Level Control of Coupled Tanks System using Feedback Linearization Control Theory

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Abstract—This paper manages the level control of coupled tanks framework utilizing criticism linearization control. An input linearization control method is proposed for the framework. The proposed control methods ensure the asymptotic dependability of the shut circle framework. To show the created control procedures the framework's execution is checked by MATLAB programming. The reproduction results demonstrate that the proposed control systems function admirably.

Key words: Feedback Linearization Control, Level Control, Coupled Tanks System

I. INTRODUCTION

Amid the most recent three decades, variable structure frameworks and sliding mode control have gotten critical intrigue and have turned out to be settled examination zones with extraordinary potential for distinctive applications. Variable structure frameworks with a sliding mode control system were talked about in the Soviet writing [1], [2], and have been broadly created as of late. Extensive studies of variable structure control given in [3], [4]. The striking element of sliding mode control gets from the property of heartiness to organized and unstructured vulnerabilities once the framework enters the sliding mode. Note that the framework power is not ensured until the sliding mode is come to. The principle disadvantage of sliding mode control is "gabbing" which can energize undesirable high recurrence elements. Diverse routines for prattling lessening have been accounted for. One methodology which puts a limit layer around the exchanging surface such that the hand-off control is supplanted by an immersion capacity [6].

Sliding mode control procedure is a sort of variable structure control where the motion of a nonlinear framework is changed by exchanging irregularly on time on a foreordained sliding surface with a rapid, nonlinear criticism [7]. Really, the methodology of sliding mode controller outline has two stages: the first step is to acquire a sliding surface for wanted stable elements and the second step is about getting the control law that gives to achieve this sliding surface. The framework directions are touchy to parameter varieties and unsettling influences amid the coming to mode while they are inhumane in the sliding mode [8].

The broken way of the control activity in sliding mode control is asserted to result in exceptional heartiness highlights for both framework adjustment and yield following issues. A decent execution additionally incorporates heartlessness to parameter varieties and dismissal of unsettling influences. Variable structure frameworks has been connected in numerous control fields which incorporate robot control [9], engine control [10, 11], flight control [12], and procedure control [13].

In this paper, we propose an input linearization control for the coupled tanks framework. Reenactment results are exhibited to show the adequacy of the proposed control systems.

The paper is composed as takes after: In segment II the dynamic model of the coupled tanks framework is gotten. In area III an input linearization control procedure is proposed for the framework. The consequences of reproduction are displayed in area IV. At long last finishing up comments and references are given in segment V and VI.

II. DYNAMIC MODEL OF COUPLED TANKS SYSTEM

The mechanical assembly, appeared in Fig. 1, comprises of two indistinguishable tanks coupled by an opening [15]. The information is supplied by a variable pace pump which supplies water to the first tank. The hole permits the water to stream into the second tank and thus out to a store. The objective of the control problem is to adjust the inlet flow rate $q(t)$ so as to maintain the level in the second tank, $h_2(t)$ close to a desired set point level, h_{2d}.

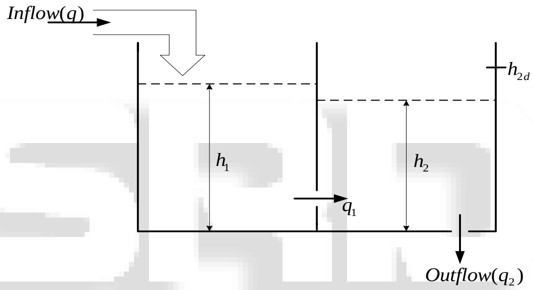

Fig. 1. Model of coupled tanks system

The dynamic model of the coupled tanks system can be written as-

$$\left.\begin{aligned} C\frac{dh_1}{dt} &= q - q_1 \\ C\frac{dh_2}{dt} &= q_1 - q_2 \end{aligned}\right\}$$

where

$$\left.\begin{aligned} q_1 &= c_{12}\sqrt{2g(h_1 - h_2)} \quad \text{for } h_1 > h_2 \\ q_2 &= c_2\sqrt{2gh_2} \quad \text{for } h_2 > 0 \end{aligned}\right\}$$

and

$h_1(t)$: the level in the first tank;
$h_2(t)$: the level in the second tank;
$q(t)$: the inlet flow rate;
$q_1(t)$: the flow rate from tank 1 to tank 2;
$q_2(t)$: the flow rate out of tank 2;
g: the gravitational constant;
C: the cross-section area of tank 1 and tank 2;
c_{12}: the area of coupling orifice;
c_2: the area of the outlet orifice;

For the coupled tanks system, the fluid flow rate, q, into tank 1, cannot be negative because the pump can only pump water into the tank. Therefore, the constraint on the inflow rate is given by

$$q \geq 0 \qquad (3)$$

Now the governing dynamical equations of the coupled tanks system can be written as (Chang -1990)

$$\left.\begin{aligned} \dot{h}_1 &= -\frac{c_{12}}{c}\sqrt{2g|h_1-h_2|}\,sgn(h_1-h_2)+\frac{q}{c} \\ \dot{h}_2 &= \frac{c_{12}}{c}\sqrt{2g|h_1-h_2|}\,sgn(h_1-h_2)-\frac{c_2}{c}\sqrt{2gh_2} \end{aligned}\right\} \quad (4)$$

At equilibrium, for constant water level set point, the derivatives must be zero, i.e.,

$$\dot{h}_1 = \dot{h}_2 = 0 \quad (5)$$

Thus, at equilibrium, the following algebraic equations must hold:

$$\left.\begin{aligned} -\frac{c_{12}}{c}\sqrt{2g|h_1-h_2|}\,sgn(h_1-h_2)+\frac{Q}{c} &= 0 \\ \frac{c_{12}}{c}\sqrt{2g|h_1-h_2|}\,sgn(h_1-h_2)-\frac{c_2}{c}\sqrt{2gh_2} &= 0 \end{aligned}\right\} \quad (6)$$

Where Q is the equilibrium inflow rate. From eqs. (6), and to satisfy the constraint in eq. (3) on the input flow rate, we should have $sgn(h_1 - h_2) \geq 0$, which implies

$$h_1 \geq h_2 \quad (7)$$

Therefore, in order to satisfy the constraint in eq. (3) on the input inflow rate for given values of plant parameter c_{12} and c_2, the liquid levels in the tanks must satisfy the constraint in eq. (7). In addition, for the case when $h_1 = h_2$, the system model is decoupled. Thus, the general model suggested in [5] reduces to the model used in this paper which is the most widely used model in the literature when modeling the coupled tanks system.

Let

$$z_1 = h_2 > 0,\ z_2 = h_1 - h_2 > 0,\ Z = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix},\ u = q(t)$$

and

$$a_1 = \frac{c_2\sqrt{2g}}{c},\quad a_2 = \frac{c_{12}\sqrt{2g}}{c}$$

The output of the coupled tanks system is taken to be the level of the second tank. Therefore, the dynamic model in eqs. (1) and (2) can be written as

$$\left.\begin{aligned} \dot{z}_1 &= -a_1\sqrt{z_1}+a_2\sqrt{z_2} \\ \dot{z}_2 &= a_1\sqrt{z_1}-2a_2\sqrt{z_2}+\frac{u}{c} \\ y &= z_1 \end{aligned}\right\} \quad (8)$$

The objective of the control scheme is to regulate the output $y(t) = z_1(t) = h_2(t)$ to a desired value h_{2d}. It is easy to show using eqs. (8) that if $y(t) = z_1(t)$ is regulated to a desired value h_{2d}, then $z_2(t) = h_1(t) - h_2(t)$ will be regulated to the value $\frac{a_1^2}{a_2^2}h_{2d}$.

The dynamic model of the coupled tanks system is highly nonlinear. Therefore, we will define a transformation so that the dynamic model given in eq. (8) can be transformed into a form facilitates the control design.

Let $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$, and define the transformation $x = T(z)$ such that

$$\left.\begin{aligned} x_1 &= z_1 \\ x_2 &= -a_1\sqrt{z_1}+a_2\sqrt{z_2} \end{aligned}\right\} \quad (9)$$

the inverse transformation $z = T^{-1}(x)$ is such

$$\left.\begin{aligned} z_1 &= x_1 \\ z_2 &= \left(\frac{a_1\sqrt{x_1}+x_2}{a_2}\right)^2 \end{aligned}\right\} \quad (10)$$

It can be checked that we can write the dynamic model in eq. (8) as

$$\left.\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= \frac{a_1a_2}{2}\left(\frac{\sqrt{z_1}}{\sqrt{z_2}}-\frac{\sqrt{z_2}}{\sqrt{z_1}}\right)+\frac{a_1^2}{2}-a_2^2+\frac{a_2}{2c}\frac{1}{\sqrt{z_2}}u \end{aligned}\right\} \quad (11)$$

Where the values of z_1 and z_2 in eq. (11) are function of x_1 and x_2 as given by eq. (10).

Hence, the dynamic model of the system can be written as

$$\left.\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= f + \emptyset u \\ y &= x_1 \end{aligned}\right\} \quad (12)$$

Where

$$f = \frac{a_1a_2}{2}\left(\frac{\sqrt{z_1}}{\sqrt{z_2}}-\frac{\sqrt{z_2}}{\sqrt{z_1}}\right)+\frac{a_1^2}{2}-a_2^2$$

$$\emptyset = \frac{a_2}{2C}\frac{1}{\sqrt{z_2}}$$

The dynamic model in eq. (12) will be used to design control techniques for the coupled tanks system.

III. FEEDBACK LINEARIZATION CONTROL

In this section, we will design a feedback linearization controller for the coupled tanks system. Let h_{2d} be the desired output level of the system. i.e.

$$y_d = h_{2d}$$

To generate a direct relationship between the output y and the input u, let us on differentiate the output equation w.r.t. time, we get

$$\dot{y} = \dot{x}_1 = x_2$$

Since $\dot{y}$ is still not directly related to the input u, let us differentiate again. We obtain

$$\ddot{y} = \dot{x}_2$$

$$\ddot{y} = f + \emptyset u, \quad (13)$$

Which represent an explicit relationship between y and u.

$$\text{if } u = \frac{1}{\emptyset}(-f + v) \quad (14)$$

Where v is the new input to be determined, the nonlinearity in the eq. (13) is canceled, and we obtain a simple linear double- integrator relationship between the output and the new input v,

$$\ddot{y} = v \quad (15)$$

The design of controller v is given by

$v = \ddot{y}_d - k_1e - k_2\dot{e}$, where $e = y - h_{2d}$=tracking error

$v = \ddot{y}_d - k_1(x_1 - y_d) - k_2(\dot{x}_1 - \dot{y}_d)$, since $\dot{y}_d = \ddot{y}_d = 0$

$v = -k_1(x_1 - h_{2d}) - k_2(x_2)$

therefore feedback controller given by

$$u = \frac{1}{\emptyset}[-f - k_1(x_1 - h_{2d}) - k_2(x_2)]$$

$$u = \frac{2C\sqrt{z_2}}{a_2}\left[-\frac{a_1a_2}{2}\left(\frac{\sqrt{z_1}}{\sqrt{z_2}}-\frac{\sqrt{z_2}}{\sqrt{z_1}}\right)-\frac{a_1^2}{2}+a_2^2-k_1(x_1-h_{2d})-k_2(x_2)\right] \quad (16)$$

Where k_1 and k_2 are positive constants, which asymptotically stabilizes the output of the system $y(t) = h_2(t)$ to its desired value h_{2d}.

IV. SIMULATION RESULTS

Simulation of presented control techniques has been done using MATLAB software. Results are shown in Figures given below.

The dynamic model of the system has taken from [15], in which area of the orifices $c_{12} = 0.58\,cm^2$ and $c_2 = 0.24\,cm^2$ are given. The cross-section area of tank 1 and tank 2 are found to be $208.2\,cm^2$. The gravitational constant is $981\,cm/s^2$. The desired value of the output of the system is taken to be $h_{2d} = 5\,cm$.

A. With Feedback Linearization Controller:

The controller parameters used in the simulations are taken to be $k_1 = 0.005$ and $k_2 = 0.15$. Figs. 2 and 3 shows the simulation results when the controller is used without input saturation. Fig. 2 shows that the output $y(t) = h_2(t)$ converges to its desired value h_{2d} in about 107 s. Fig. 3 shows the control effort in which is high. Figs. 4 and 5 shows the simulation results when the controller is used with input saturation. Fig. 2 shows that the output $y(t) = h_2(t)$ converges to its desired value h_{2d} in about 118 s. Fig. 3 shows the control effort in which chattering is evident.

The input constraint for the system is in the range: $-50cm^3/s \leq u \leq 50cm^3/s$.

B. Without Input Saturation

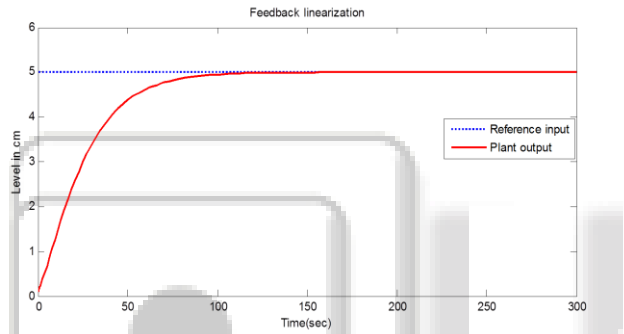

Fig. 2. Liquid level in tank 2 by using feedback linearization

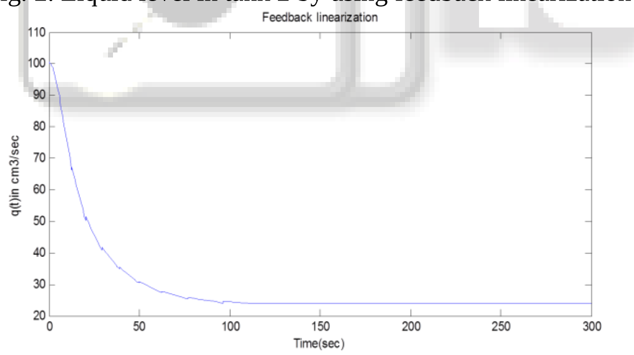

Fig.3. Liquid flow rate in Tank 1 by using feedback linearization

C. With Input Saturation

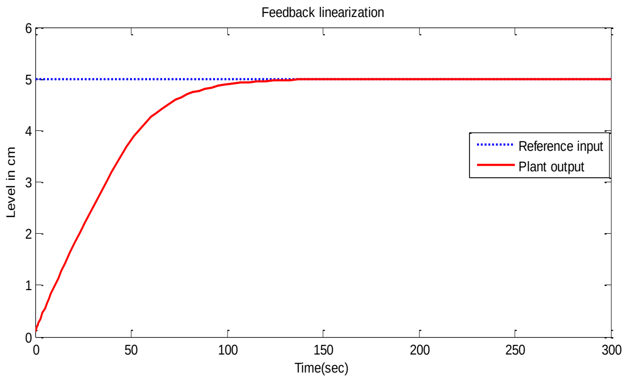

Fig. 4: Liquid level in tank 2 by using feedback linearization

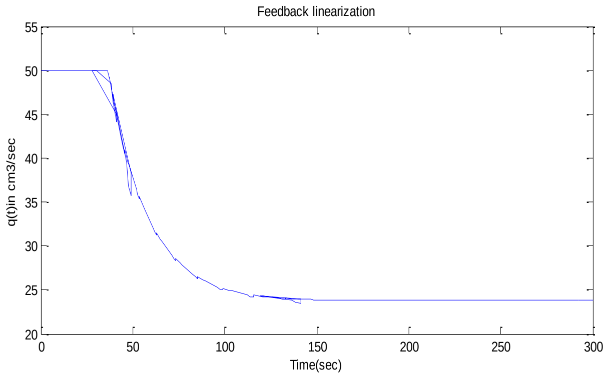

Fig. 5: Liquid flow rate in Tank 1 by using feedback linearization

V. CONCLUSION

In this paper, the feedback linearization control technique is proposed for level control of coupled tanks system. The simulation result shows that the proposed control techniques work very well. The simulation result indicates that the controller with feedback linearization gives better result.Fig.4 and 5 shows that chattering as well as control effort is greatly reduced. With this controller, the desired level is reached and maintained within the finite time.

REFERENCES

[1] S. V. Emelyanov, Variable Structure Control Systems Moscow, U.S.S.R.: Nauka, 1967.
[2] V. I. Utkin, Sliding Modes and Their Application in Variable Structure Systems (in Russian). Moscow, U.S.S.R.: Nauka.
[3] R. A. DeCarlo, S. H. Zak, and G. P. Matthews, “Variable structure control of nonlinear multivariable systems: A tutorial,” Proc. IEEE, vol. 76, pp. 212–232, Mar. 1988.
[4] J. Y. Hung, W. B. Gao, and J. C. Hung, “Variable structure control: A survey,” IEEE Trans. Ind. Electron., vol. 40, pp. 2–22, Feb. 1993.
[5] Khan MK, Spurgeon SK, “Robust MIMO water level control in interconnected twin-tanks using second order sliding mode control,” Control Eng Pract 2006;14:375–86.
[6] J. J. Slotine and S. S. Sastry, “Tracking control of nonlinear systems using sliding surfaces, with application to robot manipulators,” Int. J. Control, vol. 38, pp. 465–492, 1983.
[7] K.D. Young, V.I. Utkin, and U. Ozguner, “A Control Engineer’s Guide to Sliding Mode Control,” IEEE Transactions on Control Systems Technology, vol. 7, no. 3, May 1999, pp.328-342.
[8] J. Y. Hung, W. Gao, and J. C. Hung, “Variable structure control: A survey,” IEEE Transactions on Industrial Electronics, vol. 40, no.1, 1993, pp. 2-22.
[9] Sira-Ramirez H, Ahmad S, Zribi M, “Dynamical feedback control of robotic manipulators with joint flexibility,” IEEE Trans Syst Man Cybernet 1992;22:736–47.
[10] Zribi M, Sira-Ramirez H, Ngai A, “Static and dynamic sliding mode control schemes for a permanent magnet stepper motor,” Int J Control 2001;74:103–17.

[11] Alrifai MT, Zribi M, Sira-Ramirez H, "Static and dynamic sliding mode control of variable reluctance motors," Int J Control 2004;77:1171–88.
[12] Sira-Ramirez H, Zribi M, Ahmad S, "Dynamical sliding mode control approach for vertical flight regulation in helicopters," IEE Proc—Control Theory Appl 1994;141:19–24.
[13] Kantor JC, "Nonlinear sliding-mode controller and objective function for surge tanks," Int J Control 1999;50:2025–47.
[14] Slotin, J.J. and Li, W., Applied Nonlinear Control, printice-Hall of India pvt. limited, New Delhi, 1991.
[15] N.B.Almutairi. Mohamed Zribiproposed, "Sliding mode control of coupled tanks" Mechatronics Volume 16, Issue 7, September 2006, pages 427-441.