Multi-Attribute Group Decision Making based on Bidirectional Projection Measure

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Abstract— This paper develops a method for solving the multiple attribute group decision making problems with the interval valued neutrosophic information. Initially, propose a non-linear equation using Euclidean distance measures based on maximizing deviation method. The non-linear equations solved by Lagrange function to obtain attribute weights. Further, that ranking the alternatives based on bidirectional projection measures from Ideal Interval Neutrosophic Estimates Reliability Solution (IINERS) and Ideal Interval Neutrosophic Estimates Un-reliability Solution (IINEURS) to select the best alternative. Finally, an illustrative example demonstrates the application of the proposed method. The effectiveness and advantages of the proposed method are shown by the comparative analysis with existing relative methods.

Key words: Interval Valued Neutrosophic Sets (IVNS), Euclidean Distance Measure, Maximizing Deviation Method, Bidirectional Projection Measures, Group Decision Making

I. Introduction

The Neutrosophic Sets was developed by Smarandache [1] not only to deal with the decision information which is often incomplete, indeterminate, and inconsistent but also to include the truth membership degree, the falsity membership degree, and the indeterminacy membership degree. For simplicity and practical application, Wang proposed the single-valued NS (SVNS) and the interval-valued NS (IVNS) which are the instances of NS and gave some operations on these sets [2, 3]. Since its appearance, many fruitful results have appeared [4, 5]. On one hand, many researchers have proposed some aggregation operators of SVNS and INS and applied them to MADM problems [6–10]. On the other hand, some researchers have also proposed entropy and similarity measure of the SVNS and IVNS and applied them to MADM problems [11, 12].The above problems that are related to the attribute weights are completely known. However, with the development of the information society and internet technology, the socioeconomic environment gets more complex in many decision areas, such as capital investment decision making, medical diagnosis, and personnel examination. Only one decision maker cannot deal with the complex problems. Accordingly, it is necessary to gather multiple decision makers with different knowledge structures and experiences to conduct a group decision making. In some circumstances, it is difficult for the decision makers to give the information of the attribute weights correctly, which makes the attribute weights incompletely known or completely unknown. How to derive the attribute weights from the given neutrosophic information is an important topic. In intuitionistic fuzzy environments, many researchers have proposed some program models to obtain the incompletely known attribute weights or the completely unknown attribute weights, such as Xu proposed the deviation-based method [13], the ideal solution-based method [14], and the group consensus-based method [15] and Li et al. proposed the consistency-based method [16]. Under the neutrosophic environment, Sahin and Liu proposed the maximizing deviation method [17, 21]. In this paper, we deal with MAGDM problem which the information expressed by IVNS, and the attribute weights are completely unknown. Our aim is:

1) To determine attribute weights based on maximizing deviation method using Euclidean distance measure.
2) Aggregate these weights based on Ideal Interval Neutrosophic Estimates Reliability Solution (IINERS) and Ideal Interval Neutrosophic Estimates Un-reliability Solution (IINEURS).
3) Ranking the alternatives based on Bidirectional Projection Measures.

The rest of the paper is organized as follows. Section 2 briefly describes some basic concepts of NS, SVNS and IVNS. Section3 proposes a bidirectional projection measure based multiple attribute group decision making methodology. In sect. 4, an illustrative example is presented to demonstrate the application of the proposed method, Sect.5 the effectiveness and advantages of the proposed method are demonstrated by the comparative analysis with existing relative methods. Finally, Sect. 6contains conclusions and future work.

II. Preliminaries

A. Neutrosophic Set (NS)

Let X be a space of points (objects) and $x \in X$. A neutrosophic set A in X is defined by a truth membership function (x), an indeterminacy membership function $\mathrm{I}(x)$ and a falsity membership function $\mathrm{F}_A(x)$. (x), $\mathrm{I}(x)$ and $\mathrm{F}_A(x)$ are real standard or real nonstandard subsets of $]0^-,1^+[$. That is $T_A(x)$: $X \rightarrow]0^-,1^+[$, $\mathrm{I}_A(x)$: $X \rightarrow]0^-,1^+[$ and $\mathrm{F}_A(x)$:$X \rightarrow]0^-,1^+[$. There is no restriction on the sum of (x), $\mathrm{I}(x)$ and $\mathrm{F}_A(x)$, so $0^- \leq \sup T_A(x) \leq \sup I_A(x) \leq \sup F_A(x) \leq 3^+$.

B. Compliment of NS

The complement of a neutrosophic set A is denoted by A^c and is defined as $T_{A^c}(x) = \{1^+\} \ominus (x)$, $I_{A^c}(x) = \{1^+\} \ominus I_A(x)$ and $F_{A^c}(x) = \{1^+\} \ominus F_A(x)$ for all $x \in X$.

C. Single Valued Neutrosophic Sets (SVNS)

Let X be a universe of discourse. A single valued neutrosophic set A over X is an object having the form $A=\{\langle x, u_A(x), w_A(x), v_A(x)\rangle : x \in X\}$where $u_A(x)$:$X \rightarrow [0,1]$, $w_A(x)$:$X \rightarrow [0,1]$ and $v_A(x)$:$X \rightarrow [0,1]$ with $0 \leq u_A(x)+w_A(x)+v_A(x) \leq 3$ for all $x \in X$. The intervals (x), (x) and (x) denote the truth membership degree, the indeterminacy membership degree and the falsity membership degree of x to A, respectively.

D. Complement of SVNS

The complement of an SVNS A is denoted by A^c and is defined as (x) = (x), (x) = 1−(x), and $v_{A^c}(x) = u(x)$ for all $x \in X$. That is, $A^c = \{\langle x, (x), 1-w_A(x), u_A(x)\rangle : x \in X\}$.

E. Interval Neutrosophic Sets (INS)

Let $u_{\tilde{A}}(x) = [u_{\tilde{A}}^{-}(x), u_{\tilde{A}}^{+}(x)]$, $w_{\tilde{A}}(x) = [w_{\tilde{A}}^{-}(x), w_{\tilde{A}}+(x)]$ and $v_{\tilde{A}}(x) = [v_{\tilde{A}}-(x), v_{\tilde{A}}+(x)]$, then $\tilde{A}=\{\langle x, [u_{\tilde{A}}-(x), u_{\tilde{A}}+(x)], [w_{\tilde{A}}-(x), w_{\tilde{A}}+(x)], [v_{\tilde{A}}-(x), v_{\tilde{A}}+(x)]\rangle : x \in X\}$ with the condition, $0 \le \sup u_{\tilde{A}} + (x) + \sup w_{\tilde{A}} + (x) + \sup v_{\tilde{A}} + (x) \le 3$ for all $x \in X$. Here, we only consider the sub-unitary interval of [0, 1]. Therefore, an INS is clearly neutrosophic set.

F. Compliment of INS

The complement of an INS $\tilde{A}$ is denoted by $\tilde{A}^c$ and is defined as $u_{\tilde{A}^c}(x) = v(x)$, $(w_{\tilde{A}}-)^c(x)=1-w_{\tilde{A}}^{+}(x)$, $(w_{\tilde{A}}^{+})(x)= 1-w_{\tilde{A}}^{-}(x)$ and $v_{\tilde{A}^c}(x)=u(x)$ for all $x \in X$. That is, $\tilde{A}^c=\{\langle x, [v_{\tilde{A}}^{-}(x), v_{\tilde{A}}^{+}(x)], [1-w_{\tilde{A}}^{+}(x), 1-w_{\tilde{A}}^{-}(x)], [u_{\tilde{A}}^{-}(x), u_{\tilde{A}}^{+}(x)]\rangle : x \in X\}$.

G. INS Ranking

Suppose that $\tilde{A}_1 = ([a_1, b_1], [c_1, d_1], [e_1, f_1])$ and $\tilde{A}_2 = ([a_2, b_2], [c_2, d_2], [e_2, f_2])$ are two interval valued neutrosophic sets Then we define the ranking method as follows:

- If L $(\tilde{A}_1)$ > L $(\tilde{A}_2)$, then $\tilde{A}_1 > \tilde{A}_2$.
- If L $(\tilde{A}_1)$ = L $(\tilde{A}_2)$ and N $(\tilde{A}1)$ > N $(\tilde{A}2)$, then $\tilde{A}_1 > \tilde{A}_2$.

H. INS Distance Measuring Functions

(Ye 2014a)Let x = $([T_1^L, T_1^U], [I_1^L, I_1^U], [F_1^L, F_1^U])$, and y = $([T_2^L, T_2^U], [I_2^L, I_2^U], [F_2^L, F_2^U])$ be two INVs, then

- The Hamming distance between x and y is defined as follows

$$d_H(x,y) = \frac{1}{6}\left(\left|T_1^L - T_2^L\right| + \left|T_1^U - T_2^U\right| + \left|I_1^L - I_2^L\right| + \left|I_1^U - I_2^U\right| + \left|F_1^L - F_2^L\right| + \left|F_1^U - F_2^U\right|\right) \quad (1)$$

- The Euclidian distance between x and is defined as follows.

$$d_E(x,y) = \sqrt{\frac{1}{6}\left(\begin{matrix}(T_1^L - T_2^L)^2 + (T_1^U - T_2^U)^2 + (I_1^L - I_2^L)^2 + \\ (I_1^U - I_2^U)^2 + (F_1^L - F_2^L)^2 + (F_1^U - F_2^U)^2\end{matrix}\right)} \quad (2)$$

1) Lagrange Function

$$L(w,\prod) = \sum_{k=1}^{t}\lambda_k \sqrt{\frac{1}{6}\sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{s=1}^{m} w_j\,((\Delta_{upv})_E)} + \frac{\prod}{12}\left(\sum_{j=1}^{n} w_j^2 - 1\right)$$

We compute Partial derivatives of L as follows,

$$\frac{\partial L}{\partial w_j} = \sum_{k=1}^{t}\lambda_k \sum_{i=1}^{m}\sum_{s=1}^{m}((\Delta_{upv})_E + \prod w_j = 0\,)$$

$$\frac{\partial L}{\partial \prod} = \sum_{j=1}^{n} w_j^2 - 1 = 0$$

Where, $\prod$is Lagrange multiplier and λ_k is weight of DM

$$d_k\,(\Delta_{upv})_E = \sqrt{\begin{matrix}((u_1^L - u_2^L)^2 + (u_1^U - u_2^U)^2 + (p_1^L - p_2^L)^2 \\ +(p_1^U - p_2^U)^2 + (v_1^L - v_2^L)^2 + (v_1^U - v_2^U)^2\end{matrix}}$$

A simple and exact formula for determining the attribute weights as follows:

$$w_j^* = \frac{\sum_{k=1}^{t}\lambda_k \sum_{i=1}^{m}\sum_{s=1}^{m}((\Delta_{upv})_E)}{\sum_{j=1}^{n}(\sum_{k=1}^{t}\lambda_k \sum_{i=1}^{m}\sum_{s=1}^{m}((\Delta_{upv})_E)}$$

I. IRNPIS

Let α and β be the collection of benefit attributes and cost attributes, respectively. $R_s^+$is the Interval Relative Neutrosophic Positive Ideal Solution (IRNPIS). $R_s^+ = [r_{s_1}^+, r_{s_2}^+, \ldots, r_{s_n}^+]$ is defined as a solution in which every component $r_{s_j}^+ = < T_j^+, I_j^+, F_j^+ >$ is characterized by $T_j^+ = \{(\max\{T_{ij}\})$ | i-th attribute ∈ α , $(\min\{T_{ij}\})$| i-th attribute ∈$\beta\}$

J. IRNNIS

The Interval Relative Neutrosophic Negative Ideal Solution (IRNNIS). $R_S^- = [r_{s_1}^-, r_{s_2}^-, \ldots, r_{s_n}^-]$ is defined as a solution in which every component $r_{s_j}^- = < T_j^-, I_j^-, F_j^- >$ is characterized by $T_j^- = \{(\min\{T_{ij}\})$ | i-th attribute ∈ α , $(\max\{T_{ij}\})$| i-th attribute ∈β}

- $I_j^- = \{(\max\{I_{ij}\})$ | i-th attribute ∈ α , $(\min\{I_{ij}\})$| i-th attribute ∈ β},
- $F_j^- = \{(\max\{F_{ij}\})$ | i-th attribute ∈ α , $(\min\{F_{ij}\})$| i-th attribute ∈ β},
- In the neutrosophic decision matrix $D_S = < T_{ij}, I_{ij}, F_{ij} >$ for i=1,2,…,n and j=1,2,…,m

K. The ideal interval neutrosophic estimates reliability solution (IINERS) and the ideal interval neutrosophic estimates un-reliability solution (IINEURS) for interval neutrosophic decision matrix

For an interval neutrosophic decision making matrix $D_S = < T_{ij}, I_{ij}, F_{ij} >$ for i=1,2,…,n and j=1,2,…,m . T_{ij}, I_{ij}, F_{ij} are the degrees of membership, degree of indeterminacy and degree of non-membership of the alternative A_i satisfying the attribute C_j. The interval neutrosophic estimate reliability solution (see definition 2.13, and 2.14) can be determined from the concept of SVNS cube [22].

The ideal interval neutrosophic estimates reliability solution (IINERS) is presented as follows

$$Y_S^+ = \langle y_{S1}^+, y_{S2}^+, y_{S3}^+, y_{S4}^+\rangle = \begin{bmatrix}\langle max_i\{U_{i1}\}, min_i\{P_{i1}\}, min_i\{V_{i1}\}\rangle, & \langle max_i\{U_{i2}\}, min_i\{P_{i2}\}, min_i\{V_{i2}\}\rangle, \\ \langle max_i\{U_{i3}\}, min_i\{P_{i3}\}, min_i\{V_{i3}\}\rangle, & \langle max_i\{U_{i4}\}, min_i\{P_{i4}\}, min_i\{V_{i4}\}\rangle,\end{bmatrix}$$

The ideal interval neutrosophic estimates unreliability solution (IINEURS) is presented as follows

$$Y_S^- = \langle y_{S1}^-, y_{S2}^-, y_{S3}^-, y_{S4}^-\rangle = \begin{bmatrix}\langle min_i\{U_{i1}\}, max_i\{P_{i1}\}, max_i\{V_{i1}\}\rangle, & \langle min_i\{U_{i2}\}, max_i\{P_{i2}\}, max_i\{V_{i2}\}\rangle, \\ \langle min_i\{U_{i3}\}, max_i\{P_{i3}\}, max_i\{V_{i3}\}\rangle, & \langle min_i\{U_{i4}\}, max_i\{P_{i4}\}, max_i\{V_{i4}\}\rangle,\end{bmatrix}$$

III. PROPOSED METHOD

- Step 1: Let X^k or $A^{(k)} = (a_{ij}^{(k)})_{m \times n}$ be an interval neutrosophic decision matrix, where $a_{ij}^{(k)} = ([u_{ij}^{-(k)}, u_{ij}^{+(k)}], [p_{ij}^{-(k)}, p_{ij}^{+(k)}], [v_{ij}^{-(k)}, v_{ij}^{+(k)}])$ is an attribute value, given by the decision maker d_k, for the alternative A_I with respect to the attribute C_j.
- Step 2: Using the attribute values given by decision makers, determined the attribute weights by given formula:

$$w_j^* = \frac{\sum_{k=1}^{t}\lambda_k \sum_{i=1}^{m}\sum_{s=1}^{m}((\Delta_{upv})_E)}{\sum_{j=1}^{n}(\sum_{k=1}^{t}\lambda_k \sum_{i=1}^{m}\sum_{s=1}^{m}((\Delta_{upv})_E)}$$

Where, λ_k = weight of DM d_k

$$(\Delta_{upv})_E = \sqrt{((u_1^L - 1)^2 + (u_1^U - 1)^2 + (p_1^L)^2 + (p_1^U)^2 + (v_1^L)^2 + (v_1^U)^2}$$

Here Euclidean distance measured from each alternative A_i with respective to the attribute C_j to Positive Ideal Solution (PIS) assuming {[1 ,1], [0, 0] ,[0 , 0]}

- Step 3: A weighted alternative decision matrix is obtained by calculating $y^i_{kj}=[\ y^{li}_{kj}\ ,\ y^{ui}_{kj}\ ] = [w_j x^{li}_{kj}, w_j x^{lu}_{kj}\ ]$ (k = 1, 2,…, t; j = 1, 2,…, n; i = 1, 2,…, m)for X^i (i = 1, 2, …, m),
- Step 4: Identifythe Ideal Interval Neutrosophic Estimates Reliability Solution (IINERS) according to following equation:

$$Y_S^+ = \langle y_{S1}^+, y_{S2}^+, y_{S3}^+, y_{S4}^+\rangle = \begin{bmatrix} \langle max_i\{U_{i1}\}, min_i\{P_{i1}\}, min_i\{V_{i1}\}\rangle, & \langle max_i\{U_{i2}\}, min_i\{P_{i2}\}, min_i\{V_{i2}\}\rangle, \\ \langle max_i\{U_{i3}\}, min_i\{P_{i3}\}, min_i\{V_{i3}\}\rangle, & \langle max_i\{U_{i4}\}, min_i\{P_{i4}\}, min_i\{V_{i4}\}\rangle, \end{bmatrix}$$

And the Ideal Interval Neutrosophic Estimates Unreliability Solution (IINEURS) is as follows:

$$Y_S^- = \langle y_{S1}^-, y_{S2}^-, y_{S3}^-, y_{S4}^-\rangle = \begin{bmatrix} \langle min_i\{U_{i1}\}, max_i\{P_{i1}\}, max_i\{V_{i1}\}\rangle, & \langle min_i\{U_{i2}\}, max_i\{P_{i2}\}, max_i\{V_{i2}\}\rangle, \\ \langle min_i\{U_{i3}\}, max_i\{P_{i3}\}, max_i\{V_{i3}\}\rangle, & \langle min_i\{U_{i4}\}, max_i\{P_{i4}\}, max_i\{V_{i4}\}\rangle, \end{bmatrix}$$

- Step 5: The bidirectional projection measure between each weighted alternative decision matrix Y^i (i = 1, 2,…, m) and IINERSY^+ IINEURSY^-can be calculated by

$$\text{BProj}\ (Y^i,Y^+) = \frac{\|Y^i\|\|Y^+\|}{\|Y^i\|\|Y^+\|+|\|Y^i\|-\|Y^+\||Y^i.Y^+},$$

Where

$$\|Y^i\| = \sqrt{\sum_{k=1}^t v_k \sum_{j=1}^n \left[\left(y_{kj}^{li}\right)^2 + \left(y_{kj}^{ui}\right)^2\right]}$$

$$\|Y^+\| = \sqrt{\sum_{k=1}^t v_k \sum_{j=1}^n \left[\left(y_{kj}^{l+}\right)^2 + \left(y_{kj}^{u+}\right)^2\right]}$$

$$Y^i.Y^+ = \sum_{k=1}^t v_k \sum_{j=1}^n \left[y_{kj}^{li} y_{kj}^{l+} + y_{kj}^{ui} y_{kj}^{u+}\right]$$

And also, bidirectional projection measures from IINEURS

$$\text{BProj}\ (Y^i,Y^-) = \frac{\|Y^i\|\|Y^-\|}{\|Y^i\|\|Y^-\|+|\|Y^i\|-\|Y^-\||Y^i.Y^-},$$

Where

$$\|Y^i\| = \sqrt{\sum_{k=1}^t v_k \sum_{j=1}^n \left[\left(y_{kj}^{li}\right)^2 + \left(y_{kj}^{ui}\right)^2\right]}$$

$$\|Y^-\| = \sqrt{\sum_{k=1}^t v_k \sum_{j=1}^n \left[\left(y_{kj}^{l-}\right)^2 + \left(y_{kj}^{u-}\right)^2\right]}$$

$$Y^i.Y^- = \sum_{k=1}^t v_k \sum_{j=1}^n \left[y_{kj}^{li} y_{kj}^{l-} + y_{kj}^{ui} y_{kj}^{u-}\right]$$

- Step 6: Determine the Relative Correlation Coefficients based on bidirectional projection measures by following equation:

$$RCC_i = \frac{\text{BProj}\ (Y^i,Y^+)}{\text{BProj}\ (Y^i,Y^+)+\text{BProj}\ (Y^i,Y^-)}$$

- Step 7: Then, the alternatives are ranked in a descending order according to the values of RCC_i. The greater value of RCC_i means the better alternative.

IV. ILLUSTRATIVE EXAMPLE

A. *Step 1*

The decision making problem is adapted from [25]. Suppose that an organization plans to implement ERP system. The first step is to format project team that consists of CIO and two senior representatives from user departments. By collecting all information about ERP vendors and systems, project team chooses four potential ERP systems Ai (i = 1, 2, 3, 4) as candidates. The company employs some external professional organizations (experts) to aid this decision making. The project team selects four attributes to evaluate the alternatives: (1) $C1$: function and technology, (2) $C2$: strategic fitness, (3) $C3$: vendors' ability, and (4) $C4$: vendor's reputation. Suppose that there are three decision makers, denoted by $D1$, $D2$, $D3$, whose corresponding weight vector is λ = (1/3, 1/3, 1/3). The four possible alternatives are to be evaluated under these four attributes and are in the form of IVNNs for each decision maker, as shown in the following:

1) *Interval valued neutrosophic decision matrix:*

D1=

{{[0.4, 0.5] , [0.2, 0.3] , [0.3, 0.5]}	{[0.3, 0.4] , [0.3, 0.6] , [0.2, 0.4]}	{[0.2, 0.5] , [0.2, 0.6] , [0.3, 0.5]}	{[0.5, 0.6] , [0.3, 0.5] , [0.2, 0.5]}}
{{[0.6, 0.7] , [0.1, 0.2] , [0.2, 0.3]}	{[0.1, 0.3] , [0.1, 0.4] , [0.2, 0.5]}	{[0.4, 0.5] , [0.2, 0.5] , [0.3, 0.7]}	{[0.2, 0.4] , [0.1, 0.4] , [0.3, 0.3]}}
{{[0.3, 0.4] , [0.2, 0.3] , [0.3, 0.4]}	{[0.3, 0.6] , [0.2, 0.3] , [0.2, 0.5]}	{[0.2, 0.7] , [0.2, 0.4] , [0.3, 0.6]}	{[0.2, 0.6] , [0.4, 0.7] , [0.2, 0.7]}}
{{[0.2, 0.6] , [0.1, 0.2] , [0.1, 0.2]}	{[0.2, 0.5] , [0.4, 0.5] , [0.1, 0.6]}	{[0.3, 0.5] , [0.1, 0.3] , [0.2, 0.2]}	{[0.4, 0.4] , [0.1, 0.6] , [0.1, 0.5]}}

D2=

{{[0.4, 0.6] , [0.1, 0.3] , [0.2, 0.4]}	{[0.3, 0.5] , [0.1, 0.4] , [0.3, 0.4]}	{[0.4, 0.5] , [0.2, 0.4] , [0.1, 0.3]}	{[0.3, 0.6] , [0.3, 0.6] , [0.3, 0.6]}}
{{[0.3, 0.5] , [0.1, 0.2] , [0.2, 0.3]}	{[0.3, 0.4] , [0.2, 0.2] , [0.1, 0.3]}	{[0.2, 0.7] , [0.3, 0.5] , [0.3, 0.6]}	{[0.2, 0.5] , [0.2, 0.7] , [0.1, 0.2]}}
{{[0.5, 0.6] , [0.2, 0.3] , [0.3, 0.4]}	{[0.1, 0.4] , [0.1, 0.3] , [0.3, 0.5]}	{[0.5, 0.5] , [0.4, 0.6] , [0.3, 0.4]}	{[0.1, 0.2] , [0.1, 0.4] , [0.5, 0.6]}}
{{[0.3, 0.4] , [0.1, 0.2] , [0.1, 0.3]}	{[0.3, 0.3] , [0.1, 0.5] , [0.2, 0.4]}	{[0.2, 0.3] , [0.4, 0.5] , [0.5, 0.6]}	{[0.3, 0.3] , [0.2, 0.3] , [0.1, 0.4]}}

D3=

{{[0.1,0.3] , [0.2,0.3] , [0.4,0.5]}	{[0.3,0.3] , [0.1,0.3] , [0.3,0.4]}	{[0.2,0.6] , [0.3,0.5] , [0.3,0.5]}	{[0.4,0.6] , [0.3,0.4] , [0.2,0.3]}}
{{[0.3,0.6] , [0.3,0.5] , [0.3,0.5]}	{[0.3,0.4] , [0.3,0.4] , [0.3,0.5]}	{[0.3,0.5] , [0.2,0.4] , [0.1,0.5]}	{[0.1,0.2] , [0.3,0.5] , [0.3,0.4]}}
{{[0.4,0.5] , [0.2,0.4] , [0.2,0.4]}	{[0.2,0.3] , [0.1,0.1] , [0.3,0.4]}	{[0.1,0.4] , [0.2,0.6] , [0.3,0.6]}	{[0.4,0.5] , [0.2,0.6] , [0.1,0.3]}}
{{[0.2,0.4] , [0.3,0.4] , [0.1,0.3]}	{[0.1,0.4] , [0.2,0.5] , [0.1,0.5]}	{[0.3,0.6] , [0.2,0.4] , [0.2,0.2]}	{[0.2,0.4] , [0.3,0.3] , [0.2,0.6]}}

W_4=0.2636

B. *Step2*

Attribute weights are obtained from Mat lab using maximizing deviation method with Euclidean distance measures as follows.

W_1= 0.2211, W_2=0.2593, W_3=0.2560, W_4=0.2636

C. *Step 3*

{{[0.0622 , 0.1232] , [0.6740, 0.7663], [0.7690 , 0.8457]}	{[0.0883 0.1209] [0.6338 0.8121] [0.7108 0.7885]}
{{[0.0997 , 0.1809], [0.6779 ,0.7751], [0.7259 , 0.8047]}	{[0.0587 0.1105] [0.6623 0.7558] [0.6623 0.8086]}
{{[0.1040, 0.1396], [0.7006 , 0.7852] , [0.7475, 0.8166]}	{ [0.0507 0.1302] [0.5946 0.6898] [0.7108 0.8216]}
{{[0.0559 , 0.1266] , [0.6779 ,0.7505], [0.6010 , 0.7475]}	{[0.0507 0.1209] [0.6937 0.8355] [0.5946 0.8384]}
{[0.0716 0.1764] [0.6901 0.8403] [0.6931 0.8107]}	{[0.1227 0.2145] [0.7281 0.8360] [0.6826 0.8245]}}
{[0.0834 0.1893] [0.6901 0.8236] [0.6931 0.8806]}	{[0.0445 0.1044] [0.6579 0.8550] [0.6856 0.7312]}}
{[0.0602 0.1710] [0.7173 0.8549] [0.7347 0.8549]}	{[0.0571 0.1227] [0.6896 0.8678] [0.7207 0.8591]}}
{[0.0749 0.1412] [0.6969 0.7937] [0.7446 0.7726]}	{[0.0858 0.1122] [0.6579 0.7952] [0.5895 0.8360]}}

Table 1: Weighted alternative decision matrix

D. Step 4

Ideal Interval Neutrosophic Estimates Reliability Solution (IINERS) for instance first attribute is as follows

{[0.1040 , 0.1809] , [0.6740 , 0.7505], [0.6010 , 0.7475]}

And the Ideal Interval Neutrosophic Estimates Unreliability Solution (IINEURS) for instance first attribute is as follows

{[0.0559 0.1232] , [0.7006 0.7852], [0.7690 0.8457]}

– Step 5: The bidirectional projection measure between each weighted alternative decision matrix Y^i (i = 1, 2,…, m) and IINERS Y^+&IINEURSY^-

Alternative	Y^+	Y^-
A1	3.7642	4.0467
A2	3.8834	4.1494
A3	3.7397	4.0401
A4	4.0162	4.3071

– Step 6: Relative Correlation Coefficients based on bidirectional projection measures for alternatives A_i

A_1=0.4819
A_2=0.4834
A_3=0.4807
A_4=0.4825

– Step 7: Then, the alternatives are ranked in a descending order according to the values of RCC_i. The greater value of RCC_i means the better alternative.

Here, $A_2 > A_4 > A_1 > A_3$

V. Comparative Analysis and Discussion

The results obtain from two examples with partially known and completely unknown weights are compared to Sahin and Liu [24] and Liu and Luo [25] methods.

1) Sahin and Liu [24] developed score and accuracy discrimination functions for MCGDM problem after proposing two aggregation operators. The unknown weights of attributes are determined by constructing linear equation based on maximizing deviation method. The attribute weights are obtained by solving linear equation using Lagrange technique. Then individual decision matrixes are grouped with aid of geometric weighted aggregation operator. For each alternative weighted aggregated neutrosophic values are calculated using obtained attribute weights to aggregated group decision matrix. Therefore the ranking of each alternative is based on score and accuracy functions applied to alternative weighted aggregated neutrosophic values.
2) Liu and Luo [25] proposed weighted distance from positive ideal solution to each alternative based linear equation for determining unknown weights of attributes after observing some drawback for MAGDM under SVNS. The linear function aims to minimize overall weighted distance from PIS where attribute weights are unknown. The partially known or unknown conditions are subjected to proposed linear equation and solved using any linear programming technique results weights of attributes. Then ranking of alternatives given based on weighted hamming distance from PIS. The proposed model also extended to IVNS.
3) Proposed method aimed to enhance results accuracy, flexible to operate and effectiveness. In table 2 example is evaluated with completely unknown weights. The unknown weights of attributes are determined by constructing linear equation based on maximizing deviation method using Euclidean distance measure. The attribute weights are obtained by solving linear equation using Lagrange technique. And ranking the alternatives based on bidirectional projection measures. Then the proposed method given similar results to [24] and [25]. Therefore the proposed method is accurate, flexible and effective.

Type of Problem	Sachin and Liu [24]	Liu and Luo [25]	Proposed Method
Completely Unknown weights	$A2 \succ A4 \succ A1 \succ A3$	$A2 \succ A4 \succ A1 \succ A3$	$A_2 > A_4 > A_1 > A_3$

Table 2: Comparisons of Methods

VI. Conclusion

Therefore, present method for solving the multiple attribute group decision making problems with the interval valued neutrosophic information was developed. Initially, a non-linear equation using Euclidean distance measures based on maximizing deviation method developed. Then the non-linear equations solved by Lagrange function to obtain attribute weights. Further, that rank the alternatives based on bidirectional projection measures from IINERS and IINEURS to select the best alternative. Finally, an illustrative example demonstrated the application of the developed method, and then the effectiveness and rationality of the developed method are demonstrated by the comparative analysis with existing relative methods.

In the future work, we shall extend the bidirectional projection method to other decision data, such as refined neutrosophic numbers and neutrosophic sets, and develop the applications such as pattern recognition and medical diagnosis.

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