Contact Stress Analysis of Mitre Bevel Gear Pairs by Numerical and Analytical Approach

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Abstract— The gears are used for a wide range of industrial applications. Their function is to convert input provided by prime mover into an output with lower speed and corresponding higher torque. Contact stress is generally the deciding factor for the determination of the requisite dimensions of gears. Research on gear action has confirmed fact that beside contact pressure, sliding velocity, viscosity of lubricant as well as other factors such as frictional forces, contact stresses also influence the formation of pits on the tooth surface. So thorough study of contact stress developed between the different matting gears are mostly important for the gear design. Current Analytical methods of calculating gear contact stresses use Hertz's equations, which were originally derived for contact between two cylinders. So for contact stress it is necessary to develop and to determine appropriate models of contact elements, and to calculate contact stresses and tooth bending stress of mitre bevel gear using ABAQUS and compare the results with Hertzian theory.

Key words: Contact stress, Hertz equation, ABAQUS, Tooth bending stress

I. INTRODUCTION

Gearing is one of the most critical components in a mechanical power transmission system and in most industrial rotating machinery.It is possible that gears will predominate as the most effective means of transmitting power in future machines due to their high degree of reliability and compactness. In addition the rapid shift in the industry from heavy industries such as shipbuilding to industries such as automobile manufacture and office automation tools will necessitate a refined application of gear technology.

Designing highly loaded bevel gears for power transmission systems that are both strong and quiet requires analysis methods that can easily be implemented and also provide information on contact and bending stresses, along with transmission errors. Finite element method is capable of providing this information, but time needed to create such a model is large.

Transmission error is usually due to two main factors. The first one is caused by manufacturing inaccuracy and mounting errors. Gear designers often attempt to compensate for transmission error by modifying the gear teeth. Second type of error is caused by elastic deflections under load. Among the types of gearbox noise, one of the most difficult to control is gear noise generated at the tooth mesh frequency.

In this project first the finite element models and solution methods needed for the accurate calculation of three dimensional bevel gear contact stresses and gear bending stresses were determined. Then the contact and bending stresses calculated using hyper view and are compared to the results obtained from existing methods. The main purpose of this project is to develop a model to study and predict the transmission error model including the contact stresses, and the torsional mesh stiffness of gears in mesh using the Hyper mesh software package based on numerical method. Our aim is to reduce the amount of contact stresses and tooth bending stress of the bevel gears, and thereby reduce the amount of noise generated.

II. OBJECTIVES

The main objective of this project work is

1) Several iterations will be carried by changing the profile and shape of the gear.
2) Several iterations will be carried by changing the profile with three different material of the gear.
3) Selection of the suitable variant considering the results and cost.
4) Linear Static analysis for the all model variants to calculate stress, displacement, PEEQ (Equivalent plastic strain) and contact stresses.
5) Theoretical calculation of Gears (contact stress and tooth bending stress) are compared with the numerical results.

III. METHODOLOGY

A. *Geometric Modeling:*

A standard mitre bevel gear is imported from SAE (Society of Automotive Engineers), different views are shown in figure and some of the specifications of the imported bevel gear which imported are show in bellow table.

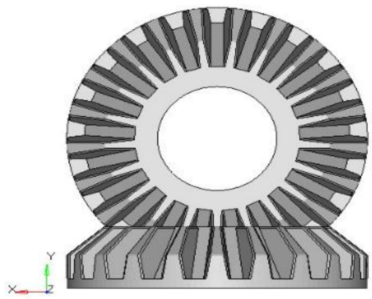

Fig. 1: Front view of bevel gear

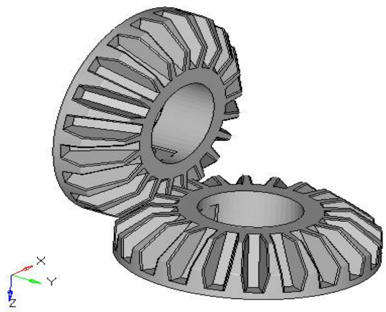

Fig. 2: Isometric view of bevel gear

Sl No	Parameters of the gear	Value
1	Number of teeth (Z)	21
2	Diametral Pitch (DP)	43
3	Circular Pitch (mm)	1.8557
4	Module (mm)	2.0
5	Pressure Angle (α)	20^0
6	Pitch Circle Diameter(mm)	12.405
7	Correction Factor	0.1000
8	Addendum(mm)	0.650
9	Full Depth(mm)	1.329
10	Dedendum(mm)	0.679
11	Outside Diameter(mm)	13.324
12	Pitch Cone Angle	45^0
13	Face width	0.1347

Table 1: Imported Bevel gear specifications

B. *Modified Mitre Bevel Gear*

The gear micro geometry modification for the tooth profile is the tip relief. This modification is very important for proper gear mesh and engagement process, especially when assembly deflection is significant. For the mating pair of teeth under load, it is not possible to have the next tip enter contact in the pure involute position because there would be sudden interference corresponding to the elastic deflection and the corner of the tooth tip would gouge into the mating surface.

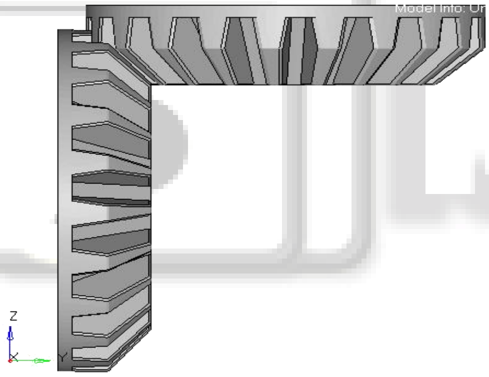

Fig. 3: modified design of bevel gear

C. *Materials Properties*

Materials used for the analysis of the gear pair are and their properties are tabulated in bellow table.

Material Name	Young's modulus (E) N/mm²	Poisson's Ratio(υ)	Density (ρ) ton/mm³
AISI 9310 steel	206842.7	0.30	$8.03E^{-9}$
AISI 1018 carbon steel	205000	0.29	$7.8E^{-9}$
AISI E9310H	200000	0.30	$7.85E^{-9}$

Table 2: Materials and their properties

D. *FE Modelling of Bevel Gear*

Elements used C3D4 is used for 3-D modelling of solid structure. Elements that have nodes only at their corners, such as the 4-node tetra element use linear interpolation in each direction and are often called linear elements or first order elements. The element is defined four nodes having three degrees of freedom at each node such as translation in the nodal X, Y, and Z directions. The elements have plasticity, creep, swelling, stiffening, large deflections, and large strain capabilities.

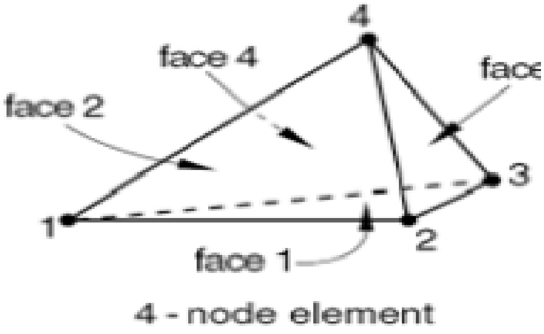

Fig. 4: Nodal and face numbering of the element types used

A three dimensional gear contact model has been modelled using non-linear finite element analysis

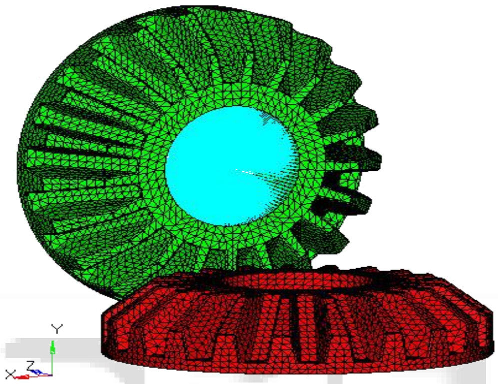

Fig. 5: Meshing of Bevel Gear

E. *Loading and Boundary Conditions*

The proper specification of boundary conditions is just as important for analysis. The improper specification of the boundary conditions leads to incorrect answers. One gear is constrained in all Degree of freedom (red color gear) and another gear (green color gear) is constrained in x, y, z, Rx, Ry and Torque of 70lb-in=43.74 N-m applied at gear center.

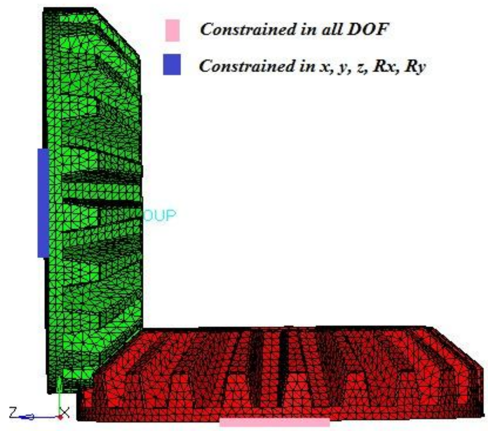

Fig. 6: Loading and Boundary Condition of the Bevel gear

IV. RESULTS AND DISCUSSION

A. *Base Model (With AISI 9310 Steel Material)*

The stress, displacement and PEEQ contours have been obtained for the torque mentioned, following figures shows the Stress, displacement and PEEQ contours for torque of 70lb-in=43.74 N-m applied at gear center.

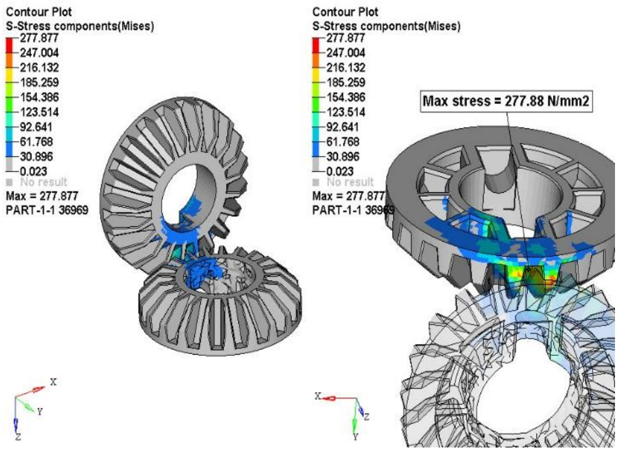

Fig. 7: Diagram of stress distribution before modification

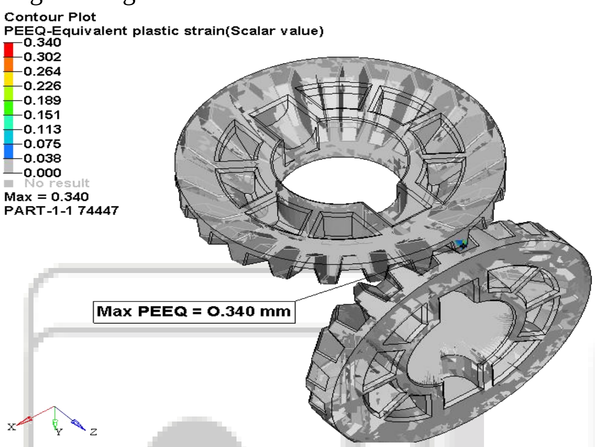

Fig. 8: PEEQ of Bevel gear before modification

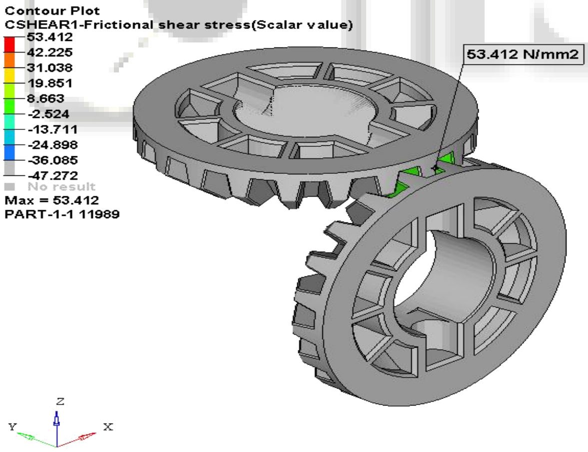

Fig. 9: Contact stress of Bevel gear before modification

And the results of other material changes are tabulated in the bellow table

Material	Stress (N/mm^2)	PEEQ	Contact stress (N/mm^2)
AISI 1018 carbon steel	278.04	0.329	53.842
AISI 9310 steel	277.88	0.34	53.412
AISI E9310H	278.15	0.251	54.859

Table 3: Tabulation of results for Different Materials

Above table shows the results of the bevel gear for base design with change in three different materials i.e., AISI 1018 Carbon steel, AISI 9310 Steel and AISI E9310H.

From above table it is concluded that displacement and stress of the tooth of the bevel gear for base design yields approximately same value. But Equivalent Plastic Strain (PEEQ) for material AISI E9310H is less when compared with other two different materials viz., AISI 1018 Carbon steel, AISI 9310 Steel.

B. Design Change-1

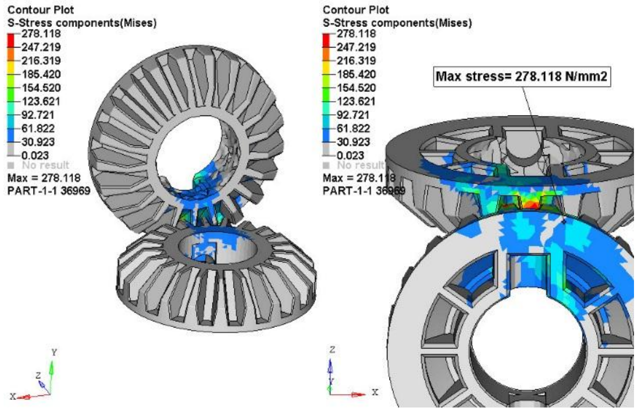

Fig. 10: Stress distribution of Bevel gear for Design-1

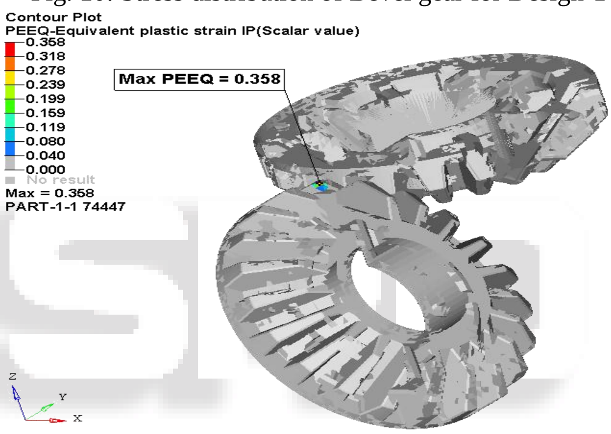

Fig. 11: PEEQ of Bevel gear for Design-1

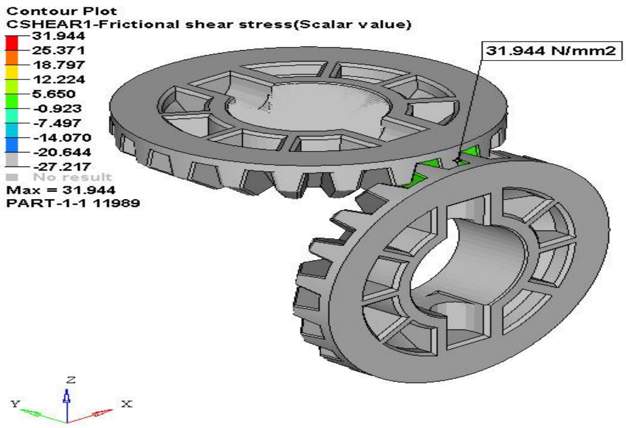

Fig. 12: Contact stress of Bevel gear for Design-1

Design Change	Base Design	Design-1
Stress N/mm^2	277.88	278.19
PEEQ	0.34	0.358
Contact stress N/mm^2	53.412	31.944

Table 4: Tabulation of results of Design changes made for Material AISI9310 STEEL

Above table shows the results of the bevel gear for design changes made for material AISI 9310 Steel. From above table it is concluded that displacement and Equivalent Plastic Strain (PEEQ) of the tooth of the bevel gear for base design and Design change_1 yields approximately same value. But Stress for both designs are different for same material AISI 9310 Steel.

V. GRAPHICAL REPRESENTATIONS OF THE RESULTS

A. *Tooth Bending Stress Variations*

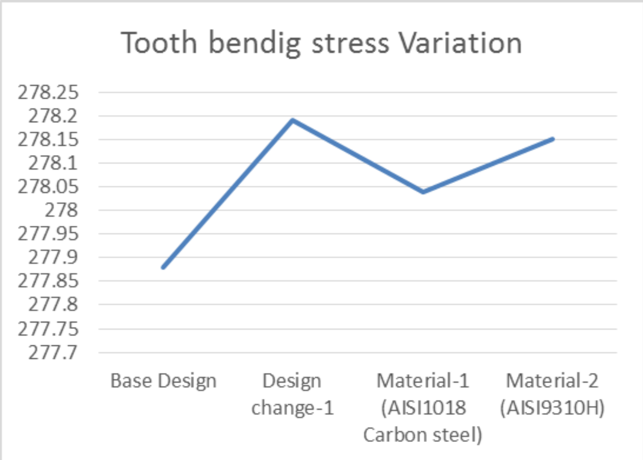

Fig. 13: Tooth bending stress variation with design and material change

B. *PEEQ Variation*

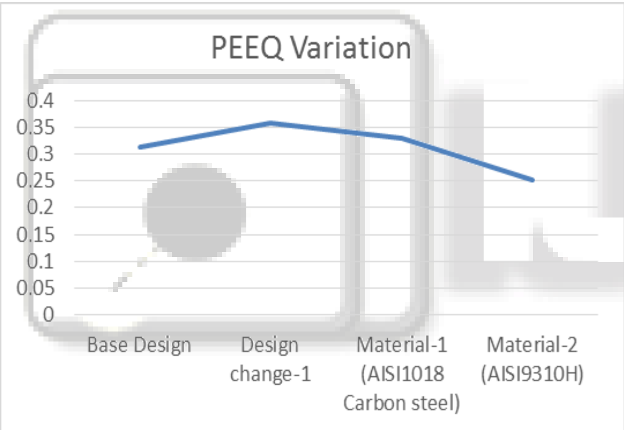

Fig. 14: PEEQ variation with design and material change

C. *Graphical Representations of PEEQ Vs. Tooth Bending Stress*

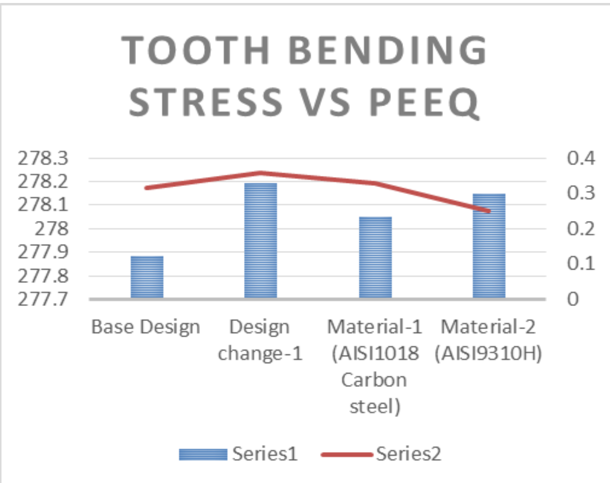

Fig. 15: Tooth bending stress vs PEEQ

D. *Contact Stress Variations*

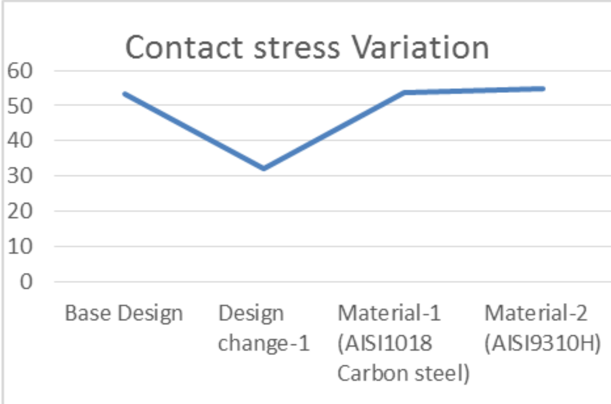

Fig. 16: Contact stress variations with material and Design change

VI. THEORETICAL CALCULATIONS

A. *Theoretical Calculations for Contact Stress*

Surface fatigue failure due to many repetitive contact stresses occurring in the gear tooth surface at the time of power transmission. If pair of teeth of gears is in contact subjected to cyclic type of loading the contact stresses are induce on the gear tooth surface are higher than fatigue strength of the gear the tooth get brake. The method of calculating gear contact stress by Hertz's equation originally derived for contact between two cylinders by using Hertz's contact stress theory.

The Hertz equations can be used to calculate the contact stresses induced in tooth surfaces of two mating bevel gears. The contact stresses of such gears approximately can be taken to be equivalent to the contact stresses of cylinders having the same radii of curvature at the contact point as the load transmitting gears. The Hertzian contact stress equation is given by

$$P_p=\left\{\sqrt{\frac{Ft}{b*d}*\frac{u+1}{u}}\right\}*Y_m*Y_p$$

Where P_p= Contact stress
F_t=tangential force
b=Width of teeth
d= Diameter of the gear
u= Gear Ratio = $d_2 \div d_1$ (d1 and d2 are pitch circle diameters of the gears)
Y_m= Material coefficient
Y_p= Pitch point coefficient

For bevel gear $Ft=\frac{1000*T}{d}$

Where T= Torque d= Diameter of the gear

Material coefficient $Y_m=\sqrt{\frac{0.35*2E1E2}{E1+E2}}$

Where E_1= Young's modulus of gear
E_2= Young's modulus of pinion

Pitch point coefficient $Y_p=\sqrt{\frac{1}{(cos\alpha)*(cos\alpha)*(tan\alpha)}}$

Where α= Pressure angle

Input parameters for calculations	Value	Symbol
Module	2.0	m
Gear Ratio	1	u
Number of teeth on pinion	21	z

Pressure Angle	20^0	α
Material of pinion	AISI 9310 Steel	
Material of gear	AISI 9310 Steel	

Table 5: Input parameters for theoretical calculations

Material coefficient
Y_m=269.063
Pitch point coefficient
Y_p =1.76
Torque (T) = 70lb-in = 43.74N-m
Width of teeth (b) = 2.4mm
Diameter of the gear (d) = 43mm Gear ratio (u) =1
Pressure angle (α) =20^0

$$Ft = \frac{1000 * T}{d}$$

F_t= 1017209.302 N

$$P_p = \left\{\sqrt{\frac{Ft}{b*d} * \frac{u+1}{u}}\right\} * Y_m * Y_p$$

After substituting all the values in the above equation
P_p = 66.4804 MPa

B. Tooth Bending Stress Calculations for Bevel Gear

The standard method for determining induced bending stresses in bevel gears comes from the American Gear Manufacturers Association and is based on the equation below.

$$\sigma = \frac{Wt * Ka}{Kv} * \frac{p}{F} * \frac{Ks * Km}{J}$$

Where
σ =Max bending stress in tooth (Compare to gear material strength to determine if the gear will break).
W_t = Transmitted tangential load. Can be calculated as
W_t = Torque$*\frac{Pitch\ diameter}{2}$
K_a= Application factor (Accounts for probability of greater than design load occurrences. This is not something we expect so neglect this and set K_a= 1).
K_v = Dynamic factor (Accounts for dynamic effects and velocity of tooth contact. For static loading, which is an assumption we will make since we are dealing with relatively low speeds, neglect this term and set K_v= 1).
P = Diametrical pitch. $P = \frac{N}{D}$
F = Face width of the teeth.
K_s = Size factor (Accounts for unusually sized gears. Not applicable for normal gears. Set K_s = 1).
K_m = Load distribution factor (Accounts for shaft misalignment and shaft bending. Build your gearbox carefully, so you can neglect this term and set K_m = 1).
J = Geometry factor (Similar to Lewi's equation used in spur gear. Obtained from chart based on number of teeth on gear and pinion).
After setting all the above parameters simplified AGMA equation becomes
$\sigma = W_t * \frac{P}{F} * \frac{1}{J}$
Now input parameters for the calculations are
Face width = 0.1347mm
Number of teeth = 21
Pitch circle diameter =43mm
Diametral pitch = P = $\frac{21}{43}$=0.4883
From design data handbook equation 23.115
Lewis form factor for 20^0involute tooth profile $y = 0.154 - \frac{0.912}{z\vartheta}$

Where $z\vartheta = \frac{Z2}{cos\delta}$

Where pitch cone angle δ= tan^{-1}(i) and

$$i = \frac{Z2}{Z1} = \frac{21}{21} = 1$$

Pitch cone angle δ= tan^{-1}(1)

$\delta = 45^0$

$$z\vartheta = \frac{Z2}{cos\delta} = 29.698$$

$z\vartheta = 29.698$
y = 0.1232

Tangential load $W_t = T * \frac{PD}{2}$
W_t= 940.41 N

Now maximum tooth bending stress
$\sigma = W_t * \frac{P}{F} * \frac{1}{J}$
After substituting all the values in the above equation
σ = 278.30 N/mm^2
Maximum tooth bending stress σ = 278.30 N/mm^2.

C. Comparison of Theoretical and FEM Results

Sl No	Stress	Theoretical value	FEM Value	% Variation
1	Tooth bending stress	278.30 N/mm^2	278.04 N/mm^2	0.1%
2	Contact stress	65.48 N/mm^2	55.859 N/mm^2	15%

Table 6: Comparison of Theoretical and FEA values

VII. CONCLUSION AND FUTURE SCOPE

A. Conclusions

– With the materials considered for the analysis, AISI E9310H material yields less Equivalent Plastic Strain (PEEQ) when compared with the other two materials considered.
– For the design changes made for the material AISI 9310 Steel, base design yields less stress when compared with the Design change-1.
– Contact stress of the standard imported bevel gear can be reduced by slightly changing the standard teeth dimension.
– The calculation of maximum stresses in a bevel gear at tooth root is three dimensional problems. The accurate evaluation of stress state and distribution of stress is complex task. The stresses produced at any discontinuity are different in magnitude from those calculated by elementary formulae.
– There is fairly good agreement between experimental and finite element results. The error in maximum contact stress is found to be 15 %.

B. Future Scope

Fatigue life estimation of the bevel gear can be carried out using FEM. Thermo-couple analysis of the bevel gear can be carried out with suitable data for different materials.

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