M_{c\max}.40/(qa^2)$$

$$\bar{\sigma}_{xx} = \sigma_{xx\max}.(qa^4/h^2) \qquad \bar{\sigma}_{yy} = \sigma_{yy\max}.(qa^4/h^2)$$

$$\bar{\sigma}_{xy} = \sigma_{xy\max}.(qa^4/h^2) \qquad \bar{\sigma}_{xz} = \sigma_{xz\max}.(qa^4/h^2)$$

Unless until specified, the material properties are taken as: $E1 = 25, E2 = 1, \nu = 0.3, G1 = G2 = 0.5, G3 = 0.2$

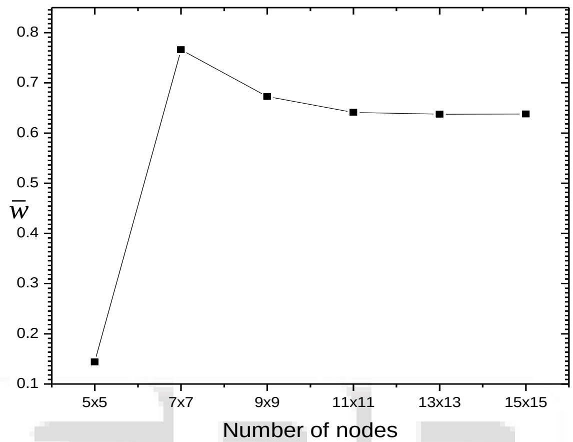

Fig. 3: Convergence study for deflection $\bar{w}$ of a simply supported skew plate (a/h = 100)

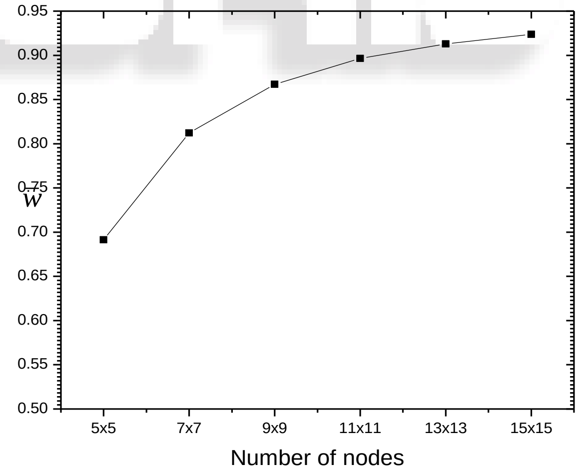

Fig. 4: Convergence study for deflection $\bar{w}$ of a simply supported skew plate (a/h = 10)

From Fig.3 and Fig.4, it can be seen that a good convergence is achieved for both the cases of thin to thick plate. The convergence is within 2% for nodes more than 11x11.

Other numerical examples have been also considered and the results obtained for different values of span to thickness ratio is shon in Table-1 to Table 9 and for different orthotropy ratio is shown in Table-10 to Table-18.

	Skew angle				
a/h	90	75	60	45	30
5	4.6541	4.2451	3.6919	2.9444	1.6869
10	4.8915	4.6009	3.9219	3.0191	1.502

20	4.9865	8.9319	3.9499	3.0074	1.5052
30	5.0043	4.4615	3.8778	2.9271	4.8608
40	5.0256	4.474	3.7745	2.8003	1.4051
50	5.0453	4.4537	3.6646	2.6594	1.3541
100	5.1821	4.2861	3.1908	2.1972	1.1823

Table 1: Effect of span to thickness ratio on M_{xx} of a square orthotropic skew plate

	Skew angle				
a/h	90	75	60	45	30
5	0.4249	0.3876	0.697	0.6396	0.4947
10	0.3127	0.3596	0.7581	0.527	1.0218
20	0.2863	0.426	0.4259	0.5775	0.5387
30	0.2771	0.2491	0.4202	0.5948	0.7641
40	0.2731	0.2379	0.3968	0.5741	0.5056
50	0.271	0.235	0.3714	0.538	0.4738
100	0.2871	0.2157	0.273	0.3883	0.346

Table 2: Effect of span to thickness ratio on Myy of a square orthotropic skew plate

	Skew angle				
a/h	90	75	60	45	30
5	0.4241	0.4319	0.3328	0.2885	0.206
10	0.3404	0.4293	0.3124	0.3171	0.7762
20	0.2945	0.5167	0.3372	0.4186	0.1689
30	0.2843	0.3036	0.3097	0.4991	1.1866
40	0.2802	0.2881	0.3048	0.4362	0.1492
50	0.2787	0.281	0.2953	0.3211	0.1407
100	0.2879	0.2513	0.2417	0.1974	0.1042

Table 3: Effect of span to thickness ratio on Mxy of a square orthotropic skew plate

	Skew angle				
a/h	90	75	60	45	30
5	0.4249	0.5979	1.089	1.498	1.2277
10	0.3127	0.5105	1.1018	1.5152	1.0965
20	0.2863	0.4991	1.0768	1.4945	1.0945
30	0.2771	0.4533	1.0548	1.4495	3.625
40	0.2731	0.4532	1.0276	1.3872	1.0223
50	0.271	0.4519	0.9989	1.3199	0.9857
100	0.2871	0.4517	0.874	1.0937	0.8619

Table 4: Effect of span to thickness ratio on Mnn of a square orthotropic skew plate

	Skew angle				
a/h	90	75	60	45	30
5	0.4241	1.0173	1.5028	1.3147	0.5575
10	0.3404	1.1362	1.6282	1.3772	0.59
20	0.2945	2.6767	1.6588	1.3937	0.5041
30	0.2843	1.1098	1.6298	1.3634	2.2113
40	0.2802	1.1129	1.5859	1.3052	0.4783
50	0.2787	1.1072	1.539	1.238	0.4613
100	0.2879	1.0602	1.3376	1.0204	0.4043

Table 5: Effect of span to thickness ratio on Mns of a square orthotropic skew plate

	Skew angle				
a/h	90	75	60	45	30
5	2.3547	3.0804	1.8337	1.2938	0.6561
10	2.5365	7.2928	8.511	1.3306	4.1229
20	2.6381	43.9793	3.8699	2.8476	1.1848
30	2.6607	8.1303	2.7305	4.0815	23.5292
40	2.5803	5.4762	1.9653	3.8912	0.9519
50	2.5871	4.6606	2.0375	2.7832	1.1632
100	2.6128	3.8183	2.2727	2.303	1.6534

Table 6: Effect of span to thickness ratio on $\bar{\sigma}_{xx}$ of a square orthotropic skew plate

	Skew angle				
a/h	90	75	60	45	30
5	0.4067	0.5423	0.7664	0.751	0.7487
10	0.3428	0.5014	0.8129	0.7064	1.1691
20	0.3137	1.2318	0.8589	0.9011	1.0368
30	0.2931	0.3543	0.6778	0.9961	1.4157
40	0.2799	0.3021	0.5533	0.9918	1.0557
50	0.2685	0.2855	0.4614	0.9429	0.9942
100	0.2442	0.2208	0.3667	0.6904	0.6919

Table 7: Effect of span to thickness ratio on $\bar{\sigma}_{yy}$ of a square orthotropic skew plate

	Skew angle				
a/h	90	75	60	45	30
5	0.1073	0.1381	0.188	0.197	0.1951
10	0.0855	0.2758	0.1907	0.2698	0.2538
20	0.0757	1.5652	0.3358	0.3956	0.3107
30	0.0737	0.2883	0.2904	0.4573	0.5129
40	0.0724	0.1963	0.2682	0.47	0.3412
50	0.0722	0.1682	0.2653	0.4557	0.3304
100	0.076	0.1416	0.2273	0.3455	0.2504

Table 8: Effect of span to thickness ratio on $\bar{\sigma}_{xy}$ of a square orthotropic skew plate

	Skew angle				
a/h	90	75	60	45	30
5	0.1625	0.1514	0.1302	0.1031	0.0625
10	0.1552	0.1447	0.1191	0.085	0.0416
20	0.2235	0.1664	0.1638	0.1161	0.0519
30	0.3106	0.2661	0.2218	0.1527	0.0974
40	0.4042	0.3447	0.2769	0.1836	0.0799
50	0.5003	0.4216	0.3275	0.2083	0.0922
100	1.0044	0.786	0.5239	0.31	0.1398

Table 9: Effect of span to thickness ratio on $\bar{\sigma}_{xz}$ of a square orthotropic skew plate

	Skew angle			
E1/E2	**90**	**75**	**60**	**45**
3	2.9657	2.5531	1.8754	1.1911
5	3.623	3.1433	2.3704	1.552
15	4.647	5.4293	3.4073	2.5584
20	4.7937	4.519	4.544	2.8254
25	4.8915	4.6009	3.9219	3.0191
30	4.945	4.6662	4.0451	3.1658
40	5.0624	4.7623	4.2252	3.3733

Table 10: Effect of span to thickness ratio on Mxx of a square orthotropic skew plate

	Skew angle			
E1/E2	**90**	**75**	**60**	**45**
3	1.0571	0.9452	0.9157	0.9453
5	0.7928	0.7265	0.8224	0.8328
15	0.407	0.6634	0.5046	0.6143
20	0.3494	0.4092	1.3237	0.5593
25	0.3127	0.3596	0.7581	0.527
30	0.2926	0.3373	0.6535	0.5117
40	0.2679	0.3173	0.5278	0.4926

Table 11: Effect of span to thickness ratio on M_{yy} of a square orthotropic skew plate

Skew angle				
E1/E2	**90**	**75**	**60**	**45**
3	1.0599	1.0062	0.842	0.5531
5	0.8219	0.8237	0.6996	0.4931
15	0.4506	1.0985	0.5954	0.392
20	0.4238	0.5379	0.5151	0.3504
25	0.3404	0.4293	0.3124	0.3171
30	0.3062	0.3711	0.2802	0.2905
40	0.257	0.3048	0.2387	0.2518

Table 12: Effect of span to thickness ratio on M_{xy} of a square orthotropic skew plate

Skew angle				
E1/E2	**90**	**75**	**60**	**45**
3	1.0571	0.9769	0.8315	0.6102
5	0.7928	0.8241	0.8796	0.7864
15	0.407	0.6039	1.0074	1.2841
20	0.3494	0.542	1.0887	1.4178
25	0.3127	0.5105	1.1018	1.5152
30	0.2926	0.4911	1.1192	1.5893
40	0.2679	0.4661	1.1454	1.6944

Table 13: Effect of span to thickness ratio on M_{nn} of a square orthotropic skew plate

Skew angle				
E1/E2	**90**	**75**	**60**	**45**
3	1.0599	0.7944	0.6027	0.6396
5	0.8219	0.7155	0.8607	0.5754
15	0.4506	1.5099	1.3856	1.1146
20	0.4238	1.1119	2.2147	1.2669
25	0.3404	1.1362	1.6282	1.3772
30	0.3062	1.1551	1.6892	1.4607
40	0.257	1.1824	1.7781	1.5787

Table 14: Effect of span to thickness ratio on M_{ns} of a square orthotropic skew plate

Skew angle				
E1/E2	**90**	**75**	**60**	**45**
3	1.5483	1.4533	0.8429	0.5417
5	2.2156	2.9581	1.1953	0.7859
15	2.4106	22.8126	13.8862	1.1746
20	4.1072	9.5472	18.5247	1.2798
25	2.5365	7.2928	8.511	1.3306
30	2.5878	6.6316	4.3885	1.4703
40	2.5625	6.6056	6.3167	1.3558

Table 15: Effect of span to thickness ratio on $\bar{\sigma}_{xx}$ of a square orthotropic skew plate

Skew angle				
E1/E2	**90**	**75**	**60**	**45**
3	0.6249	0.8259	1.0363	1.2714
5	0.537	0.6885	0.9429	1.1015
15	0.3906	0.9202	1.2448	0.8058
20	0.3636	0.5634	1.447	0.739
25	0.3428	0.5014	0.8129	0.7064
30	0.329	0.4699	0.7413	0.6822
40	0.312	0.4427	0.6537	0.6483

Table 16: Effect of span to thickness ratio on $\bar{\sigma}_{yy}$ of a square orthotropic skew plate

Skew angle				
E1/E2	**90**	**75**	**60**	**45**
3	0.2397	0.441	0.4912	0.5541
5	0.1856	0.4648	0.3948	0.4551
15	0.1037	1.0651	0.2706	0.3154
20	0.1006	0.3807	0.2401	0.2885
25	0.0855	0.2758	0.1907	0.2698
30	0.0792	0.2468	0.2122	0.2558
40	0.0741	0.2472	0.2572	0.2364

Table 17: Effect of span to thickness ratio on $\bar{\sigma}_{xy}$ of a square orthotropic skew plate

Skew angle				
E1/E2	**90**	**75**	**60**	**45**
3	0.501	0.4106	0.2717	0.1548
5	0.3924	0.3283	0.2292	0.138
15	0.2078	0.1993	0.144	0.1009
20	0.1758	0.1638	0.1321	0.0916
25	0.1552	0.1447	0.1191	0.085
30	0.1409	0.1318	0.1104	0.0801
40	0.1229	0.1153	0.099	0.0735

Table 18: Effect of span to thickness ratio on $\bar{\sigma}_{xz}$ of a square orthotropic skew plate

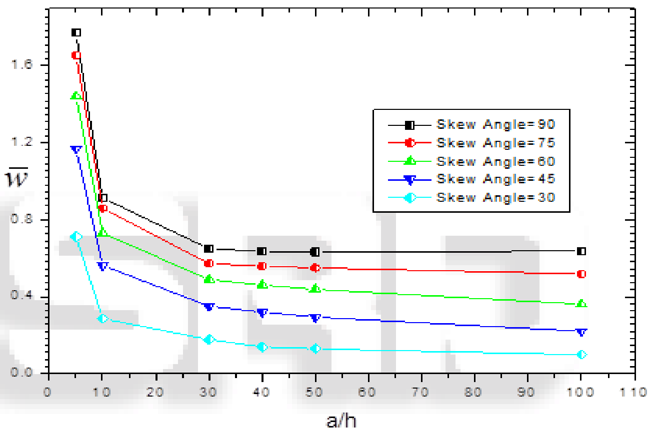

Fig. 5: Effect of span to thickness ratio for deflection $\bar{w}$ of a simply supported skew plate

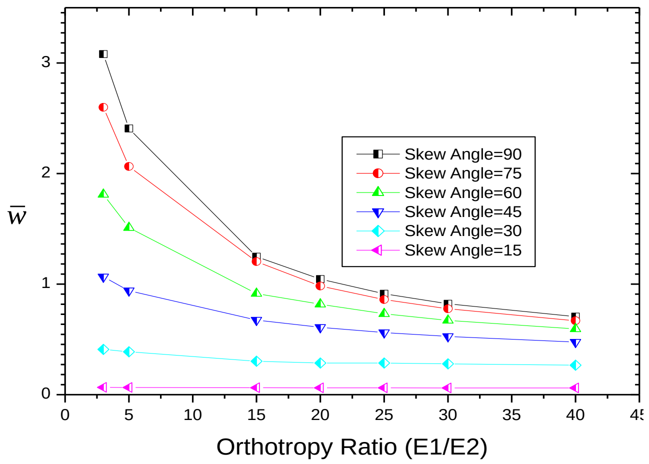

Fig. 6: Effect orthotropy ratio for deflection $\bar{w}$ of a simply supported skew plate

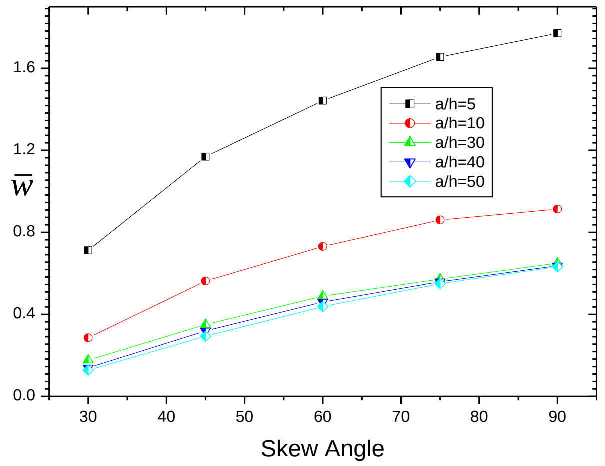

Fig. 6 Effect of thickness along skew angle for deflection $\overline{w}$ of a simply supported skew plate

Fig. 5 to Fig. 7 shows the effect of skew angle on deflection.

It is observed that as skew angle increases, the deflection decreases. The effect of span to thickness ratio seems to be negligible after a/h=40. The effect of orthotropy ratio seems to be negligible after E1/E2=30.

V. Conclusion

The present study shows that the proposed RBFs are capable to accurately predict the flexure behavior of skew plates. Effect of skewness on deflection, moments and stresses is obtained. It is found that all the parameters decreases as skewness increases. Effect is more prominent for thick plates as compared to thick plate. The study further can be extended for orthotropic, laminated and FGM plates.

References

[1] Liew K.M. and Han J.B. (1997): Bending analysis of simply supported shear deformable skew plates. Journal of Engineering Mechanics, vol.123, pp.214-221.
[2] Muhammad T. and Singh A.V. (2004): A p-type solution for the bending of rectangular, circular, elliptic and skew plates. International Journal of Solids and Structures, vol. 41, pp.3977-3997.
[3] Butalia, T. S., Kant, T., Dixit, V.D., “Performance of heterosis element for bending of skew rhombic plates,” Computers and Structures, 34(1), 23-49, 1990.
[4] Srinivasan R. S. and Ramachandran S. V. (1976): Stability of Generally Orthotropic Skew Plate. Journal of the Engineering Mechanics Division, Vol. 102, No. 3, pp. 569-572
[5] Daripa R. and Singha M.K. (2009): Influence of corner stresses on the stability characteristics of composite skew plates. International Journal of Non-Linear Mechanics, vol. 44, No. 2, pp.138-146.
[6] Ferreira A.J.M., Roque C.M.C., Jorge R.M.N.(2005):Free vibration analysis of symmetric laminated composite plates by FSDT and radial basis functions. Comput. Methods Appl. Mech. Engrg. 194 (2005) 4265–4278
[7] C. Sarath Bab and T. Kant(1999), “Two shear deformable finite element models for buckling analysis of skew fibre-reinforced composite and sandwich panels. Composite Structures 46 (1999) 115±124

